# Control Systems

# CONTROL SYSTEMS

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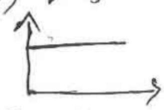
→ In control systems we don't study about the design of the system. Just we are discuss about the performance of a system.

→ For finding the performance of a control system we are using two mathematical analyses. First one is Transfer fun", and state space analysis.

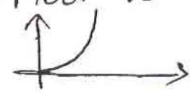
→ For any system we are applying some signal (or) disturbance as an input and then we can observe the performance & response of the system.

→ When a switch is ON (or) OFF, it will take minimum time i.e zero time. So this will indicates "step signal".



→ When we control the regulator of a fan, it will shows Ramp signal.

→ Controlling (or) moment of lift from ground floor to top floor & viceversa will shows parabollic signal.



→ When a person is touches, then we react & give a powerful punch to him. If he recover after sometime then he is stable otherwise unstable. This signal is called "Impulse".

Stability is not only Bounded input Bounded output concept. It can also defined as when a system is subiected to sudden shocks, if the system is recover to its original state, then the system is said to be "stable", otherwise system is unstable.

## Part-1 :- Introduction to CONTROL SYSTEM :-

1. Consider a liquid level control system whose control objective is to maintain water level in the tank at a prescribed height.

2. Controller is an automatic device with error signal E(S) as input and controller o/p P(S) effecting the dynamics of the plant to achieve control objective.

   Therefore controller o/p

   $$\boxed{P = f(e)} \quad \text{where } e = \text{Error}$$



3. The different modes of controller operation are proportional, proportional + integral and proportional + integral + derivative.

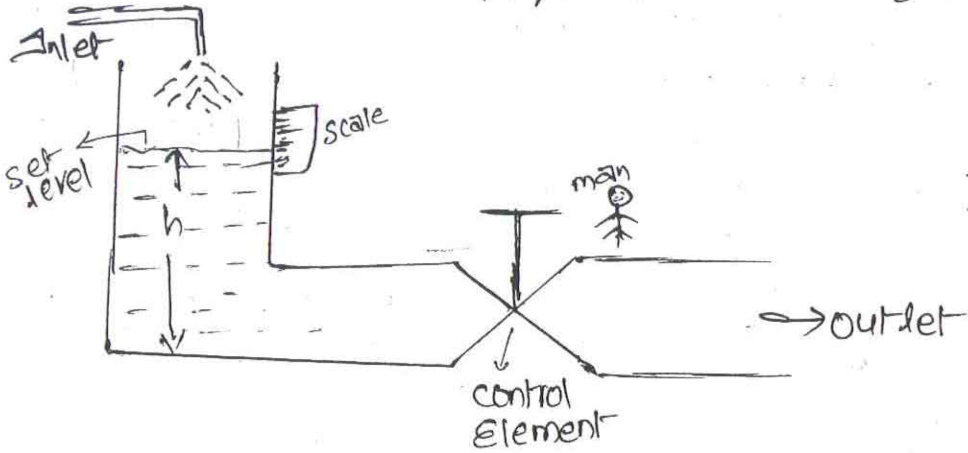
4. There are two Basic control loop configurations.

(a) Closed loop (or) Feedback control system:-

In this configuration the changes in the output are measured through feedback and compared with the input (or) a set point to achieve the control objective.

"Feedback implies measurement [sensors (or) Transeducers]".



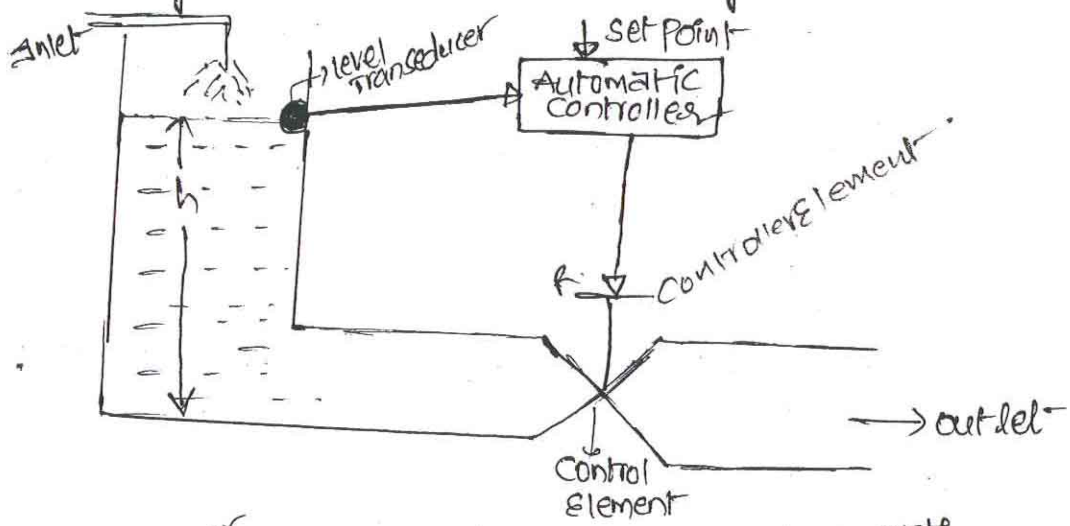



It is a manual control system. Since by observing the scale if the water level is deviated from set point then man will operate the control elem and make it to set level.

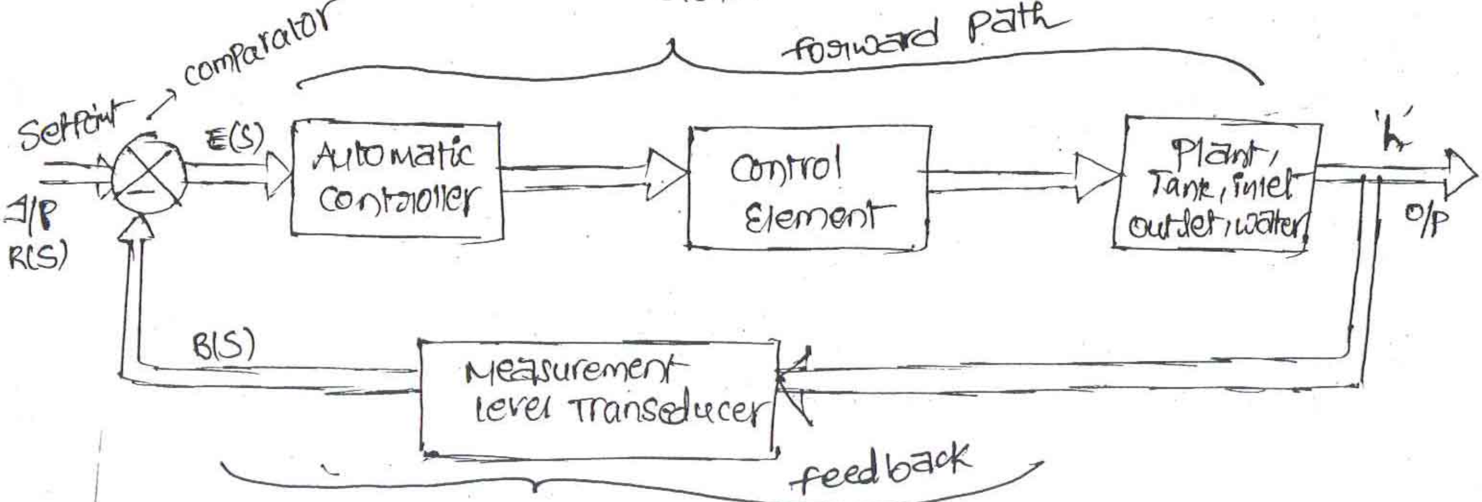
Manual Control System



For making it automatic control system we have to use "Automatic Controller". Basically automatic controller will operates only for electrical signal. So for converting any phy Non-Electrical quantity into electrical quantity we have to use "Transeducer."





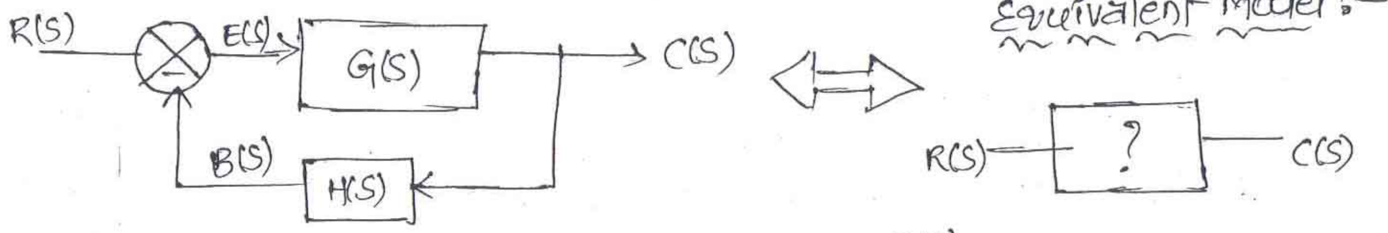



Note:- Basic thing in the entire control scheme is measurement. We measure the water level and then which is given back to the system as feedback.

"so feedback is nothing but measurement."



## CONTROL CANNONICAL FORM :-




$$E(S) = R(S) - B(S)$$

$$\therefore C(S) = E(S)\,G(S)$$

$$\frac{C(S)}{G(S)} = R(S) - B(S)$$

$$\frac{C(S)}{G(S)} = R(S) - C(S)\,H(S)$$

$$\frac{C(S)}{G(S)} = \frac{R(S) - C(S)\,H(S)}{1}$$

$$\Rightarrow C(S) = R(S)G(S) - C(S)G(S)\,H(S)$$

$$\Rightarrow C(S)\left[1 + G(S)H(S)\right] = R(S)\,G(S)$$

$$\Rightarrow \boxed{\frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S)H(S)}}$$

$\therefore$ Equivalent model:-


## Openloop control System :-

(Basically open loop c.s is a conditional c.s)

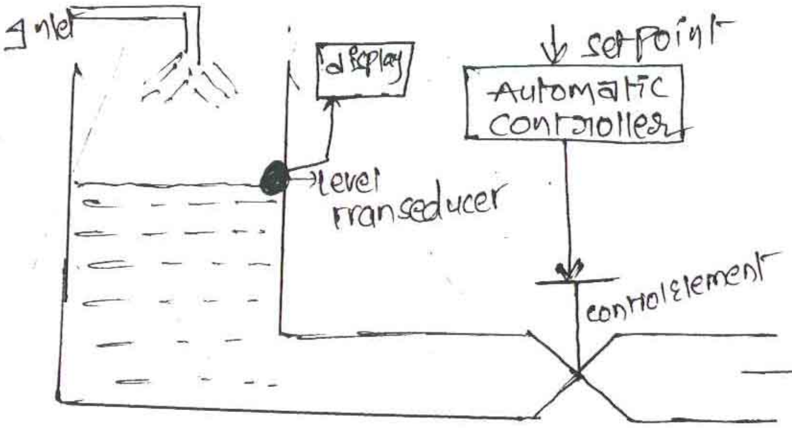
* It is a conditional control system formulated under the condition that the system is "not subjected to any type of disturbance".
* In this configuration, the feedback (or) measurement is not connected to the forward path (or) controller [open loop].
* Feed back in an openloop system except for displaying the information about the o/p as no major significance. This insignificantness of feed back is termed as removal of Feed back.
* openloop systems are more stable than closed loop systems. [without disturbance]. because the effect of feed back in closed loop systems is that it introduces delays or lags does loocking over all speed of closedloop system - "slow" (or) "sluggish".

Performance Analysis is not applicable for openloop systems because they are highly stable systems.



NOTE:-

* Always the F.B is Negative, that gives whether the water level is high (or) low to the set value.
* Every Analysis is meant only for closed loop control system since openloop control systems are more stable. we will analyse the system under disturbance condition only like our live example we will not go for doctor under healthy conditions.
* All we are conditional stable systems. How? it means that if the class is up to 12:30 then we are intrested to sit & listen. If the class is extended to 4:30 then we are sit, not listen. so that we are conditional stable. Similarly openloop control systems are conditional stable system under no distum.




→ In openloop control system we dont Eliminate the feedback i.e Just remove the connection b/w Automatic Controllers & level Transeducer.

→ In OLCS also FB exist, but no connection to comparator.

→ Just display the water level. not given signal to Automatic Controller.

Openloop control system

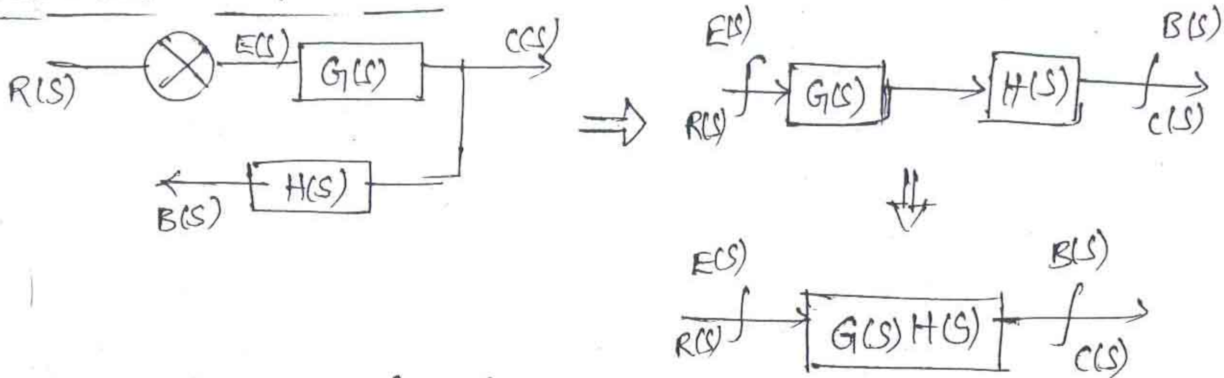
→ (Here the inlet flowrate is Constant, then there is no disturbance) ⇒ Display = setpoint.

Ex:- We all are closed loop control system, a blind man is an openloop control system. let us consider a blindman and a normal Person are walking on a road with no disturbance. so both are having same speed. Now when a car is parking besides the road. so Normal man stops and see his face. But blind-man will moves. Hence openloop control systems are Faster than closedloop control system.

In the closed loop Automatic control system some lag may be occurs i.e transportation lag, controller lag etc. } which will cause for the CLCS is slow.

When we use ~~this~~ controller(s) to the traffic signal due to delay in the system the entire traffic will be face ~~diff~~ difficulty (just analyse).

## Representation of OLCS :-

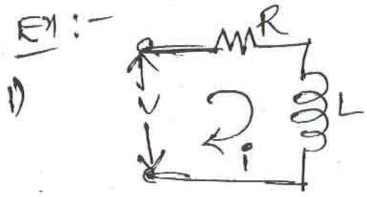



## Concept of Transfer function :-

It is a mathematical model representing control system relating i/p & o/p in the form of ratio i.e $\frac{o/p}{i/p}$ :

Ex :-

1)



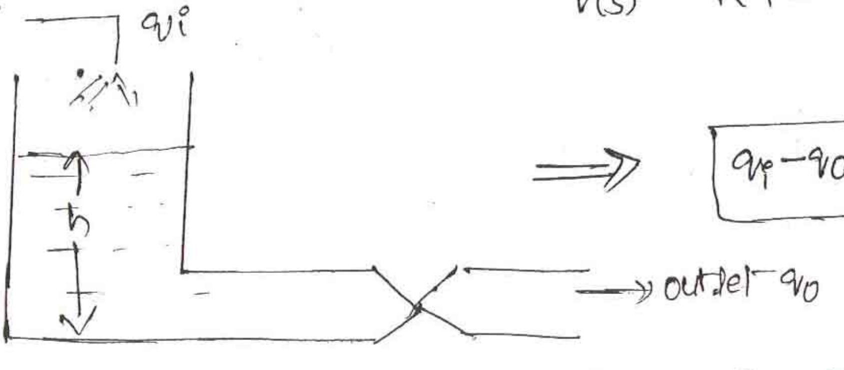

$$V = Ri + L\frac{di}{dt}$$

Apply laplace transform

$$V(S) = RI(S) + SL\,I(S) - \underbrace{\cancel{LI(0)}}$$ (this was not known i.e without initial condn)

$$\Rightarrow \frac{I(S)}{V(S)} = \frac{1}{R+LS} = \frac{1/L}{S + R/L} \ldots$$

2)




$$\Rightarrow \boxed{q_i - q_0 \propto \frac{dh}{dt}}$$

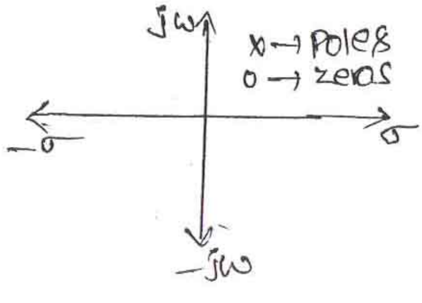
water flowrate

note:- for any system the first & foremost eqn is differential terms as shown in Ex!(1). Similarly write $q_i - q_0 \propto \frac{dh}{dt}$.

## Standard Representation :-

$$T.F = F(S) = \frac{C(S)}{R(S)} = \frac{K(S+z_1)(S+z_2)}{(S+P_1)(S+P_2)} \rightarrow (1)$$

where K = System gain




S-plane

**Poles (or) Singularities :-** These are the critical frequns which will makes the T.F = ∞.

**Zeros :-** These are the critical frequns which will make the T.F = 0.

from Equⁿ ①

→ the Transfer function of Linear Time invarient (L.T.I) system may be defined as the ratio of laplace transform of output to the laplace transform of input under the assumption that initial conditions are relaxed.

→ Poles and zeros are those critical frequencies which makes the Transfer Function infinity (or) zero.

## Singularity functions & Transfer functions :-

The standard Time domine test signals are relavent examples of singularity functions to define the Transfer funⁿ of L.T.I system.

**Step signal :-**

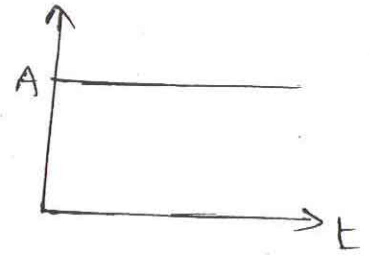
$r(t) = A\,u(t)$

$u(t) = 1 \; ; \; t>0$

$\quad = 0 \; ; \; t<0$

Laplace Transform

Now $R(s) = \frac{A}{s}$




note:
In control systems, Always we are taking response at $t>0$ only.
Not at $t=0, t<0$.
Thats why we write
✓ $t>0$ ; $t \geq 0$ ✗

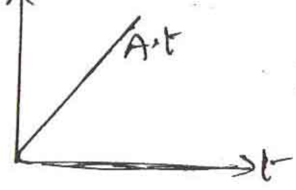
**Ramp signal :-**

$r(t) = At \; ; \; t>0$

$\quad = 0 \; ; \; t<0$

L.T:-

$R(s) = \frac{A}{s^2}$



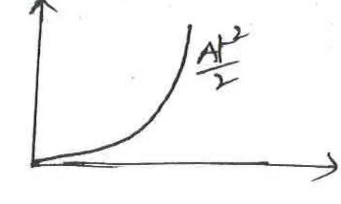

**Parabollic signal :-**

$r(t) = \frac{At^2}{2} \; ; \; t>0$

$\quad = 0 \; ; \; t<0$

L.T:-

$R(s) = \frac{A}{s^3}$

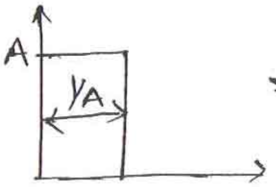



**Impulse signal :-** it is most powerful (or) important signal out of all signals.

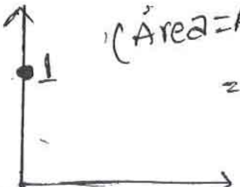
$r(t) = 1 \; ; \; t=0$

$\quad = 0 \; ; \; t \neq 0$

L.T:- $R(s) = 1$




⇒




$\left(Area = A \cdot \frac{1}{A} = 1\right)$

Note:- Due to sudden shocks, if the system response is recover, then it is stable at the time of impulse response $= 0$.

Note :-

* $\frac{d}{dt}$(Parabollic Response) = Ramp Response

* $\frac{d}{dt}$(Ramp Response) = step Response

* $\frac{d}{dt}$(step response) = Impulse Response

* $\int$ Impulse = step
* $\int$ step = ramp
* $\int$ ramp = Parabollic

let $F(S) = \frac{C(S)}{R(S)} = T.F$

$\Rightarrow C(S) = R(S) \cdot F(S)$

let $R(S) =$ Impulse signal $= 1$

$\therefore \Rightarrow C(S) = F(S) \times 1 = T.F$ ....

Note:-
If $R(S) =$ Impulse signal $\Rightarrow$ $C(S) =$ Impulse response
$R(S) =$ step $\Rightarrow$ $C(S) =$ step resp"
$R(S) =$ ramp $\Rightarrow$ $C(S) =$ ramp res"
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$$\boxed{\therefore \; L\left[\begin{matrix}\text{Impulse}\\ \text{Response}\end{matrix}\right] = T.F = \frac{C(S)}{R(S)}}$$

"WEIGHTING SIGNAL"

similarly Impulse Response $= L^{-1}[T.F]$

(Pb) Impulse Response of a system is $c(t) = -te^{-t} + 2e^{-t}$ $(t>0)$ Its OLTF is ?

ⓐ $\frac{2S+1}{(S+1)^2}$ ⓑ $\frac{2S+1}{S+1}$ ⓒ $\frac{2S+1}{S^2}$ ⓓ $\frac{2S+1}{S}$

Sol:-

$$T.F = \frac{C(S)}{R(S)} = L[\text{Impulse Response}] = \frac{-1}{(S+1)^2} + \frac{2}{S+1}$$

$$T.F = \frac{C(S)}{R(S)} = \frac{-1+2(S+1)}{(S+1)^2} = \frac{(2S+1)}{(S+1)^2} \;//...$$

This answer will be correct when the quesn is given like the Impulse Response of a openloop C.S. But in the pblm he given I.R of a system i.e CLCS'

Now $\frac{G(S)}{1+G(S)H(S)} = \frac{2S+1}{(S+1)^2} = \frac{C(S)}{R(S)}$

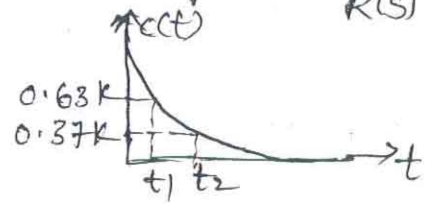
But we have to find OLTF. i.e $G(S)H(S) = ?$

Put $H(S) = 1$.

$$\frac{G(S)}{1+G(S)} = \frac{2S+1}{(S+1)^2} \Rightarrow G(S)[(S+1)^2 - 2S - 1] = 2S+1$$

$$\Rightarrow G(S) = \frac{2S+1}{S^2} \;//....$$

(Pb) The Impulse Response of a system having T.F $\frac{C(S)}{R(S)} = \frac{K}{S+\alpha}$ is shown in figure. find '$\alpha$' value?




ⓐ $t_1$ ⓑ $\frac{1}{t_1}$ ⓒ $t_2$ ⓓ $\frac{1}{t_2}$

sol:- $\frac{C(S)}{R(S)} = \frac{K}{S+\alpha}$

$\Rightarrow$ I.R $= Ke^{-\alpha t}$

At $t = t_2$:-

$\therefore Ke^{-\alpha t_2} = 0.37K$ $\quad [\because 0.37 = e^{-1}]$

$\Rightarrow e^{-\alpha t_2} = e^{-1}$

$\Rightarrow \alpha t_2 = 1$

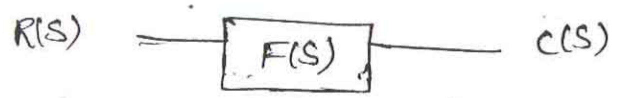
$\Rightarrow \boxed{\alpha = \frac{1}{t_2}}$ //...

(Pb) A certain control system has i/p r(t) and o/p c(t). If the i/p is first passed through a block whose T.F is $e^{-s}$ & then applied to the system. The modified o/p will be

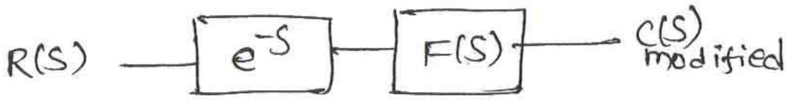
(a) $c(t)\,u(t-1)$ ⓑ $c(t-1)\,u(t)$

ⓒ $c(t-1)\,u(t-1)$ ⓓ $c(t)\cdot u(t)$

sol:-




$\Rightarrow C(S) = F(S)R(S)$




$C_m(S) = F(S)\cdot e^{-S}\cdot R(S)$

$= \underbrace{F(S)\,R(S)}\,e^{-S}$

$C_m(S) = C(S)\,e^{-S}$

$\boxed{L^{-1}\,F(S)\cdot e^{-as} = f(t-a)\cdot u(t-a)}$ //...

$\Rightarrow \therefore C_m(t) = c(t-1)\,u(t-1)$

(Pb) A unit step response is $y(t) = te^{-t}$ $(t>0)$, its T.F will be

ⓐ $1/(s+1)^2$ ⓑ $1/s+1$ ⓒ $s/(s+1)^2$ ⓓ $s/s+1$

sol:- I. T.F $= \frac{C(S)}{R(S)} = \left(\frac{1}{(S+1)^2}\right)/(1/s) = \boxed{\frac{S}{(S+1)^2}}$ $\Rightarrow$

T.F

laplace(I.R) = T.F

L(I.R) = T.F

$C(S) = S/(S+1)^2$

(or)

II. $\frac{d}{dt}$(step) = impulse response.

$y'(t) = -te^{-t} + e^{-t}$

$L[y'(t)] = L[-te^{-t} + e^{-t}]$

$$I.R = \frac{-1}{(s+1)^2} + \frac{1}{(s+1)} = \frac{s}{(s+1)^2} \text{ //...}$$

(16) The step response of system is $c(t) = 5 - \frac{4}{8}e^{-2t} + 8e^{-t}$. The steady state gain of T.F in time constant form will be

(a) $-7.5$ (b) $7.5$ (c) $-7$ (d) $7$

Sol:

$$c'(t) = 0 + \frac{4}{8} \times 2\, e^{-2t} - 8e^{-t}$$

$$= e^{-2t} - 8e^{-t}$$

$$L[c'(t)] = \frac{1}{s+2} - \frac{8}{s+1} = \frac{s+1-8(s+2)}{(s+2)(s+1)}$$

$$T.F = \frac{-7s-15}{(s+2)(s+1)} = \frac{-7(s+\frac{15}{7})}{(s+2)(s+1)} \text{ //}$$

If Time constant form is not given then Answer is $-7$.

$$T.F = \frac{-\cancel{7}\left(\frac{7s}{15}+1\right)\frac{15}{\cancel{7}}\, 7.5}{\cancel{2}(1+s/2)(1+s)} = \frac{-7.5\left(1+\frac{7s}{15}\right)}{(1+\frac{s}{2})(1+s)} \text{ //...}$$

$$\boxed{\therefore \text{gain}(K) = -7.5}$$

(17) What is the openloop DC gain of a unity -ve feedback system having closed loop T.F $\frac{s+4}{s^2+7s+13}$ ? (a) $\frac{4}{13}$ (b) $\frac{4}{9}$ (c) 4 (d) 13

Sol:-

$$\frac{G(s)}{1+G(s)H(s)} = \frac{s+4}{s^2+7s+13}$$

$$\frac{G(s)}{1+G(s)\cdot 1} = \frac{s+4}{s^2+7s+13}$$

$$(s^2+7s+13)\, G(s) = (s+4) + (s+4)G(s)$$

$$\Rightarrow G(s) = \frac{s+4}{s^2+6s+9} \text{ //...}$$

Put $s=0$ (since for DC $f=0 \Rightarrow s=0$)

$$G(0) = K = \frac{0+4}{0+0+9} = \frac{4}{9} \text{ //...}$$

(OR)

$$OLTF = \frac{CLTF}{1-CLTF}$$

$$= \frac{\frac{s+4}{s^2+7s+13}}{1-\frac{s+4}{s^2+7s+13}}$$

$$G(s)H(s) = \frac{s+4}{s^2+6s+9} \text{ //...}$$

$$G(0) = \frac{4}{9} \text{ //...}$$

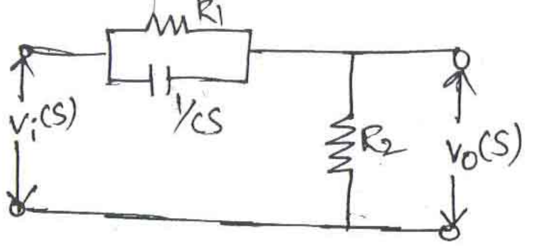
## Introduction to COMPENSATORS :-

Compensators in control systems are used for improving the performance specifications i.e transient and steady state characteristics. They are 3 types.

### 1. LEAD COMPENSATOR :-

It is used for improving "transient state char" (or) "speed of response of system".

Network Representation :-




$$V_i(S) = I(S)\left[\frac{R_1}{R_1CS+1} + R_2\right]$$

$$\Rightarrow V_i(S) = I(S)\left[\frac{R_1+R_2+R_1R_2CS}{R_1CS+1}\right]$$

Now

$$V_o(S) = I(S)\cdot R_2$$

$$\boxed{\therefore \frac{V_o(S)}{V_i(S)} = \frac{R_2(R_1CS+1)}{R_1+R_2+R_1R_2CS}}$$

$T = R_1C$, $\alpha$ = Attenuation constant $= \frac{R_2}{R_1+R_2}$ $[\alpha<1]$

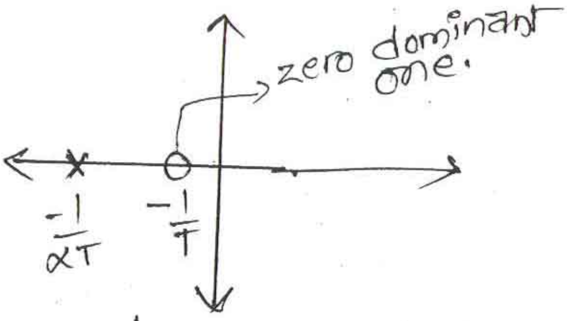
$$T.F = \frac{R_2(R_1CS+1)}{(R_1+R_2)\left(1+\frac{R_1R_2CS}{R_1+R_2}\right)}$$

note: R & C should be +ve values.

$$\Rightarrow \boxed{\frac{V_o(S)}{V_i(S)} = \frac{\alpha(1+TS)}{(1+\alpha TS)}}$$

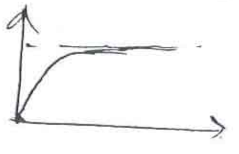
zero at $S = \frac{-1}{T}$

Pole at $S = \frac{-1}{\alpha T}$




NOTE:- Adding a zero to a system T.F interms of compensators represents "LEAD COMPENSATOR".

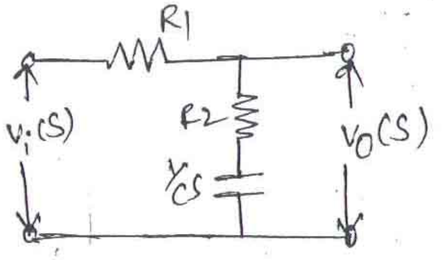

→ before lead compensator applied



→ After " " .

## LAG COMPENSATOR:-

It is used for improving the steady state characteristics of the system i.e elimination of steady state error b/w input and output.




$$V_i(S) = I(S)\left[R_1 + R_2 + \frac{1}{CS}\right]$$

$$= I(S)\left[\frac{R_1CS + R_2CS + 1}{CS}\right] \quad \text{I} \ldots$$

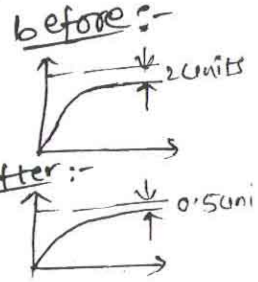
$$V_o(S) = I(S)\left[R_2 + \frac{1}{CS}\right] = I(S)\left(\frac{R_2CS+1}{CS}\right)$$

$$\rightarrow \ldots$$

$$\boxed{T.F = \frac{V_o(S)}{V_i(S)} = \frac{R_2CS+1}{R_1CS + R_2CS+1}} \quad \text{II} \ldots$$

$$T = R_2C \; ; \; \beta = \frac{1}{\alpha} = \frac{R_2 + R_1}{R_2} \quad (\beta > 1)$$

$\beta$ → lag parameter

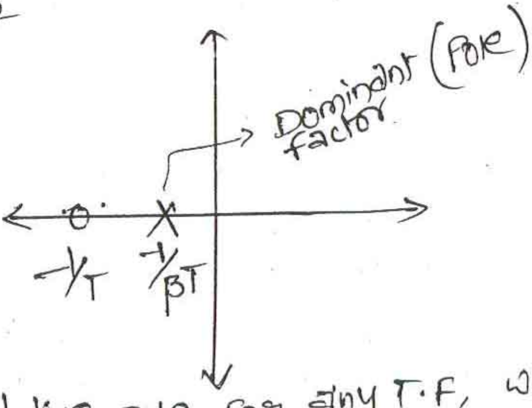



$$T.F = \frac{1 + R_2CS}{R_2CS\left(\frac{R_1+R_2}{R_2}\right)+1}$$

$$\therefore \boxed{\frac{V_o(S)}{V_i(S)} = \frac{1+TS}{1+\beta TS}}$$

zero at $S = -1/T$

Pole at $S = -1/\beta T$



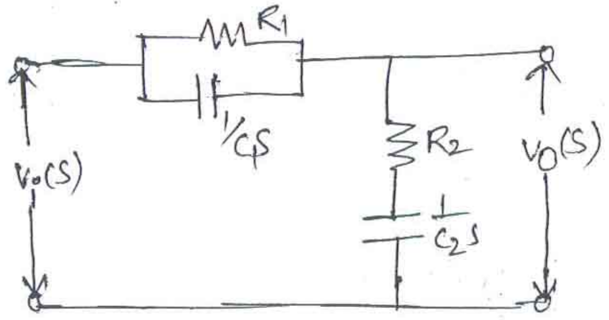

If specifically asked for adding pole for any T.F, we will go for lag compensator.

**NOTE:-** Adding the pole for the system T.F interms of compensators represent lag compensator.

## LEAD - LAG (or) LAG - LEAD COMPENSATOR :-

It is used for improving both steady state & Transient response characteristics of the system.

It exhibits both Magnitude & Phase characteristics of lead & lag compensators in its frequency response.




lead-lag compensator N/W

**LEAD**

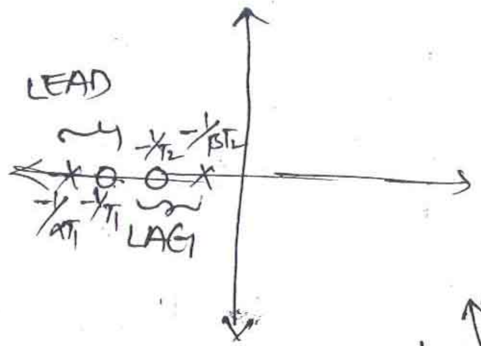
$T_1 = R_1 C_1$

$\alpha = \frac{R_2}{R_1+R_2}$

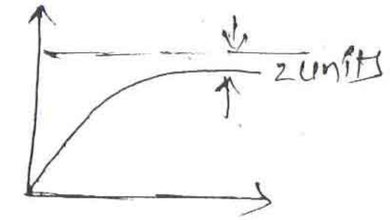
**LAG**

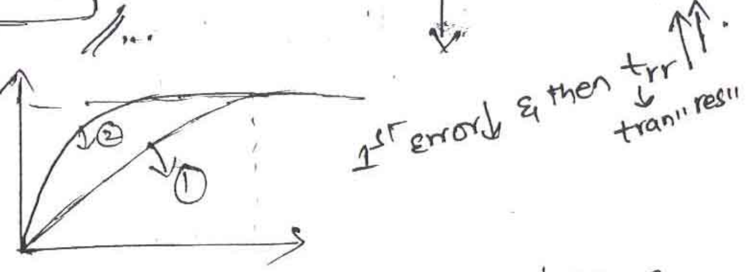
$T_2 = R_2 C_2$

$\beta = \frac{R_1+R_2}{R_2}$




$$\boxed{\frac{V_0(S)}{V_i(S)} = \frac{\alpha(1+T_1S)(1+T_2S)}{(1+\alpha T_1 S)(1+\beta T_2 S)}}$$




In any control system first elimination of error is done & then next speed of response is improved. (It is a perfect design of system) i.e first lag compensator and then lead compensators are used simultaniously.

We don't have two comparators. for lead - lag & lag - lead compensators both have same N/W. This is standard N/W there is no change in N/W. So while in case of filters (Band pass, Band stop) we may interchange the N/W & $R_1C_1$ & $R_2C_2$ combinations. But here we have only one N/W i.e first lead N/W $(R_1C_1)$, & then lag $(R_2C_2)$. This is similar to PID controller we don't have DIP, IDP... likethat. So the N/W is standard...

## MECHANICAL SYSTEMS :-

Any mechanical system having ~~is~~ mass & friction. Friction we can't see, so it is represented by as Damper. Similarly Force required to move duster and iron piece is not same. It depends on stiffness. so it is represents as spring. since springs are classified into 2 types.

1. Hard
2. soft

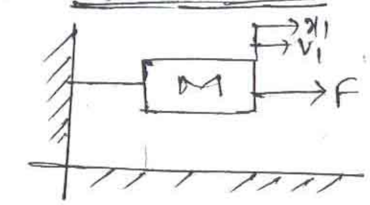
Similarly Hardness and softness of object is decided, in which moment will be done.

## 1. MECHANICAL TRANSLATIONAL SYSTEM :—

Input = Force
output = Linear displacement($x$) (or) Linear velocity. ($v$)

The 3- Ideal Elements are          note:- These 3 elements are can't seen, only can feel.

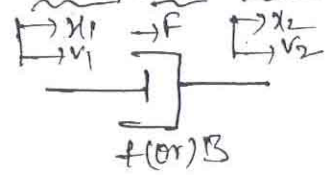
### 1. MASS ELEMENT :-




ⓐ $F = M\frac{dv}{dt}$ ($\because F = ma$)

ⓑ $F = M\frac{d^2x}{dt^2}$

### 2) Damper element :-




ⓐ $F = f(v_1 - v_2) = fv$

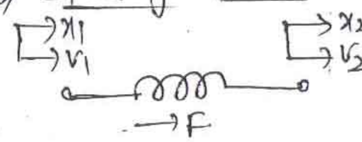
where $v = (v_1 - v_2)$

ⓑ $F = f\frac{d}{dt}(x_1 - x_2)$

$= f\frac{dx}{dt}$

where $x = x_1 - x_2$

### 3) Spring Element :-




ⓐ $F = K\int(v_1 - v_2)dt$

$= K\int v\,dt$

ⓑ $F = K(x_1 - x_2) = K$

where

$v = v_1 - v_2$

$x = x_1 - x_2$

note:-

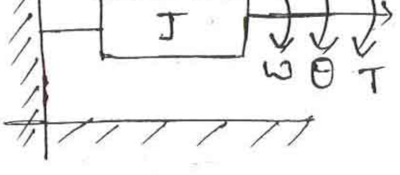
The Damper element is represented by a cylinder with piston why should we prefer like that only? explanation is when a duster (or) Iron piece is placed in our hand. If some force is applied to move the object, only object will moves. but the hand will not moves & stationary. Thats why we are representing it as piston with cylinder.

## (2) Mechanical Rotational System :—

Input = Torque ($T$)
output = Angular Displacement($\theta$) (or) Angular velocity ($\omega$)

The Three Ideal Elements are

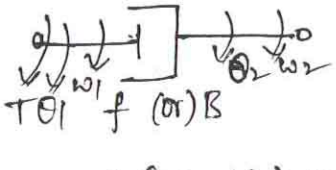
### 1) Inertial element:




ⓐ $T = J\frac{d\omega}{dt}$

ⓑ $T = J\frac{d^2\theta}{dt^2}$

### 2) Damper Element



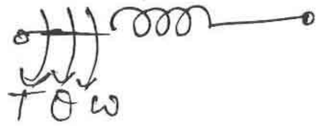

ⓐ $T = f(\omega_1 - \omega_2) = f\omega$

where $\omega = \omega_1 - \omega_2$

ⓑ $T = f\frac{d}{dt}(\theta_1 - \theta_2) = f\frac{d\theta}{dt}$

where $\theta = \theta_1 - \theta_2$

### 3) Torsional Spring Element



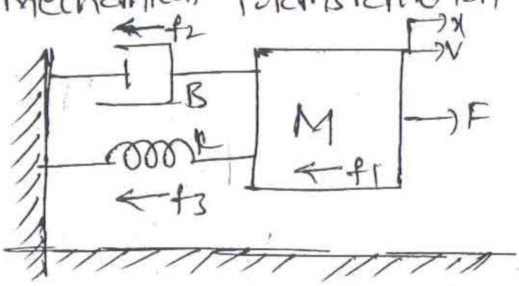

ⓐ $T = K\int\omega\,dt$

ⓑ $T = K\theta$

# ANALOGOUS SYSTEMS :-

Electrical equivalents of mechanical elements are termed as Analogous systems.

Mechanical Translational

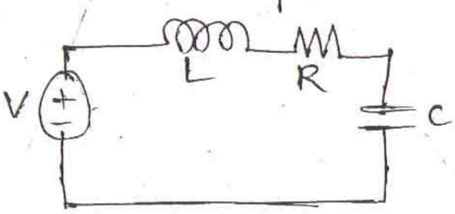



$$F = f_1 + f_2 + f_3$$

$$F = M\frac{dv}{dt} + Bv + K\int v\,dt$$

$$F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx \rightarrow ①$$

Electrical System :-



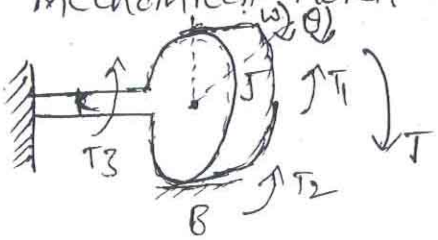

$$v = v_1 + v_2 + v_3$$

$$= L\frac{di}{dt} + iR + \frac{1}{C}\int i\,dt$$

$$i = \frac{dq}{dt} \quad (q = \text{charge})$$

$$v = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} \rightarrow ③$$

Mechanical Rotational

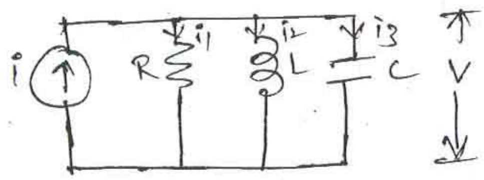



$$T = T_1 + T_2 + T_3$$

$$T = J\frac{d\omega}{dt} + B\omega + K\int \omega\,dt$$

$$= J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta \rightarrow ②$$

Electrical System :-




$$I = i_1 + i_2 + i_3$$

$$I = C\frac{dv}{dt} + \frac{v}{R} + \frac{1}{L}\int v\,dt$$

$$v = \frac{d\phi}{dt} \quad (\phi = \text{flux})$$

$$I = C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{\phi}{L} \rightarrow ④$$

Comparing eq ①, ②, ③, ④

| | | | |
|---|---|---|---|
| F | T | V | I |
| M | J | L | C |
| B | B | R | 1/R |
| K | K | 1/C | 1/L |
| V | ω | i | V |
| x | θ | q | φ |

1. F-T-V Analogy
2. F-T-I Analogy

Mass & Spring elements are known as "conservative elements".

Explanation: Mass and spring elements both related to L & C. An electrical system 'L' & 'C' are energy stored parameters. So that mass & spring also "energy storing" and "energy conservating elements".

NOTE:- In order to enhance (or) reduce the torque we are connecting some device b/w the electrical motor and load.

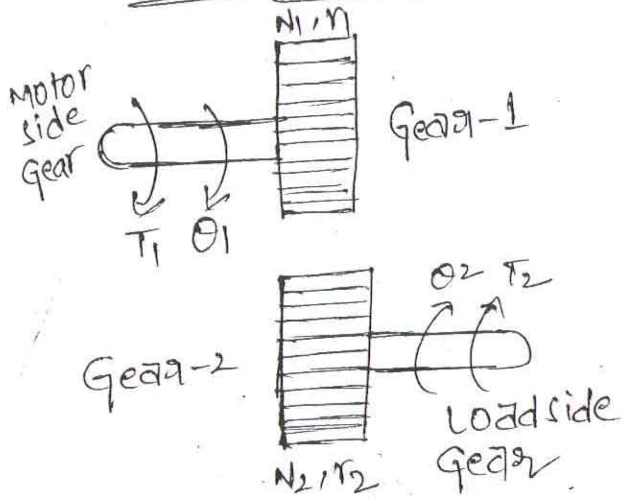
GEARS :-

Mechanical Gears are those devices which are used for stepup (or) stepdown speed (or) Torque.

They are used as intermediate elements b/w electrical motor & load.

they are analogous to electrical transformers (or) Amplifiers and Buffers.

## Dynamics of Gear Train :-




N = NO: of Teeth on the circumferance of Gear wheel.

r = radious of Gear wheel (m)

θ = Angular Displacement (rad)

T = Torque on Gearwheels (N-m)

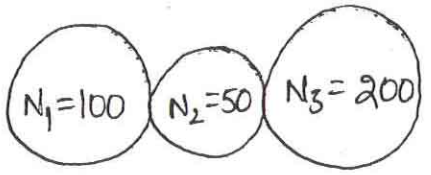
$$\boxed{\frac{N_1}{N_2} = \frac{T_1}{T_2} = \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{\omega_2}{\omega_1} = \frac{a_2}{a_1}}$$

for Two Gearwheels

for 'n' gear whells connected in series.

$$\boxed{\frac{N_x}{N_y} = \frac{T_x}{T_y} = \frac{r_x}{r_y} = \frac{\theta_y}{\theta_x} = \frac{\omega_y}{\omega_x} = \frac{a_y}{a_x}}$$

Ex:




① $T_1 = 10$ N-m find $T_2$ & $T_3$?

② $\omega_1 = 20$ rad/sec find $\omega_2$ & $\omega_3$?

Sol:- ① $\frac{N_1}{N_2} = \frac{T_1}{T_2} \Rightarrow \frac{100}{50} = \frac{10}{T_2} \Rightarrow T_2 = 5$ N-m

$\frac{N_1}{N_3} = \frac{T_1}{T_3} \Rightarrow \frac{100}{200} = \frac{10}{T_3} \Rightarrow T_3 = 20$ N-m

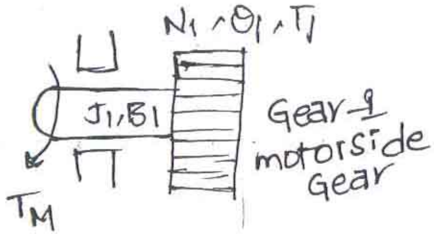
② $\frac{N_1}{N_2} = \frac{\omega_2}{\omega_1} \Rightarrow \omega_2 = \frac{100}{50} \times 20 = 40$ rad/sec (C.C.W)

$\frac{N_1}{N_3} = \frac{\omega_3}{\omega_1} \Rightarrow \omega_3 = \frac{100}{200} \times 20 = 10$ rad/sec (C.W)

Observations :-

1) $N_1 > N_2 \Rightarrow T\downarrow, \omega\uparrow$

2) $N_1 = N_2 \Rightarrow$ There is no change in speed (or) Torque.

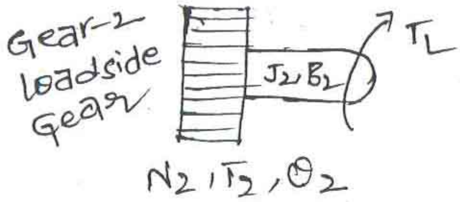



$T_M$ = Motor Torque

$T_1$ = Torque on Gear-1 due to $T_M$ (N-m)

$T_2$ = Torque " Gear-2 due to $T_1$ (N-m)

$T_L$ = Torque " Gear load due to $T_2$ (N-m)




$$T_M = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + T_1$$

$$T_2 = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_L$$

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$$\frac{N_1}{N_2} = \frac{T_1}{T_2} \Rightarrow T_1 = \left(\frac{N_1}{N_2}\right) T_2$$

$$T_1 = \frac{N_1}{N_2} \cdot J_2 \frac{d^2\theta_2}{dt^2} + \frac{N_1}{N_2} B_2 \frac{d\theta_2}{dt} + \frac{N_1}{N_2} T_L$$

$$T_m = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + \frac{N_1}{N_2} J_2 \frac{d^2\theta_2}{dt^2} + \frac{N_1}{N_2} \cdot B_2 \frac{d\theta_2}{dt} + \frac{N_1}{N_2} \cdot T_L \quad \rightarrow (1)$$

Equivalent friction and inertia for motor side Gear

$$\frac{N_1}{N_2} = \frac{\theta_2}{\theta_1} = \frac{\dot{\theta}_2}{\dot{\theta}_1} = \frac{\ddot{\theta}_2}{\ddot{\theta}_1} \Rightarrow \ddot{\theta}_2 = \left(\frac{N_1}{N_2}\right)\ddot{\theta}_1 \quad \&$$

$$\ddot{\theta}_1 = \left(\frac{N_1}{N_2}\right)\dot{\theta}_1$$

$$T_M = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right)^2 \cdot J_2 \frac{d^2\theta_1}{dt^2} + \left(\frac{N_1}{N_2}\right)^2 B_2 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) \cdot T_L$$

$$T_M = \left[J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2\right] \frac{d^2\theta_1}{dt^2} + \left[B_1 + \left(\frac{N_1}{N_2}\right)^2 \cdot B_2\right] \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

$$\boxed{\begin{aligned} J_{eq_1} &= J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2 \\ B_{eq_1} &= B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2 \end{aligned}}$$ ***

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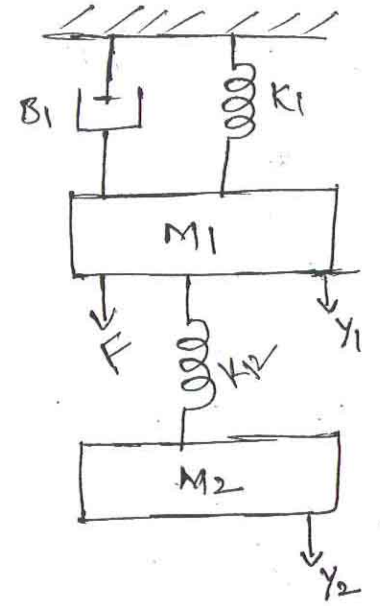
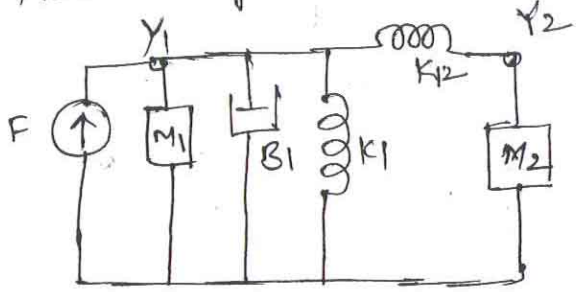
//...

## NODAL METHOD :-

* No: of Nodes = No: of displacements.
* Take an additional node which is reference node.
* connect the mass (or) inertia elements between the Principle node & reference node.
* connect the spring & damping elements either b/w the principle node & reference depending on their position.
* obtain the nodal diagram & write the describing nodal equations at each node.




Nodal diagram :-




$$F = M_1\frac{d^2Y_1}{dt^2} + B_1\frac{dY_1}{dt} + K_1Y_1 + K_{12}(Y_1 - Y_2)$$

$$0 = M_2\frac{d^2Y_2}{dt^2} + K_{12}(Y_2 - Y_1)$$

(NOTE :- $f_1$ or $B_1$ → friction any one you can take)

$$\therefore T.F = \frac{Y_1(S)}{F(S)}$$

$$F(S) = (M_1S^2 + f_1 S + K_{12} + K_1)\,Y_1(S) - (K_{12})(Y_2(S))$$

$$0 = (M_2S^2 + K_{12})\,Y_2(S) - K_{12}Y_1(S)$$

$$\Rightarrow Y_2(S) = \left(\frac{K_{12}}{MS^2 + K_{12}}\right) Y_1(S) \ldots$$

$$F(S) = (M_1S^2 + f_1S + K_1 + K_{12})\,Y_1(S) - K_{12}\left[\frac{K_1^2}{M_2S^2 + K_{12}}\right] \cdot Y_1(S)$$

$$= \left[\frac{(M_1S^2 + f_1S + K_1 + K_{12})(M_2S^2 + K_{12}) - K_{12}^2}{M_2S^2 + K_{12}}\right] Y_1(S)$$

$$\boxed{\frac{Y_1(S)}{F(S)} = \frac{M_2S^2 + K_{12}}{[(M_1S^2 + f_1S + K_1 + K_{12})(M_2S^2 + K_{12}) - K_{12}^2]}}$$

Order = 4...

1 Mass Element → order 2
2 Mass Element → order 4
3 Mass Elements → order 6
⋮
$\boxed{n \text{ Mass Elements} \rightarrow \text{order } 2n}$ //...

1) **Force - voltage Analogy :-**

$$V_1 = L_1 \frac{d^2 q_1}{dt^2} + R \frac{dq_1}{dt} + \frac{q_1}{C_1} + \frac{q_1 - q_2}{C_{12}}$$

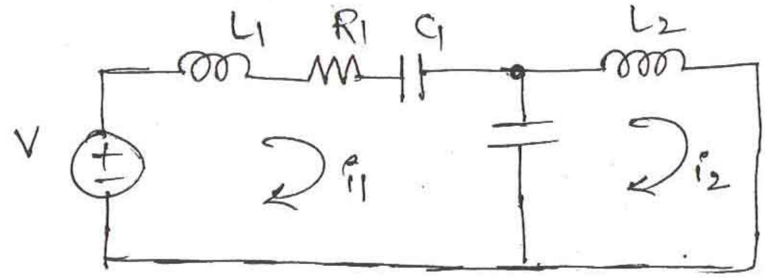
$$0 = L_2 \frac{d^2 q_2}{dt^2} + \frac{q_2 - q_1}{C_{12}}$$

$$i_1 = \frac{dq_1}{dt} \Rightarrow q_1 = \int i_1 \, dt$$

$$i_2 = \frac{dq_2}{dt} \Rightarrow q_2 = \int i_2 \, dt$$

$$V = L_1 \frac{di}{dt} + Ri + \frac{1}{C_1} \int i_1 dt + \frac{1}{C_{12}} \int (i_1 - i_2) dt$$

$$0 = L_2 \frac{di_2}{dt} + \frac{1}{C_{12}} \int (i_2 - i_1) \, dt$$




II. **Force - current Analogy :-**

$$I = C_1 \frac{d^2 \phi_1}{dt^2} + \frac{1}{R_1} \cdot \frac{d\phi_1}{dt} + \frac{1}{L_1} \phi_1 + \frac{\phi_1 - \phi_2}{L_{12}}$$

$$0 = C_2 \frac{d\phi_2}{dt} + \frac{\phi_2 - \phi_1}{L_{12}}$$

$$V_1 = \frac{d\phi_1}{dt}, \quad \phi_1 = \int V_1 \, dt$$

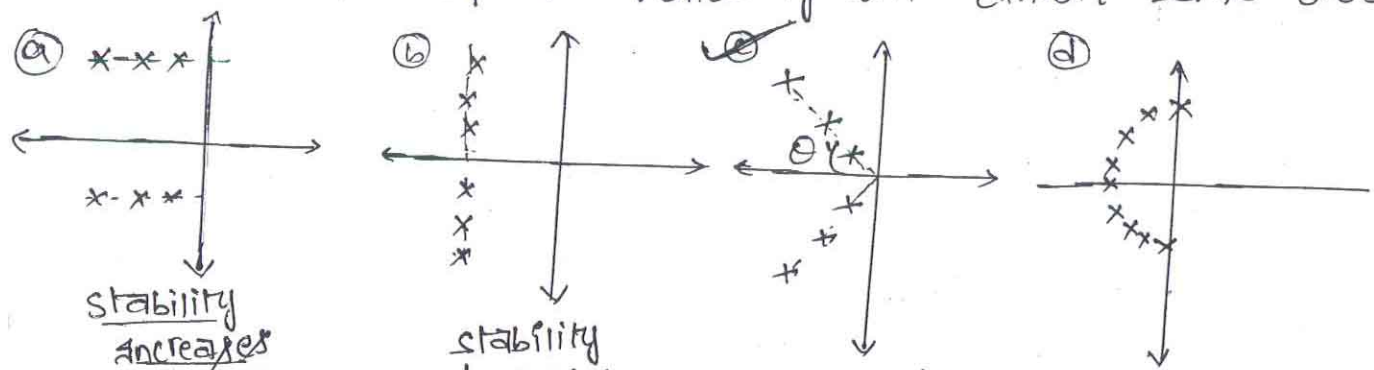
$$V_2 = \frac{d\phi_2}{dt}, \quad \phi_2 = \int V_2 \, dt$$

$$I_1 = C_1 \frac{dV_L}{dt} + \frac{V_1}{R_1} + \frac{1}{L_1} \int V_1 dt + \frac{1}{L_{12}} \int (V_1 - V_2) dt$$

$$0 = C_2 \frac{dV_2}{dt} + \frac{1}{L_{12}} \int (V_2 - V_1) \, dt$$

(Pb)*** The root locations of three underdamped systems are shown. which of the following will exhibit same overshoot?

(a) (b) ~~(c)~~ (d)



stability increases as the poles are moving like this

stability decreases ...

stability remains same ...

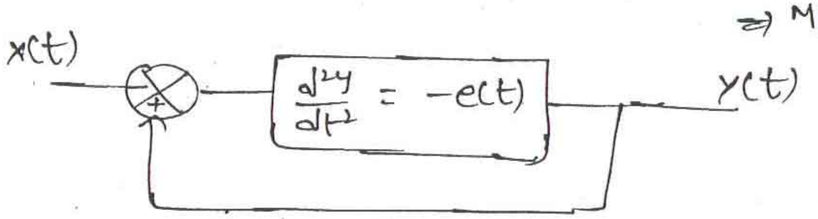
Now, $M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$

from fig (c) $\Rightarrow$ $\theta$ is same. But W.K.T $\cos\theta = \zeta$

$\Rightarrow$ so '$\zeta$' also same

$\Rightarrow M_p$ is same.

(Pb)



(a) $\sin t$ ~~(b)~~ $-\sin t$
(c) $\cos t$ (d) $-\cos t$

for $x(t) = t\,u(t)$ ; $e(t)$ ?

Sol:-

$e(t) = -x(t) + y(t) \Longrightarrow E(S) = -X(S) + Y(S)$

$E(S) = -\frac{1}{S^2}U(S) + Y(S)$

$-e(t) = \frac{d^2y}{dt^2}$

$-E(S) = S^2Y(S)$ ...

$\Rightarrow Y(S) = \frac{-1}{S^2}E(S)$

$E(S) = -\frac{1}{S^2}U(S) - \frac{1}{S^2}E(S)$

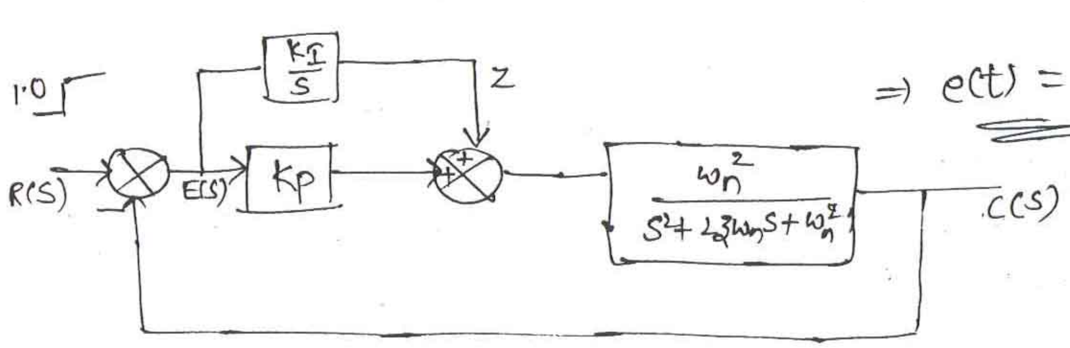
$\Rightarrow E(S)\left[\frac{1}{S^2}+1\right] = -\frac{1}{S^2}U(S)$

$\Rightarrow E(S)\left[\frac{(S^2+1)}{\cancel{S^2}}\right] = -\frac{U(S)}{\cancel{S^2}}$

$\Rightarrow E(S) = -\frac{1}{S^2+1}U(S)$

$\Rightarrow e(t) = -\sin t$ ...

(Pb)



steady state value of z?

(a) 0 ~~(b)~~ 1 (c) $\infty$ (d) 0.707

steady state value of z is $\lim_{s \to 0}$

Need Expression for z.

$$z = E(s) \cdot \frac{K_I}{s} = [R(s) - C(s)] \frac{K_I}{s}$$

$$R(s) = 1/s \; ; \; G(s)H(s) = \left(K_p + \frac{K_I}{s}\right) \cdot \left[\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right]$$

$$z = \frac{s\left[\frac{K_I}{s}\right]}{1 + \frac{(K_p s + K_I)\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}}$$

$$z = \frac{\frac{1}{s} \cdot \frac{K_I}{\cancel{s}} [\cancel{s}] (s^2 + 2\zeta\omega_n s + \omega_n^2)}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2) + (K_p s + K_I)\omega_n^2}$$

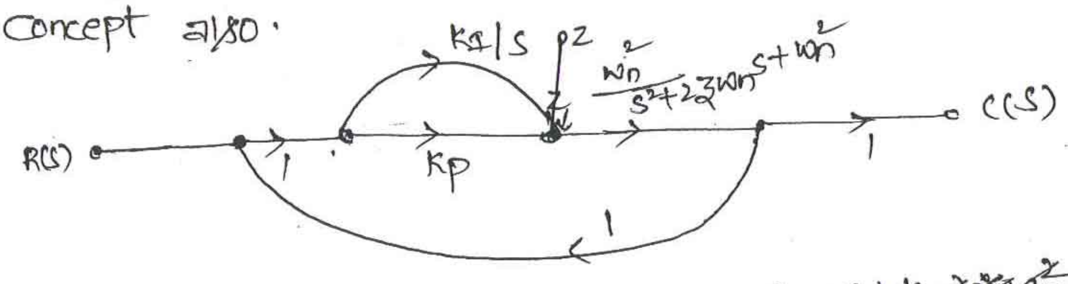
$$\lim_{s \to 0} s \cdot z = \lim_{s \to 0} \frac{\cancel{s} \cdot \frac{K_I}{\cancel{s}} (s^2 + 2\zeta\omega_n s + \omega_n^2)}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2) + (K_p s + K_I)\omega_n^2}$$

$$= \frac{\cancel{K_I \cdot \omega_n^2}}{\cancel{K_I \omega_n^2}}$$

$$= 1 \text{//....}$$

(OR)

The value of z can be findout by using signal flow graph concept also.




$$\Delta = \cancel{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2) * (K_p s + K_I) * \omega_n^2}$$

$$\Delta = 1 + \left(K_p * \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} + \frac{(K_I/s) * \omega_n^2/(s^2 + 2\zeta\omega_n s + \omega_n^2)}{1}\right)$$

$$T.F = \frac{z}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{(K_p + K_I/s)}{\left[1 + \frac{K_p * \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} + \left(\frac{K_I}{s}\right) \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right]}$$

$R(s) \to 1/s$

Conventional Q:2 Page 60

Sol:-

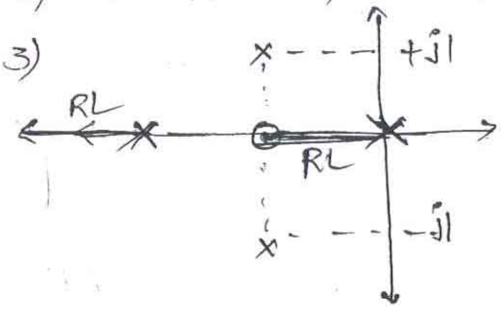
$$C.E = S(S+4)(S^2+2S+2) + K(S+1) = 0$$

for Drawing the root locus we require OLTF. So

$$OLTF = \frac{K(S+1)}{S(S+4)(S^2+2S+2)} = G(S)\,H(S)$$

rule 2) $P = 4,\ Z = 1,\ P - Z = 3$

3)




4) Asymptotes $\Rightarrow \theta_1 = 60^\circ$
$\theta_3 = 300^\circ$
$\theta_2 = 180^\circ$

5) $\sigma = \dfrac{(-4)+(-1)+(-1)-(-1)}{3}$

$= -1.6$

6) there is no chance ~~of~~ to meet the poles at any point. So that there is no B.A.P.

7) $C.E \rightarrow S^4 + 6S^3 + 10S^2 + (K+8)S + K = 0$

| | | | |
|---|---|---|---|
| $S^4$ | 1 | K | 0 |
| $S^3$ | 6 | $(K+8)$ | 0 |
| $S^2$ | $\left(\frac{52-K}{6}\right)$ | K | 0 |
| $S^1$ | $\frac{\left(\frac{52-K}{6}\right)(K+8)-K}{\left(\frac{52-K}{6}\right)}$ | 0 | 0 |
| $S^0$ | K | 0 | 0 |

$$\frac{\left(\frac{52-K}{6}\right)(K+8)-K}{\left(\frac{52-K}{6}\right)} = 0$$

$$\Rightarrow (52-K)(K+8) - 6K = 0$$

$$\Rightarrow K = 24.78,\ -16.78$$

valid one is $\therefore K = 24.78 = K_{mar}$

$K = -16.78$ ✗ since unstable.

$$A(S) = \left(\frac{52-K}{6}\right)S^2 + K = 0$$

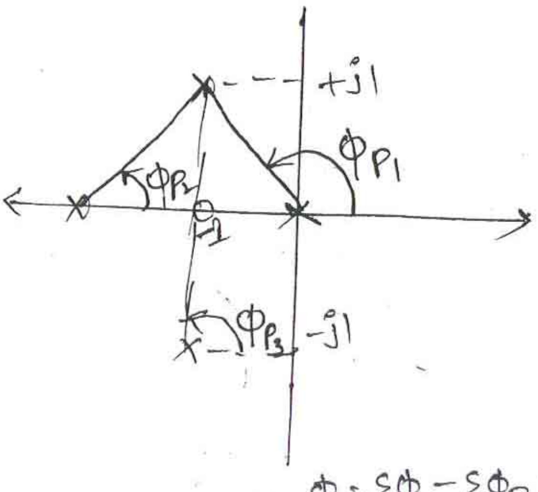
$$\Rightarrow \left(\frac{52-24.78}{6}\right)S^2 + (24.78) = 0$$

$$\Rightarrow S = \pm j2.34$$

Now the Asymptote intersect on $j\omega$ axis is,

$Y = \tan 60 \times 1.6 \Rightarrow Y = 2.77$

$$\boxed{Y \cong \pm j2.77}$$




$\phi_{P_1} = 180 - \tan[1] = 135^\circ$

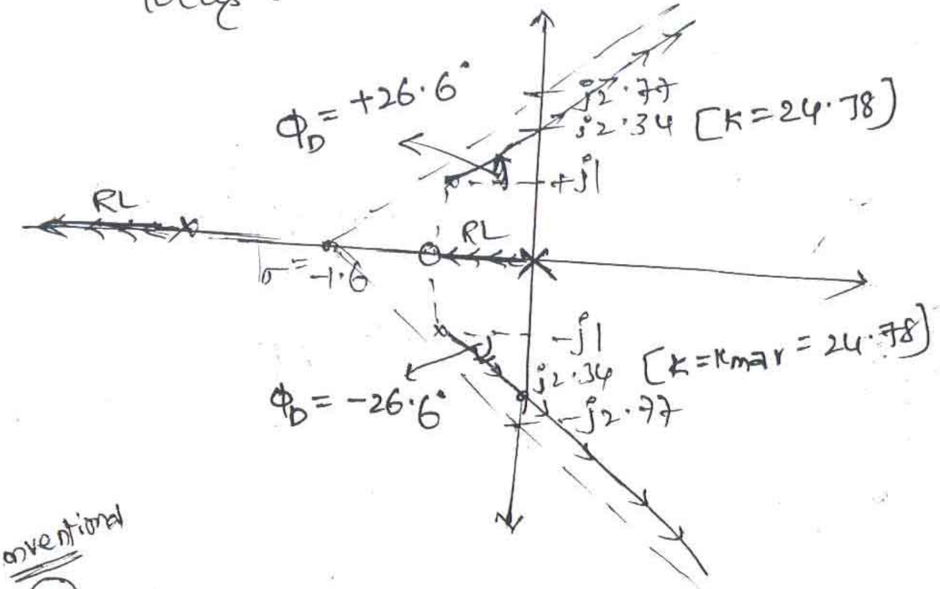
$\phi_{P_2} = \tan^{-1}\left(\dfrac{1-0}{-1-(-4)}\right) = 18.4^\circ$

$\phi_{P_3} = 90^\circ$

$\phi = \Sigma\phi_z - \Sigma\phi_p \Rightarrow \phi = 90 - [135 + 90 + 18.4] = -153.6^\circ$

$\therefore \phi_D = 180 + \phi = 180 - 153.6^\circ = \underline{26.6^\circ} \ldots$

Now with all this information we will draw the root-locus.




Range of K for stability is

$\boxed{0 < K < 24.78}$

conventional

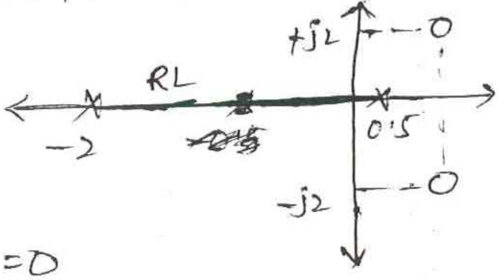
Q:3

Sol:-

$$G(s) = \frac{K[s^2 - 2s + 5]}{(s+2)(s-0.5)}$$

$\rightarrow$ 2) $P = 2, Z = 2, P - Z = 0$

3)




6) B.A.P:-

$$K = \frac{-s^2 - 1.5s + 1}{s^2 - 2s + 5}$$

$$\frac{dK}{ds} = 0 \Rightarrow 3.5s^2 - 12s + (-5.5) = 0$$

$$\Rightarrow s = 3.8, \underline{-0.4} \ldots$$

7) $s^2(1+K) + s(1.5 - 2K) + (5K-1) = 0 \rightarrow$ C.E

| | | |
|---|---|---|
| $s^2$ | $(1+K)$ | $(5K-1)$ |
| $s^1$ | $(1.5-2K)$ | 0 |
| $s^0$ | $(5K-1)$ | 0 |

$1.5 - 2K = 0 \Rightarrow K_{max} = 0.75$

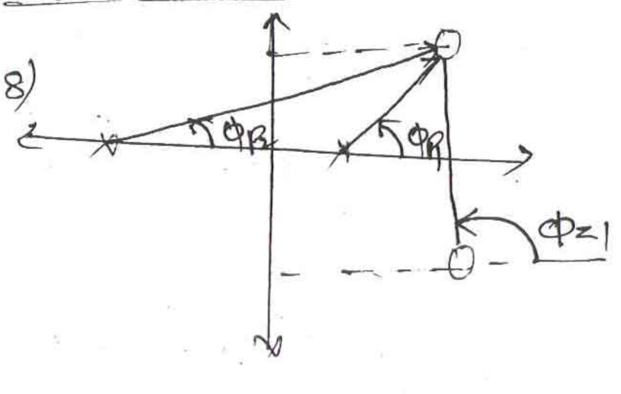
$A(s) = [1+K]s^2 + (5K-1) = 0$

$(1+0.75)s^2 + [5(0.75) - 1] = 0$

$\Rightarrow s = \pm j1.25 \ldots$

Angle of Arrival:-

8)



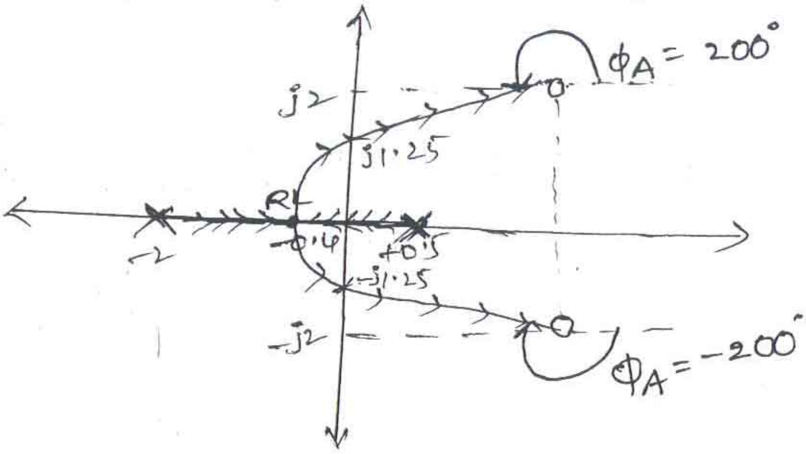

$\phi_{Z_1} = 90$

$$\phi_{P_1} = \tan^{-1}\left(\frac{2-0}{1-0.5}\right) = 76^\circ$$

$$\phi_{P_2} = \tan^{-1}\left[\frac{2-0}{1-(-2)}\right] = 34^\circ$$

$\phi_A = 180 - \phi = 180 - [90 - (76 + 34)] = \underline{200}$ ...

With all this information we will now draw the root locus dgm.




Range of K for stability

$1+K>0 \Rightarrow K>-1$ ✗

$1.5-2K>0 \Rightarrow K<0.75$ ✓

$5K-1>0 \Rightarrow K>0.2$ ✓

$\boxed{\therefore\ 0.2<K<0.75}$ is the stability limit.

At $K>-1 \Rightarrow$ eg $K=-0.5 \Rightarrow$ system unstable.

find Gain 'K' for $\xi = 0.5$ ?

sol: $\cos\theta = \xi \Rightarrow \theta = 60°$

$K = \dfrac{lP_1 \times lP_2}{lz_1 \times lz_2}$ for that we have to take appropriate scale on x-axis & y-axis.

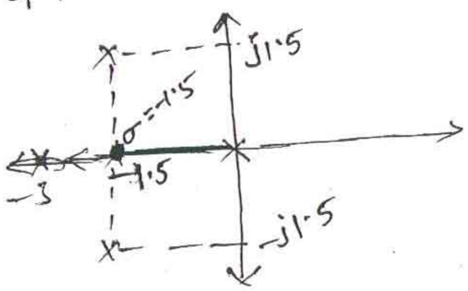
conventional

Q:1 $G(S) = \dfrac{K}{S(S+3)(S^2+3S+4.5)}$

sol: 2) $P=4,\ Z=0,\ P-Z=4$

$P = 0, -3, (-3 \pm j3)/2$

3)




4) $\theta_1 = 45°,\ \theta_2 = 135°$

$\theta_3 = 225°,\ \theta_3 = 315°$

5) $\sigma = \dfrac{0+(-3)+(-\frac{3}{2})+(-\frac{3}{2})}{4} = \cancel{-2.25}\ -1.5$

6) B.A.P: $(S^2+3S)(S^2+3S+4.5)+K=0 \Rightarrow$ C.E

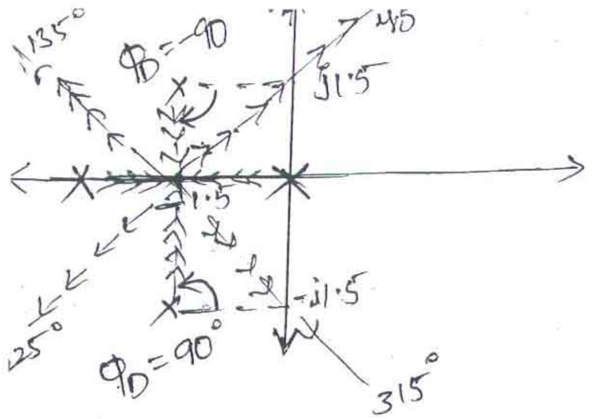
$K = -(S^4 + 6S^3 + 13.5S^2 + 13.5S)$

$\dfrac{dK}{dS} = 4S^3 + 18S + 27S + 13.5 = 0$

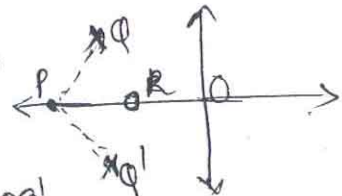
$\Rightarrow S = -1.5, -1.5, -1.5$ ...

7)

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 13.5 | K |
| $s^3$ | 6 | 13.5 | 0 |
| $s^2$ | | | |
| $s^1$ | | | |
| $s^0$ | | | |




Ex:- $PQ = PQ' = 2.6$
$RP = 1.4$
$OR = 2.0$
$OQ = 1.4 = OQ'$

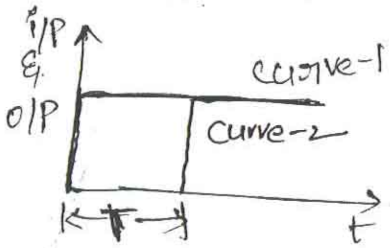



Then the value of 'K' at point 'P' is

Sol:- Gain $K = \dfrac{\text{Product of lengths of poles}}{\text{Product of length of zeros}}$

$$= \frac{(PQ)(PQ')}{PR} = \frac{(2.6)(2.6)}{1.4} = 4.8$$

# Analysis of systems having Dead Time (or) Transportation lag :-




we are taking an i/p as shown in fig.
But the o/p will occurs with a delay of time 't'.

note :-
for 'y' operation we are intrested to design some of the system with an DeadTime.

If DeadTime is precent then how the T.F will be appear - we will see.

for curve 1 :-
$$o/p \; y(t) = i/p \; x(t)$$

for curve 2 :-
$$o/p \; y(t) = x(t-T)$$

Apply laplace transform
$$Y(s) = e^{-sT} X(s)$$

$$\boxed{\frac{Y(s)}{X(s)} = e^{-sT}}$$

this is the T.F of a system with DeadTime.

1. Time domine Approximation:- [Time Domine analysis, R-H, RL]

$y(t) = x(t-T) = x(t) - T\dot{x}(t) + \frac{T^2}{2!}\ddot{x}(t) * \ldots\ldots$

we are taking upto 1st order only. since approximation may complex if you are taking higher orders.

$\Rightarrow y(t) = x(t) - T\dot{x}(t)$

$Y(S) = X(S)[1 - TS]$

$Y(S) = X(S)[e^{-ST}]$

$Y(S)$ $\boxed{e^{-ST} \cong (1-ST)}$

$Ex:- G(S) = \frac{Ke^{-S}}{S(S+3)} \cong \frac{K(1-S)}{S(S+3)}$ //...

Sol,

* Dead time is the one of the nonlinearity forms.
(or) Delay

therefore for any T.F, should not having the zeros on R.H.S of S-plane; since the system will tends to non linear.

If the system having the poles or zeros on RHS of S-plane (specifically zeros), then the system is called "Non-minimum Phase system".

If poles & zeros are lie on L.H.S of S-plane, shuch system is called "minimum phase system".

Ex:- Mini phace sys

$G(S) = \frac{K(1+S)}{S(S+3)}$

$\boxed{\angle G(j\omega) = -[P-Z]\,90^\circ}$ → minimum phase system criteria

$= -(2-1)\,90^\circ = -90^\circ$ ...

Now

$\angle G(S) = \frac{\angle K \cdot \angle 1+S}{\angle S \cdot \angle S+3} = \frac{\angle 0^\circ \cdot [\tan^{-1}(\omega/1)]}{(\tan^{-1}(\omega/3))\}[90^\circ]}$

$= -90 - \tan^{-1}\left(\frac{\omega}{3}\right) + \tan^{-1}(\omega)$

at $\omega = \infty \Rightarrow \boxed{\angle G(j\omega) = -90^\circ}$ So that which satisfies the mini phase sys criteria!

Any system which satisfies the minimum phase system criteria then it is called "minimum phase system".

Ex: Non-Minimum phase system.

$$G(s) = \frac{K(1-s)}{s(s+3)}$$

$$\boxed{\angle G(j\omega) \neq -(P-Z)\,90^{\circ}}$$ Non minimum phase criteria

Now

$$\angle G(j\omega) = \frac{0^{\circ}\ [360^{\circ} - \tan^{-1}(\omega)]}{[90^{\circ}]\ [\tan^{-1}(\omega/3)]}$$

$$= -\tan^{-1}(\omega) - \tan^{-1}(\omega/3) - 90$$

At $\omega = \infty \Rightarrow \angle G(j\omega) = -90-90-90 = \underline{-270} \neq 90$

so it is a non minimum phase system.

Note:- suppose if he asking that draw the root locus for above T.F. we can't draw the root locus for $(1-S)$. Since

*** "s" can not be $\underline{\underline{-ve}}$.

$\downarrow$
means $j\omega \Rightarrow$ freq"

so $(1-S)$ factor can not be taken as it is. It is our responsibility to take 's' as +ve. so it is written as

$$G(s) = \frac{-K(s-1)}{s(s+3)}$$

NOTE:-

1) Dead Time is one of the forms of Non linearity.
2) It is approximated as a zero in R.H.S of S-plane.
3) Transfer functions having poles and zeros in R.H.S of S-plane [specifically zeros] are known as "Non minimum phase functions"
4) L.T.I Transfer functions should be minimum phase function i.e their poles & zeros should lie in L.H.S of S-plane only.

5) In time domine applications since 'S' can not be $-ve$, $(1-S)$ factors should be expressed as $-(S-1)$.

It is applicable only in Timedomine, not in freqdomine

Since $\left.\begin{array}{l}(1-S) \Rightarrow \sqrt{1+\omega^2} \\ (1+S) \Rightarrow \sqrt{1+\omega^2}\end{array}\right\}$ for both magnitude is same.

Ex:-

$$G(S) = \frac{Ke^{-S}}{S(S+3)}$$

Sol:

$$G(S) \cong \frac{K(1-S)}{S(S+3)} = -\frac{K(S-1)}{S(S+3)} \text{ //...}$$

$$1+G(S)H(S) = 0$$

$$1+\left[\frac{-K(S-1)}{S(S+3)}\right] = 0$$

$$1-G(S)H(S) = 0$$

$$G(S)H(S) = 1$$

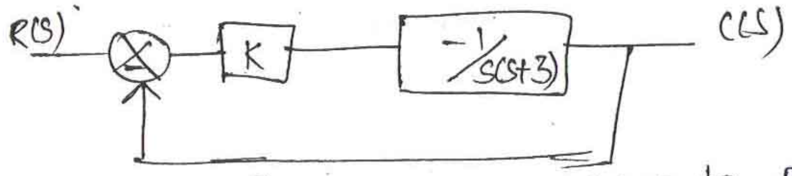
So we can not apply those rules to this function.

When ever the chareqn is $1-G(S)H(S)=0 \Rightarrow G(S)H(S)=1$ then we are not go for Normal RL rules.

when will $G(S)H(S) = -ve \Rightarrow$ for $-ve$ gain $-ve$ F.B (or) for $+ve$ gain $+ve$ F.B

In that situations we will draw complimentary (CRL) (or) Inverse (IRL) root locus.

Ex:-




now draw RL? for that we have to Draw complimentary RL. Since $G(S)H(S) = -ve$..

1) ANGLE condition:-

$$\boxed{\angle G(S)H(S) = 0^\circ = \pm[2q]\,180^\circ} \text{ //...}$$

2) MAGNITUDE condition:-

$$\boxed{|G(S)H(S)| = 1} \text{ //...}$$

Construction Rules of CRL :-

Rule NO: 1 :- The CRL is symmetrical about the real axis.

[ $G(s)H(s) = 1$ ].

Rule NO: 2 :- Same as RL.

Rule NO: 3 :- A point on real axis is said to be on CRL, if to the right side of this point the sum of openloop poles & zeros is even.

Rule NO: 4 :- Angle of Asymptotes :-

$$\theta = \frac{(2q\cancel{+1})180}{P-Z} = \boxed{\frac{(2q)180}{P-Z}\ ;\ q = 0, 1, 2\ldots}$$

Rule NO: 5 :- Centroid same as RL

Rule NO: 6 :- Break away Points same as RL.

Rule NO: 7 :- Intersection of of CRL with $j\omega$ axis same as RL.

Rule NO: 8 :- Angle of Departure / arrival.

$$\phi_D = 180 + \phi \qquad ; \text{ where } \phi = \Sigma\phi_z - \Sigma\phi_p$$
$$\phi_A = 180 - \phi$$

since $G(s)H(s) = -1$ → first angle is $180°$

NOW $G(s)H(s) = 1$ → angle is $0°$.

$$\boxed{\begin{aligned}\phi_D &= 0° + \phi \\ \phi_A &= 0° - \phi\end{aligned} \quad ; \phi = \Sigma\phi_z - \Sigma\phi_p}$$

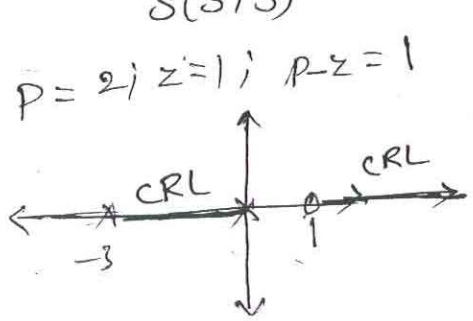
//....

Q :- $G(s) = \dfrac{K e^{-s}}{s(s+3)}$

Sol: $G(s) \simeq \dfrac{K(1-s)}{s(s+3)} = \dfrac{-K(s-1)}{s(s+3)}$

2) $P = 2$; $z = 1$; $P - z = 1$

3)




6) B.A.P :-

whenever one zero is present,

B.A.P ⇒ leftside in RL → prediction point

⇒ Right side in CRL.

4) $\theta = \frac{2(0)180}{} = 0°$ ....

| | | | |
|---|---|---|---|
| $s^4$ | 1 | $(1+K)$ | $4K$ |
| $s^3$ | 4 | $4+4K$ | 0 |
| $s^2$ | $\cancel{0}\ \varepsilon$ | $4K$ | 0 |
| $s^1$ | $\frac{(4+4K)\varepsilon-(4K)4}{\varepsilon}$ | 0 | 0 |
| $s^0$ | $4K$ | 0 | 0 |

condition for stability

$4K>0 \Rightarrow \underline{K>0}$

$$\frac{(4+4K)\varepsilon - 4(4K)}{\varepsilon} > 0$$

$$\Rightarrow \boxed{\varepsilon > \frac{16K}{4+4K}} //...$$

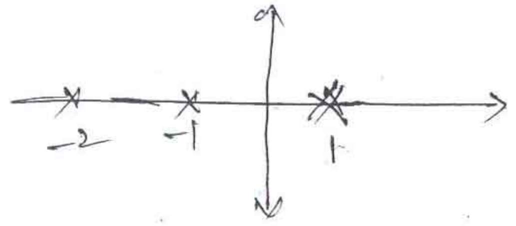
Q:4

LIMITATIONS & DIFFICULTIES in Routh Array:-

1. the co-efficients of characteristic Eqn polonomial must be real & positive.
2. NO:of sign changes of the first column of Routh Array determines the poles of in RHS of splane. But Not there locations.

[ suppose there are 2sign changes are occur, then we can say only 2 roots are on RHS of s plane, but where they are we dont know

3. When the first element of any Row is zero & the rest of the Row has atleast one Non zero term in such cases substitute small +ve Number '$\varepsilon$' inplace of zero and Evaluate Routh array incase interms of $\varepsilon$. check for sign changes by taking $\lim_{\varepsilon\to 0}$ for the first column Elements.
4. when Routh array Ends abruptly construct an auxilary Eqn A(s) & differentiated to get new co-efficients to complete the Routh array. check for multiplicity of the auxilary eqn roots on $j\omega$ axis comment on stability
5. when the system is marginally stable, to find the freqn of oscillations in the response the auxilary Eqn should be an even polomial of order 2.
6. Relative stability Analysis using Routh array Requires shifting of origin of s-plane more Negatively and is not feasible for higher order polomials.



system is unstable

# Root - Locus Technique

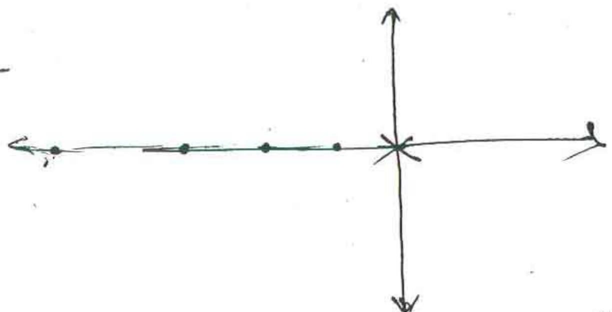
Root locus is defined as locus of closed loop poles obtained when system gain K is varied from 0 to $\infty$.

ex:- $G(S)H(S) = \frac{K}{S}$

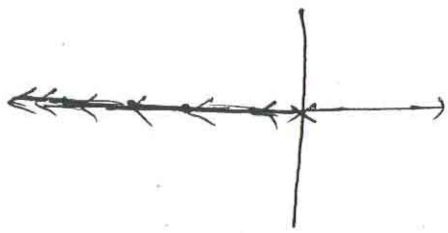
$1+G(S)H(S)=0 \Rightarrow S+K=0$

$S=-K$

| K | S=-K |
|---|---|
| 0 | 0 |
| 1 | -1 |
| 10 | -10 |
| 100 | -100 |
| ⋮ | ⋮ |
| $\infty$ | $-\infty$ |



Join all the points & the direction is mentioned below.



The information we can get from the root locus is relative stability.

In root locus we can find, what is the exact locations of the poles for a given values of Gain (K).

It is the most powerful technique to design a control system with variation of gain K.

Note:- [As the order of system increases] (or) since conventional method of drawing of root locus is very difficult. That's why we are going to construct some rules & then draw the root locus. For that we will take the char. Eqn. $1+G(S)H(S)=0 \Rightarrow G(S)H(S)=-1$

6) NO:of damped oscillations (or) NO:of cycles :-

$\omega_d = 2\pi f_d \Rightarrow \boxed{f_d = \frac{\omega_d}{2\pi} = \frac{\omega_d}{2\pi}}$ cycles/sec (or) Hz

NO:of cycles to reach 2% of Tolerence Band $\Rightarrow t_s \times f_d$

$$= \frac{4}{\zeta\omega_n} \times \frac{\omega_d}{2\pi}$$

NO:of cycles to reach 5% of T.B $\Rightarrow t_s \times \frac{\omega_d}{2\pi} = \frac{3}{\zeta\omega_n} \times \frac{\omega_d}{2\pi}$

7) Time period :-

$\boxed{T = \frac{1}{f_d}}$ sec

NOTE:-

As discussed earlier to reach the response to its final value (approximately) 5T time is required.

Let us discuss with some example. Suppose if you are taking the live example "Boiling of water or milk", when we will give step input then it will take 5T Time to heat completely.

Another Example is that, Temparature measurement by using Thermometer. when a step input is given then it will shows the final reading after 5T Time.

Time Response Analysis of Higher Order Systems :-

Still NOW, we have seen the second order systems. The standard second order system T.F is given as,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Char. eq.n $\Rightarrow s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

Ex:- $s^2 + 4s + 25 = 0$ find $\zeta$ & $\omega_n$?

NOW this can be compare with the standard sec. order system Char. eq.n. $\Rightarrow \boxed{\omega_n = 5}$

$2\zeta\omega_n = 4$

$\boxed{\zeta = \frac{4}{10}}$ 0.4

Now find $\xi$ & $\omega_n$ for $s^3+6s^2+8s+100=0$ ⇒ we can't find since how to compare this with second order system.

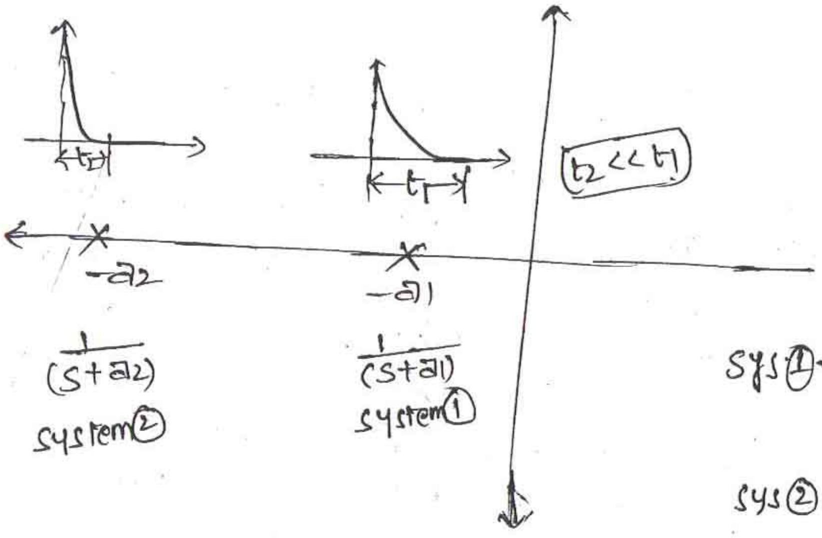
That means if 3rd order (or) more than 2nd order (any) is given we can't find $\xi$ & $\omega_n$. For that we have to follow the following Technique as shown below.

consider the third order char. eqn

$$s^3+ps^2+qs+K=0 \rightarrow (1)$$

This can be written as

$$(s+P_1)(s^2+q_1 s+K_1)=0 \rightarrow (2)$$




let us consider TWO systems System (1) & system (2) as shown in figure, where $a_1 << a_2$.

$$\text{sys (1)} \Rightarrow \frac{1}{s+a_1} = \frac{1}{a_1(1+\frac{s}{a_1})}$$

Time constant $t_1 = \frac{1}{a_1}$

$$\text{sys (2)} \Rightarrow \frac{1}{s+a_2} = \frac{1}{a_2(\frac{s}{a_2}+1)}$$

Time constant $t_2 = \frac{1}{a_2}$

since $a_2 >> a_1 \Rightarrow \boxed{t_2 << t_1}$

That means, the pole nearer to the origin will takes more time to reach final value (ss) state.

As the pole moves away from the origin, reaches quickly to the final value.

For the first system we can easily obtain the time response specifications ($t_p, t_r, t_d$ etc). But for the second system we can't calculate the time domine specifications just we can obtain the Transient response only i.e

$$c(t) = e^{-a_2 t}.$$

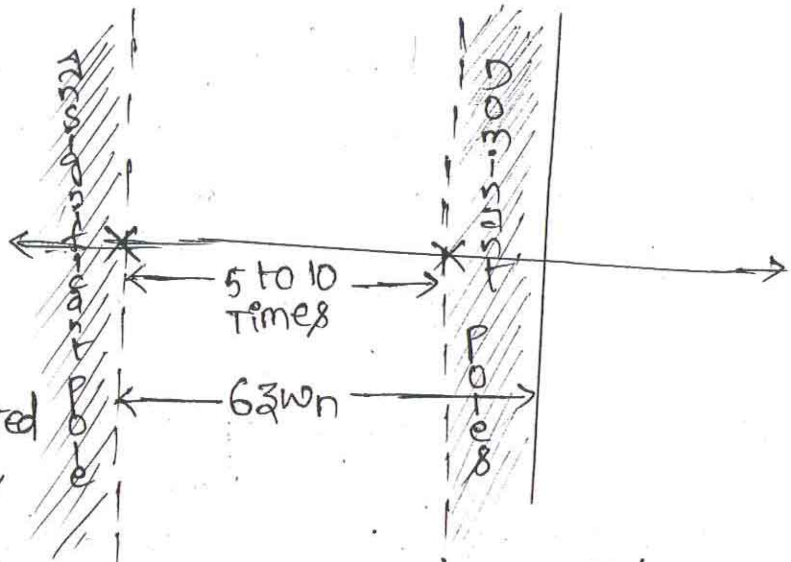
** The significance of first pole to calculate the Time response Analysis is high. ⇒ Significant Pole

The significant of the pole which is faraway from the origin, to calculate the time response Analysis is very less.
⇒ insignificant pole ...

* The poles which are nearer to the origin are called as "significant poles" (or) "slow poles" (or) "Dominant pole".

* The poles which are faraway from the origin are called as "insignificant poles" (or) "fast poles".

* To calculate the $\zeta$ & $\omega_n$ for 3rd order, Two poles should lie on Dominant region, and one pole should lie in the insignificant region. Then we can neglect the insignificant pole and then approximated to second order system, so that we can compare with standard 2nd order T.F, and then find $\zeta$ & $\omega_n$.




Here we get one doubt, if 2 poles are in Dominant region & the third pole is also lying in the Dominant region only then we can not neglect the 3rd pole. so that we can't approxi
so we can't find "$\zeta$" & "$\omega_n$".

for such systems we can find only the Time response '$c(t)$'. like $c(s) = \frac{A}{s+a_1} + \frac{B}{s+a_2} + \frac{C}{s+a_3} \Rightarrow c(t) \Rightarrow \mathcal{L}^{-1}(c(s)) = \underline{c(t)}$ ...

NOTE:-

A pole is called as insignificant pole, if it lies a distance from the Dominant pole region.

In otherwords, a insignificant pole may lies '$6\zeta\omega_n$' distance from the imaginary axis...

** The Time response Analysis of higher order system is obtained by approximating to second order system with respect to Dominant poles.

Time domine specifications obtained for approximated second order system are valid for higher order systems.

page 77 · Q:5: Ans: (a)

page 74 Q:22

$$T(S) = \frac{5}{(S+5)(S^2+S+1)} = \frac{\cancel{5}}{\cancel{5}\left(1+\frac{S}{5}\right)(S^2+S+1)}$$

$\left(1+\frac{S}{5}\right)$ → insignificant pole

$$= \frac{1}{S^2+S+1} //...$$

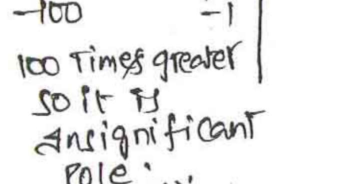
Note:- while Doing pole approximation Technique the T.F must be in Time domine form only i.e (1+ST) form.
(or) Time constant form

Note:- An approximating a higher order T.F to a Lower order T.F first convert T.F into time constant form and Eliminate the significant pole. Because the system gain of higher order and approximated lower order T.F should be same.

* Gain also plays a important role.
Gain should be same before & after Eliminating the insignificant pole:-

page 63 Q:14) T.F of the system $G(S) = \frac{100}{(S+1)(S+100)}$ for unit step i/p to the system the approximate settling time for 2%. criterion is !

Sol:- $$G(S) = \frac{100}{(S+1)(S+100)} = \frac{\cancel{100}}{\cancel{100}(1+S)\left(1+\frac{S}{100}\right)}$$




100 Times greater so it is insignificant pole....

$$= \frac{1}{1+S} //... \rightarrow \text{where } T=1$$

$$\therefore 2\% \ T.B \Rightarrow t_s = \frac{4}{\xi\omega_n} = \frac{4T}{1} = \frac{4(1)}{1} = 4\,sec$$

page 65 · Q:5 O.L.T.F $G(S) = \frac{K(S+2)}{S^3+\alpha S^2+4S+1}$, find K & α such that damping ratio is 0.2 & $\omega_n$ = 3 rad/sec.

Sol:- $$C.L.T.F = \frac{G(S)}{1+G(S)H(S)}$$

$$C.E \Rightarrow 1+G(S)H(S) = 0$$

$$1+\frac{K(S+2)}{S^3+\alpha S^2+4S+1} = 0$$

$s^3 + \alpha s^2 + (4+K)s + (2K+1) = 0 \rightarrow (1)$

$(s+a)(s^2+bs+c) = 0$

$\omega_n^2 = c = 9 \Rightarrow \omega_n = 3$ ...

$2\zeta\omega_n = b$

$2 \times 0.2 \times 3 = 1.2 = b$ ...

$(s+a)(s^2+1.2s+9) = 0$

$s^3 + s^2[a+1.2] + s[9+1.2a] + 9a = 0$ $\rightarrow (2)$

compare eq (1) & (2)

$\alpha = a + 1.2 \rightarrow (3)$

$4+K = 9+1.2a \rightarrow (4)$

$2K+1 = 9a \Rightarrow \frac{2K+1}{9} = a \rightarrow (5)$

$(4) \Rightarrow 4 + \left(\frac{2K+1}{9}\right) = 9 + 1.2\left(\frac{2K+1}{9}\right)$

$\Rightarrow \boxed{K = 7}$

$\Rightarrow a = \frac{14+1}{9} = \frac{15}{9} = 1.66$

$\alpha = a + 1.2$

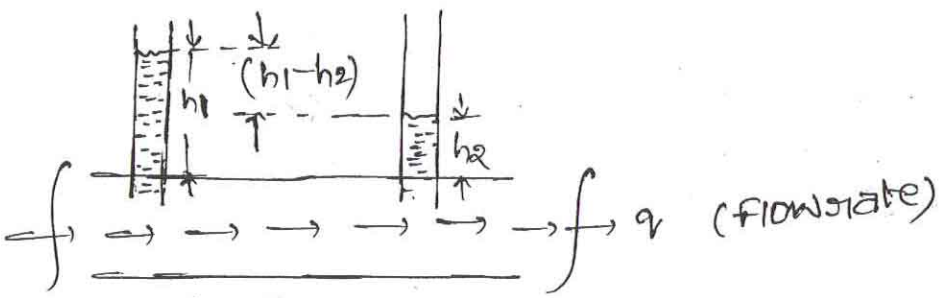
$\alpha = 1.66 + 1.2 = 2.86$

## TRANSFER FUNCTIONS FOR Physical Systems :-

The physical system contains 5 Basic Elements. They are

1) Resistance
2) capacitance
3) Time constant (R & C)
4) oscillatory → already discussed in underdamped
5) Dead line → will discuss in root locus.

### 1) Resistance Type Process :-




pipe line ....

Due to friction in the pipeline $h_2$ is less than $h_1$.

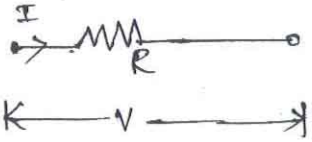
$h \propto q$

$\underline{h = Rq}$ where R= Hydraulic Resistance...

$H(S) = RQ(S)$

$$\boxed{\frac{H(S)}{Q(S)} = R} \; //\ldots$$

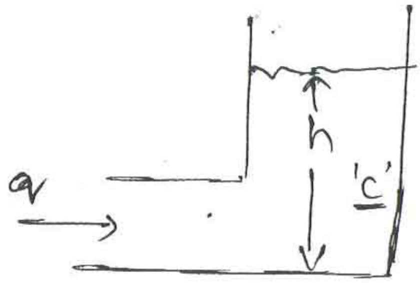
At is analogous to "electrical resistance".




$V = I \cdot R$

$$\boxed{\frac{V(S)}{I(S)} = R \; //\ldots}$$

2) Capacitance process :-




q= flow into the tank
h= hight of the tank
c = Hydraulic capacitance (Area/volume)

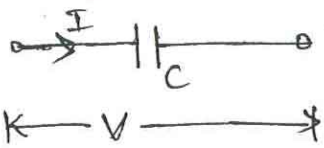
change in flowrate causes the change ine the hight level of tank.

$q \propto \frac{dh}{dt}$

$q = c \cdot \frac{dh}{dt} \ldots \Rightarrow Q(S) = C \cdot S \cdot H(S)$

$$\boxed{\frac{H(S)}{Q(S)} = \frac{1}{CS}} \; //\ldots$$

At is analogous to electrical capacitance.




$I = C\frac{dv}{dt}$

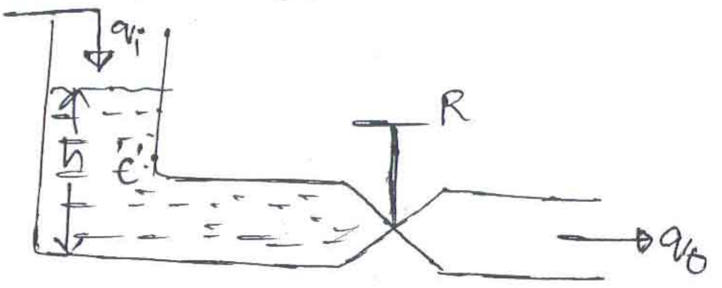
$I(S) = C \cdot S \cdot V(S)$

$$\boxed{\frac{V(S)}{I(S)} = \frac{1}{CS}} \ldots$$

* An order to analyse the larger tanks, we can go for the electrical capacitance. since at is analogous to the Hydraulic capacitance.

Ex:- Boiler tank.

3) Time Constant Process :—




Amount of water stored in tank $\propto$ net flowrate.

$$\frac{dh}{dt} \propto (q_i - q_o)$$

$$(q_i - q_o) = C \cdot \frac{dh}{dt}$$

$h = q_o \cdot R$

but $q_o = \frac{h}{R}$

$$q_i = C \cdot \frac{dh}{dt} + q_o$$

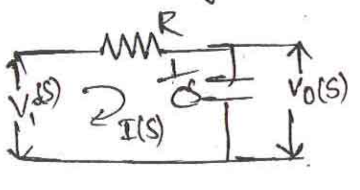
$$q_i = C \cdot \frac{dh}{dt} + \frac{h}{R}$$

$$R \cdot q_i = RC \cdot \frac{dh}{dt} + h$$

$$R \cdot Q_i(S) = RC \cdot S\,H(S) + H(S)$$

$$\Rightarrow \boxed{\frac{H(S)}{Q_i(S)} = \frac{R}{1+RCS}}$$

It is Analogous to RC Network.




$$V_i(S) = I(S)\left[R + \frac{1}{CS}\right]$$

$$V_o(S) = I(S) \cdot \frac{1}{CS}$$

$$\Rightarrow \frac{V_o(S)}{V_i(S)} = \frac{1/CS}{R + 1/CS} = \frac{1}{1+RCS}$$

$$\boxed{\therefore \frac{V_o(S)}{V_i(S)} = \frac{1}{1+RCS}}$$

Any practical system which is combination of R & C parameters will comes under "first order system".

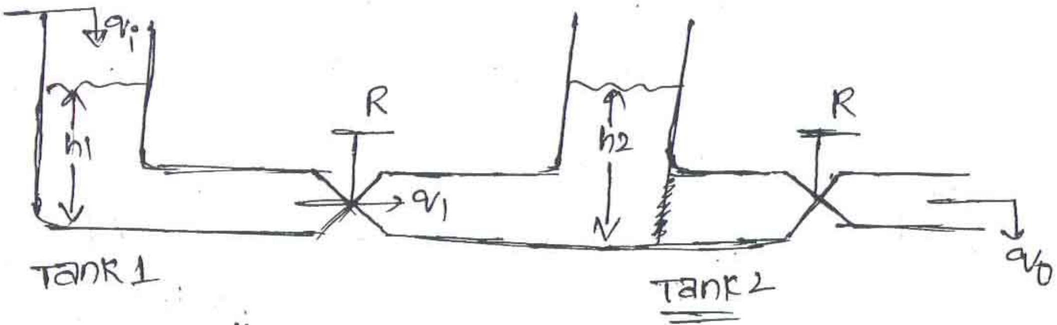
Ex:— water tank.

In the water Tank, the stored water will be analogous to capacitance & valve will be analogous to resistance, so it is first order system.

To study the loading effect b/w the two tanks, we can connect the two tanks in two configurations.

1. Interacting system
2. Non - Interacting system.

1) Interacting system :-




$$q_i - q_1 = \frac{c\,dh_1}{dt}$$

$$Q_i(S) - Q_1(S) = H(S) \cdot C \cdot S \rightarrow (1)$$

$$h_1 - h_2 = R q_1$$

$$H_1(S) - H_2(S) = R Q_1(S) \rightarrow (2)$$

$$q_1 - q_0 = \frac{c \cdot dh_2}{dt}$$

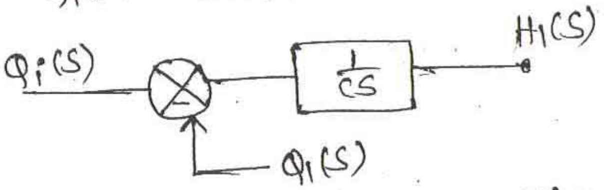
$$Q_1(S) - Q_0(S) = C \cdot S \cdot H_2(S) \rightarrow (3)$$

$$h_2 = R \cdot q_0$$

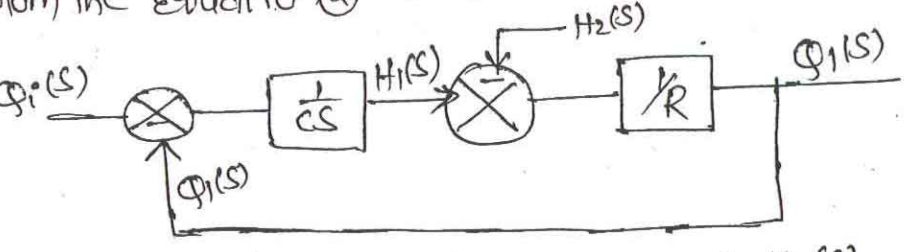
$$H_2(S) = R \cdot Q_0(S) \rightarrow (4)$$

From the first eqn.

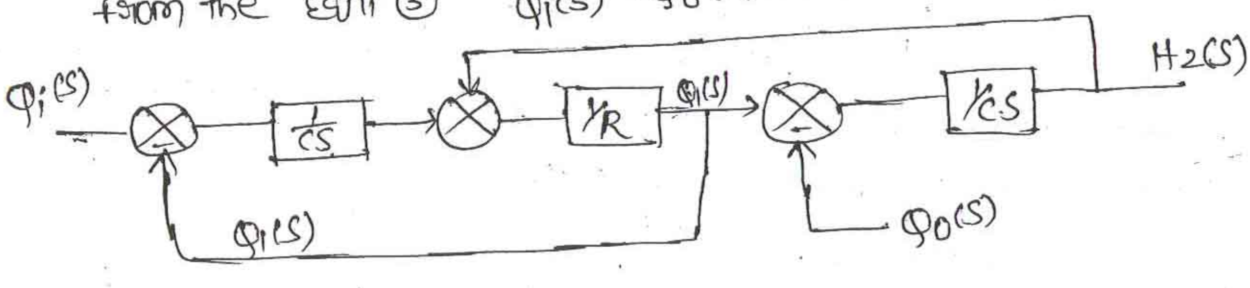
$Q_i(S)$ = overall input.




From the Equation (2) $\Rightarrow H_1(S) - H_2(S) = R Q_1(S)$

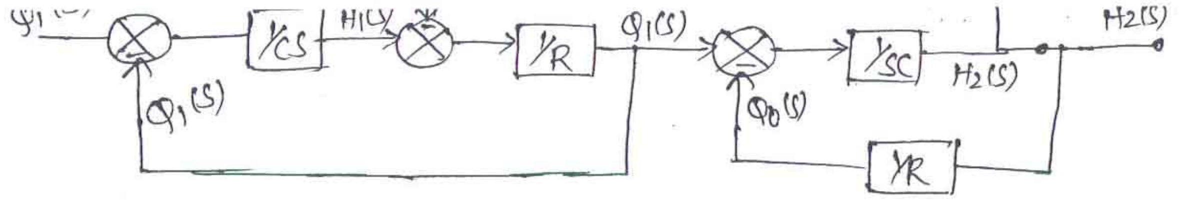



from the Eqn (3) $Q_1(S) - Q_0(S) = C \cdot S \cdot H_2(S)$




From the eqn (4) $H_2(S) = R Q_0(S)$

this can be shown as below.



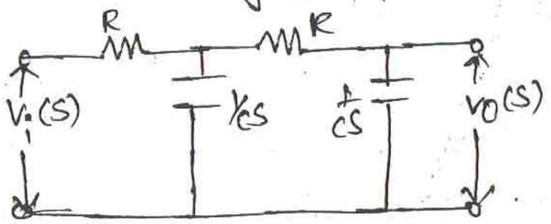
$$\frac{H_2(s)}{Q_i(s)} = \frac{\frac{1}{RC^2S^2}}{1-\left[-\frac{1}{RCS}-\frac{1}{RCS}-\frac{1}{RCS}\right]+\frac{1}{R^2C^2S^2}}$$

$$\frac{H_2(s)}{Q_i(s)} = \frac{\frac{1}{RC^2S^2}}{\frac{R^2C^2S^2+3RCS+1}{R^2C^2S^2}} = \frac{R}{R^2S^2C^2+3RCS+1}$$

* Roots are real & unequal. so over damping system.

An interacting system having identical time constants represents over damped system.

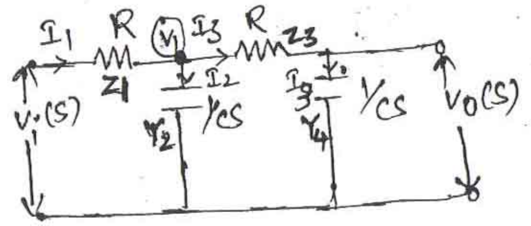
It is Analogous to:



* this N/W can be Analysed by ladder N/W model. To find the T.F advantage of this model is current through $\frac{1}{CS}$ & another $\frac{1}{CS}$ branches are can be directly findout.

* when ever two N/Ws are connected directly, then the overall T.F will never be the product of individual T.Fs like $\left(\frac{1}{RCS+1}\right)\left(\frac{1}{RCS+1}\right)$.

since they are interacting systems, the loading effects present on them.



Current through $Y_4 \Rightarrow I_3$.

$I_3 = V_0 Y_4$

$\Rightarrow I_3 = V_0 \cdot CS$

To find $V_1$: $I_3 = \frac{V_1 - V_0}{Z_3} \Rightarrow V_1 = I_3 Z_3 + V_0$

$V_1 = V_0[1+RCS]$

Now $I_2 = V_1 \cdot Y_2$

$= V_0(1+RCS) \cdot CS$

$= V_0 (CS + RC^2S^2)$

Current through '$Z_1$'

$I_1 = I_2 + I_3$

$= V_0[CS + RC^2S^2] + V_0 \cdot CS$

$I_1 = V_0[2CS + RC^2S^2]$

To find $V_i$ :-

$V_i = I_1 Z_1 + V_1$

$V_i = V_0(2CS + RC^2S^2)R + V_0(1+RCS)$

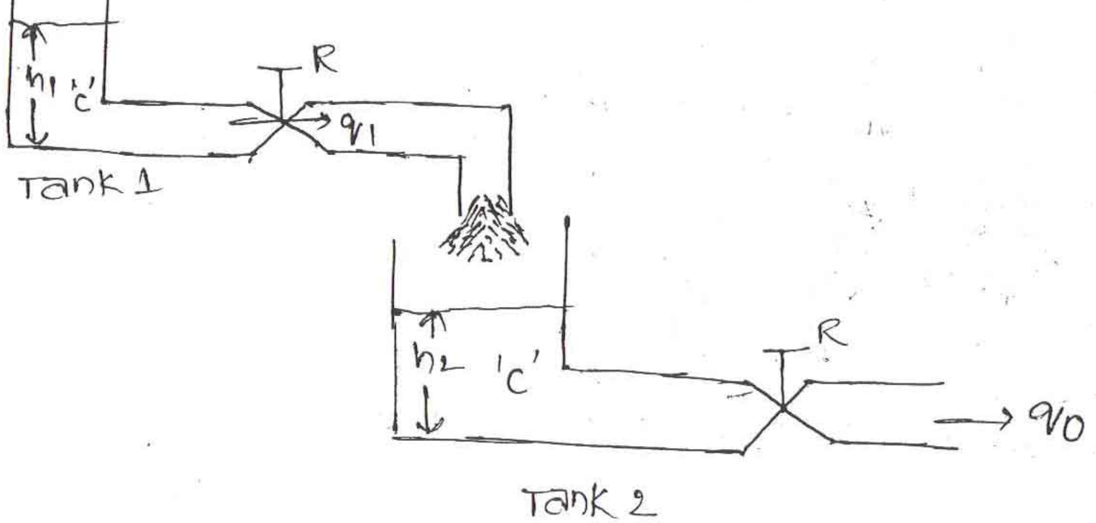
$V_i = V_0(3RCS + R^2C^2S^2 + 1]$

$$\boxed{\frac{V_0(S)}{V_i(S)} = \frac{1}{R^2C^2S^2 + 3RCS + 1}}$$

* Gain in the both systems are not same. but ~~the~~ Time-constants are same.

Non - Interacting systems :-

Representation :-




The changes in '$h_1$' causes the change in $h_2$. But the changes in Tank '$h_2$' never causes the change in '$h_1$'

that means there is no interaction. Here the interaction means that both tanks must be mutually dependent one with another. But here $h_2$ depends on $h_1$, but $h_1$ will not be depends on $h_2$. So it is Non-interacting.

* No loading is present b/w them.

Tank-1:-

$$q_i - q_1 = C\frac{dh_1}{dt}$$

$$q_i = C\frac{dh_1}{dt} + q_1$$

$$q_1 = h_1/R$$

$$q_i = C\cdot\frac{dh_1}{dt} + \frac{h_1}{R}$$

$$R\,Q_i(S) = ((SC)R + 1)\,H_1(S) \quad \rightarrow (1)$$

Tank-2:

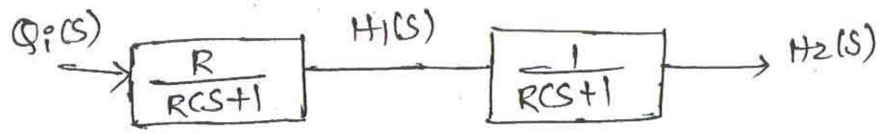
$$q_1 - q_0 = C\frac{dh_2}{dt}$$

$$q_1 = C\frac{dh_2}{dt} + q_0 \qquad \left(\because q_0 = \frac{h_2}{R}\right)$$

$$\frac{h_1}{R} = C\frac{dh_2}{dt} + \frac{h_2}{R}$$

$$h_1 = RC\frac{dh_2}{dt} + h_2$$

$$H_1(S) = [1 + RCS]\cdot H_2(S) \quad \rightarrow (2)$$




$$\boxed{\therefore \frac{H_2(S)}{Q_i(S)} = \frac{R}{(RCS+1)^2}}$$

that is Roots are real & equal. Time constants are exactly same but practically it is not possible, Because we can't made exactly two identically things.

In practical situations the Non-interacting systems i.e critically damped systems never exist, we can just imagin only not seen practically, since we can't made two systems with identical parameters (or) with identical time-constants. For example if we are consider two Tanks it is not possible to maintain the height of the water level i.e area/volume (C) by controlling valves (R) of tank 1 & tank (2).

Note:-

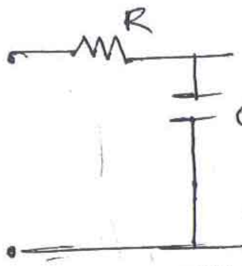
* Non interacting system having identical time constants represents a critically damped system.

page 62: Q: 62

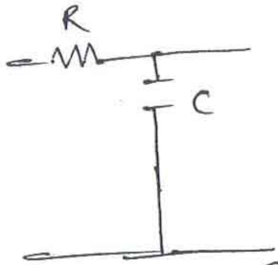
Ans: (d)

Analogous to it is:

If we connecting RC N/Ws cascading, it behaves as Non-interacting system; (so that we can obtain the T.F of resultant cascading N/w is $\frac{1}{(RCS+1)^2}$).



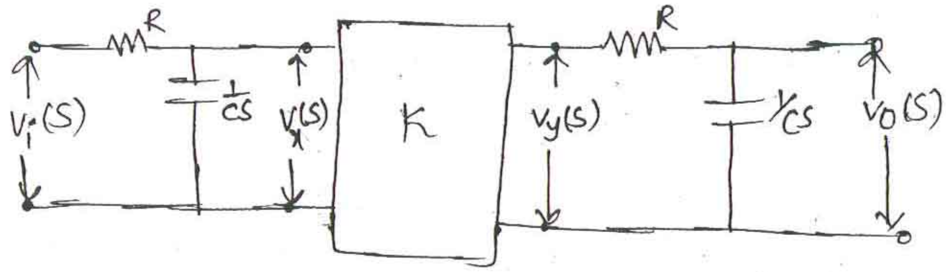

One RC N/W



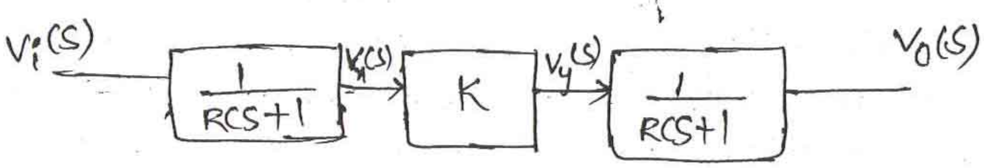

Another RC N/W.

Now we don't connect both N/Ws directly; since if we are connecting, then they will be come interacting systems.

Now for making both the N/w as non-interacting systems connect both the systems by using a coupling element, which will isolates both the N/ws.




$$\frac{V_x(S)}{V_i(S)} = \frac{1}{RCS+1} \qquad \frac{V_y(S)}{V_x(S)} = K \qquad \frac{V_o(S)}{V_y(S)} = \frac{1}{RCS+1}$$




$$\boxed{\therefore \frac{V_o(S)}{V_i(S)} = \frac{K}{(RCS+1)^2}}$$

where 'K' is coupling element. So that they can be Non interacting systems.

page 62

Q:3 which one of the following is the response $y(t)$ of causal LTI system described by $H(s) = \frac{s+1}{s^2+s+1}$ for given i/p $x(t) = e^{-t}u(t)$.

sol:-

$$\frac{Y(s)}{X(s)} = H(s) = \frac{s+1}{s^2+s+1}$$

$x(t) = e^{-t}u(t)$
$\Rightarrow X(s) = 1/s+1$

$$\therefore Y(s) = \frac{\cancel{(s+1)}}{s^2+s+1} \times \frac{1}{\cancel{(s+1)}}$$

$$Y(s) = \frac{1}{s^2+s+1} = \frac{1}{(s+1)^2+1^2}$$

$$y(t) = e^{-t}\sin t\,u(t)$$

$$y(t) = e^{-t}\sin t\; u(t)$$

Q:4

$$G(s) = \frac{10}{s^2+14s+50}$$

sol:

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\cdot \frac{1}{\cancel{s}}}{1+\frac{10}{s^2+14s+50}} = \frac{1}{1+\frac{10}{50}} = \frac{5}{6} = 0.83$$

Q:5

sol:

$L\{c(t)\} = T.F$

$$\Rightarrow T.F = \frac{12.5\times 8}{(s+5)^2+8^2}$$

$$C.E \rightarrow (s+5)^2+8^2 = 0 \Rightarrow s^2+10s+89=0$$

$\omega_n = \sqrt{89} \cong 10$ rad/sec

$2\zeta\times 10 = 10$

$\Rightarrow \zeta = 0.5 \cong 0.6$

Q:8

$$G(s) = \frac{k}{s(s+4)} \; ; \; \zeta = 0.5$$

$1+G(s)H(s) = 0 \Rightarrow s^2+4s+k=0$

$\omega_n = \sqrt{k}$

$2\zeta\omega_n = 4$
$2(0.5)\sqrt{k} = 4$ $\Rightarrow$ $\boxed{k=16}$

page 63

Q:10

$$G(s) = \frac{H(s+c)}{s(s+a)(s+b)}$$

(i) $u(t) \Rightarrow$ O/P $= 2+De^{-t}+Ee^{-3t}$

(ii) $e^{-2t}u(t) \Rightarrow Fe^{-t}+Ge^{-3t}$

sol:-

$$T.F = \frac{H(s+c)}{s(s+a)(s+b)} = \frac{k_1}{s}+\frac{k_2}{s+a}+\frac{k_3}{s+b}$$

$$= 2+De^{-t}+Ee^{-b}$$

$\Rightarrow a=1;\; b=3$

Now

$$\lim_{s\to 0} \cancel{s}\cdot\frac{H(s+c)}{\cancel{s}(s+a)(s+c)} = \lim_{t\to\infty} 2+De^{-t}+Ee^{-t}$$

$\frac{HC}{ab} = 2 \Rightarrow HC = 2[1*3] = 6$

(ii) $e^{-2t}u(t)$:—  T.F $= \frac{H(\cancel{s+c})}{(\cancel{s+2})(s+a)(s+b)}$ $\Rightarrow Ee^{-t} + Ee^{-3t}$

$\boxed{c=2}$ $\Rightarrow H=3$....

Q: 11

Sol:-

unit step response

$$c(t) = 1 - e^{-5t} - 5te^{-5t}$$

$$\Rightarrow C(s) = \frac{1}{s} - \frac{1}{s+5} - \frac{5}{(s+5)^2}$$

$$T.F = \frac{(s+5)^2 - s(s+5) - 5s}{s(s+5)^2} \Big/ \frac{1}{s}$$

$$\Rightarrow T.F = \frac{25}{s^2+10s+25} \text{ //...}$$

Q: 12

Sol:-

$$T.F = \frac{16}{s^2+4s+16}$$

$$\left.\begin{matrix} \omega_n = 4 \\ \zeta = 0.5 \end{matrix}\right\} \Rightarrow t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{2\sqrt{3}} \text{ sec //...}$$

Q: 16

Sol:-

$H(s) = \frac{1}{s+2}$ excited by $10u(t)$, the time at which the o/p reaches 99% of steady state value.

Sol:

$$c(s) = \frac{10}{s(s+2)} = \frac{5}{s} - \frac{5}{s+2} = 5[1-e^{-2t}]$$

steady state value is '5'.

99% of 5 is 4.95

$$5[1-e^{-2t}] = 4.95$$

$$\Rightarrow t = 2.3 \text{ sec}$$

Q: A certain first order system initially at rest & subjected to sudden input at $t=0$, at response reaches 1.1V in 4sec. & eventually reaches the steady state of 2V. find the Time constant?

sol:

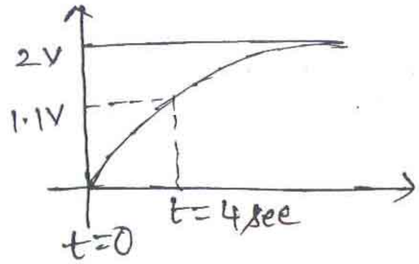
$$c(t) = K(1-e^{-t/T})$$

At $t = 4$ sec, $c(t) = 1.1V$ ....

$$1.1 = 2\left[1-e^{-4/T}\right]$$

$$\Rightarrow e^{-4/T} = 0.45$$

$$\Rightarrow T = 5 \text{ sec} ...$$




Q: The thermometer having first order dynamics of is subjecte to a sudden Temparature change of 30° to 150°C. If it has the Time constant of 5 sec. What is the Temp will be initiated by after 5 sec.

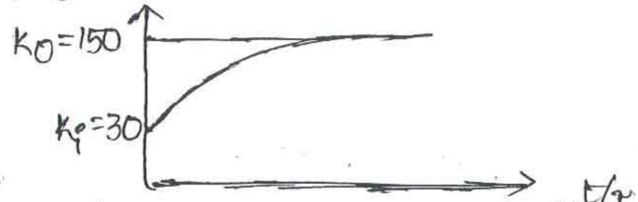
sol:-

$$x_o(t) = K_o + [K_i - K_o]e^{-t/T}$$

$$= 150 + [30-150]e^{-5/5}$$

$$= 150 + (-120)e^{-1}$$

$$= ....$$




note:-
$x_o$ = Final value + {initial - Final} $e^{-t/T}$ .....

Q:(17)

sol:-

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dx} + 2y = x(t)$$

$$\frac{Y(S)}{X(S)} = \frac{1}{s^2+3s+2}$$

$$x(t) = 2u(t)$$
$$X(S) = 2/S ...$$

$$\Rightarrow Y(S) = \frac{2}{S(S^2+3S+2)}$$

$$= \frac{A}{S} + \frac{B}{S+1} + \frac{C}{S+2}$$

$$= \frac{1}{S} - \frac{2}{S+1} + \frac{1}{S+2} = \left[1-2e^{-t}+e^{-2t}\right]u(t) ...$$

Q:(20)

$$4\frac{d^2y}{dt^2} + 8\frac{dy}{dt} + 16y(t) = 16u(t)$$

sol:

$$T.F = \frac{Y(S)}{U(S)} = \frac{16}{4s^2+8s+16} = \frac{4}{s^2+2s+4}$$

$$\omega_n = 2;\ 2\xi\omega_n = 2 \Rightarrow \xi = 0.5$$

Q: 18

sol:-

Type (1):

$$e_{ss} = \frac{A}{K}$$

let $A=1$

$$e_{ss} = \frac{1}{K} = 5\% = \frac{5}{100} = 1/20$$

$$\Rightarrow \boxed{K=20}$$

Q: (20) Type-0 :-

$$e_{ss} = \frac{1}{1+K_p} = 0.2 \xrightarrow{1/s}$$

$\Rightarrow K_p = 4$

Type-1 :

$$e_{ss} = \frac{1}{K} = \frac{1}{4} = 0.25 \ldots$$

Q: (21)

$$G(S) = \frac{10}{S^2(4+S)}$$

Type-2 :- so $e_{ss} = \frac{A}{K_a}$

$$K_a = \lim_{S \to 0} \cancel{S^2} \cdot \frac{10}{\cancel{S^2}(S+4)} = \frac{10}{4} = 2.5$$

$$e_{ss} = \frac{8 \times 4}{10} = 3.2 \text{ units}$$

(for $\frac{At^2}{2}$ ; $e_{ss} = \frac{A}{K}$

for $\cdot At^2$ ; $e_{ss} = \frac{2A}{K}$

$= \frac{2 \times 4}{10/4}$

$= 3.2$)

Q: (23)

$$r(t) = (1-t^2) \cdot 3u(t)$$

$$R(S) = \frac{3}{S} - \frac{6}{S^3}$$

$$e_{ss} = \frac{3}{1+K_p} - \frac{6}{K_A} \ldots$$

Q: (19)

Sol:- $K_V = 1000$; $\zeta = 0.5$

$$G(S) = \frac{100(K_p + K_D S)}{S(S+10)}$$

$$K_V = \lim_{S \to 0} S \cdot G(S)$$

$$1000 = \lim_{S \to 0} \cancel{S} \cdot \frac{100(K_p + K_D S)}{\cancel{S}(S+10)}$$

$$1000 = \frac{100 K_p}{10} \Rightarrow K_p = 100 \text{ rad/sec} \ldots$$

$$1 + G(S)H(S) = 0$$

$$1 + \frac{100(K_p + K_D S)}{S(S+10)} = 0$$

$$s^2 + S(10 + 100K_D) + 100K_p = 0$$

$$s^2 + S(10 + 100K_D) + (100 \times 100) = 0$$

$\Rightarrow \omega_n = 100 \text{ rad/sec} \ldots$

$$2\zeta\omega_n = 10 + 100K_D \Rightarrow 2 \times 0.5 \times 100 = 10 + 100K_D$$

$$\Rightarrow K_D = 0.9 \text{ rad/sec} \ldots$$

Q: (22)

$$\frac{1}{6} e^{-0.8t} \sin(0.6t)$$

Sol:-

$$T.F = \frac{\frac{1}{6} * 0.6}{(S+0.8)^2 + (0.6)^2} = \frac{0.1}{S^2 + 1.6S + 1} \ldots$$

$$C.E \rightarrow S^2 + 1.6S + 1 = 0$$

$\omega_n = 1 \text{ rad/sec}$

$\zeta = 0.8$

ⓒ with out 'a' keeping % Mp as 1.5, what manipulation must be done in the system, to reduce ess to 0.25.

$$G(s) = \frac{K}{s(s+6.4)}$$

$$0.25 = \lim_{s \to 0} \frac{s \cdot 1/s^2}{1 + \frac{K}{s(s+6.4)}}$$

$$0.25 = \frac{6.4}{K} \Rightarrow \boxed{K = 25.6}$$

= x =

(4)
sol:-

$$G(s) = \frac{K}{s(sT+1)}$$

$$C.E \rightarrow 1 + G(s)H(s) = 0$$

$$Ts^2 + s + K = 0$$

$$s^2 + \frac{1}{T} \cdot s + \frac{K}{T} = 0$$

$$\Rightarrow \omega_n = \sqrt{\frac{K}{T}}$$

$$\zeta = \frac{1}{2\sqrt{KT}}$$

ⓐ $\% M_p = 40\% \Rightarrow M_p = 0.4$

$$e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.4$$

$$\Rightarrow \zeta = 0.28 = \zeta_1$$

let $K = K_1$

ⓑ $\% M_p = 60\% \Rightarrow M_p = 0.6$

$$e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.6$$

$$\Rightarrow \zeta = 0.16 = \zeta_2 = 0.16$$

let $K = K_2$

$$\therefore \frac{\zeta_1}{\zeta_2} = \frac{0.28}{0.161} = \frac{1/2\sqrt{K_1T}}{1/2\sqrt{K_2T}} = \sqrt{\frac{K_2}{K_1}}$$

$$\therefore \frac{K_2}{K_1} = \left(\frac{0.28}{0.16}\right)^2 \Rightarrow K_2 \cong 3 \text{ times of } K_1$$

= x =

PART - III

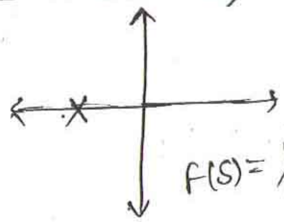
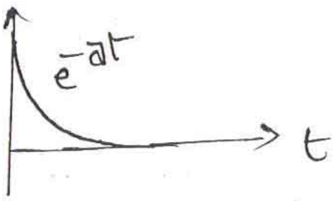
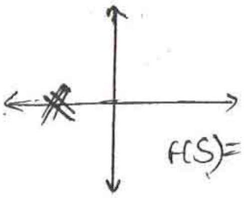
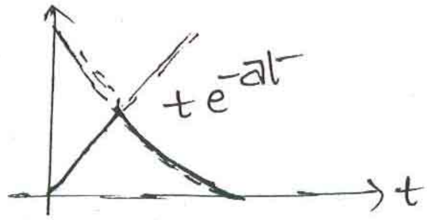
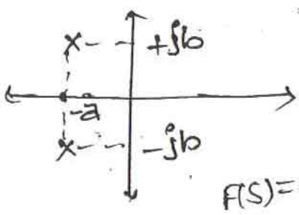
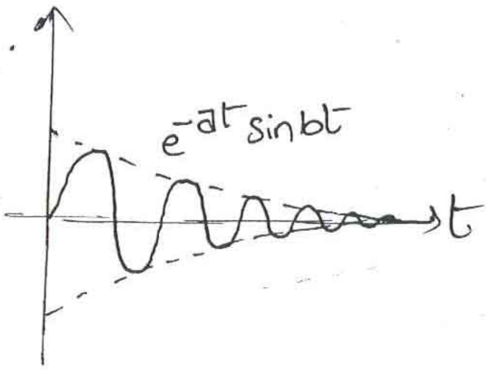
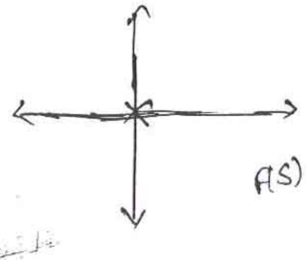
## STABILITY IN TIME DOMINE

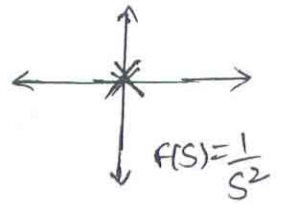
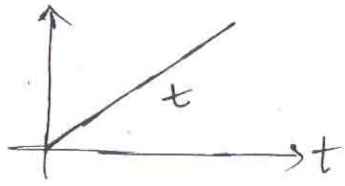
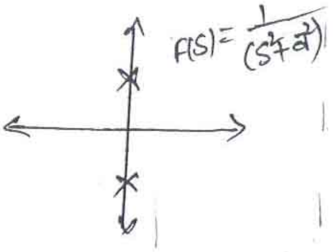
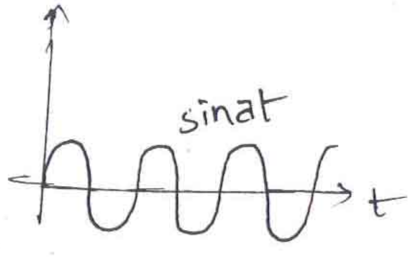
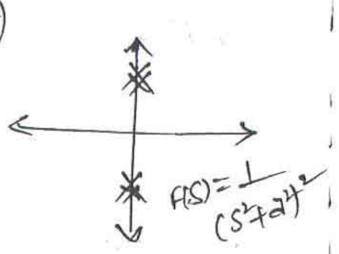
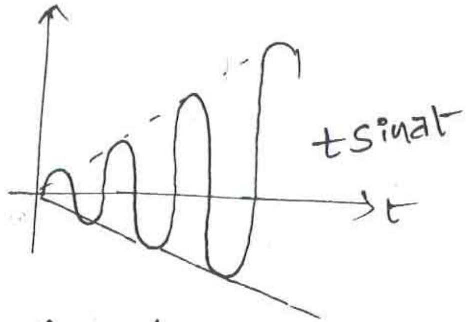
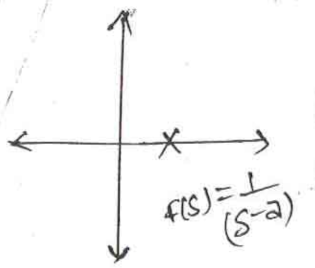
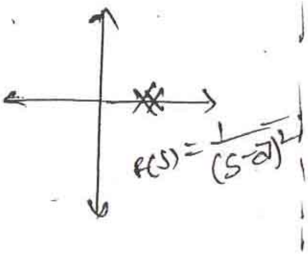
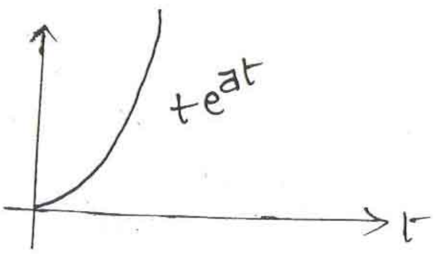
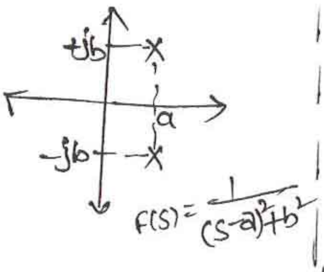
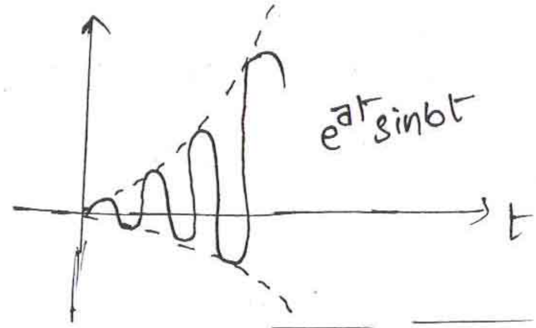
The stability of L.T.I system may be obtaine defined as when the system is subjected to bounded input, the output should be bounded.

Bounded input Bounded output implies, the impulse Response of the system should tend to zero as time 't' approaches to infinity i.e closed loop poles.

note:- when any system which is subjected to sudden disturbance then the system is said to be stable only when it recovers to its original state.

Impulse Response of stability :-

| closed loop pole locations | Stability criteria | Impulse Response |
|---|---|---|
| 1)  $F(S) = 1/s+a$ | Absolutely stable |   |
| 2)  $F(S) = \frac{1}{(s+a)^2}$ | Absolutely stable |   |
| 3)   $F(S) = \frac{1}{(s+a)^2+b^2}$ | Absolutely stable |   |
| 4)  $F(S) = \frac{1}{s}$ | Critically (or) Marginally stable |   |

| | | |
|---|---|---|
| 5)  | unstable |  |
| 6)  | critically (or) limited (or) marginally stable |  |
| 7)  | unstable |  |
| 8)  | unstable |  |
|  | unstable |  |
|  | unstable |  |

* As the order of the system increases, finding the location of poles will become difficult. So we will go for one simple method i.e Routh Hurwitz criteria.

## Routh Hurwitz criteria :-

$P(S) = S^4 + 8S^3 + 18S^2 + 16S + 5 = 0$

**Routh array :-**

| | | | |
|---|---|---|---|
| $S^4$ | 1 | 18 | 5 |
| $S^3$ | 8 | 16 | 0 |
| $S^2$ | $b_1 = 16$ | $b_2 = 5$ | 0 |
| $S^1$ | $c_1 = 13.5$ | 0 | 0 |
| $S^0$ | $d_1 = 5$ | 0 | 0 |

$$b_1 = \frac{8\times18 - 1\times16}{\boxed{8}} = 16$$

$$b_2 = \frac{8\times5 - 1\times0}{\boxed{8}} = 5$$

$$c_1 = \frac{16\times16 - 8\times5}{\boxed{16}} = 13.5$$

$$d_1 = \frac{13.5\times5 - 16\times0}{\boxed{13.5}} = 5$$

In order to examine the stability of the system, we have to consider the first column elements; If all the first column elements are positive then the system is said to be stable.

Now the question is why we have interested in first column elements for examine the stability, why don't consider the second column elements.

Answer :- They are roots of polonomial ✗
They are coefficients of polonomial ✗

** The magnitude and sign of any other elements depends on the first column elements only. Because we are dividing with first colum elements.

(Pb) $P(S) = S^5 + S^4 + 2S^3 + 2S^2 + 3S + 15$

| | | | |
|---|---|---|---|
| $S^5$ | 1 | 2 | 3 |
| $S^4$ | 1 | 2 | 15 |
| $S^3$ | 0 | -12 | |
| $S^2$ | | | |
| $S^1$ | | | |
| $S^0$ | | | |

So we can't continue the Routh array.

Now let us consider '$\varepsilon$' which is a small +ve Number. replace '0' by $\varepsilon$, and then continue the procedure.

| | | | | |
|---|---|---|---|---|
| + $s^5$ | 1 | 2 | 3 | |
| + $s^4$ | 1 | 2 | 15 | |
| + $s^3$ | $\epsilon$ (+ve) | $-12$ | 0 | |
| + $s^2$ | $\frac{2\epsilon+12}{\epsilon}$ (+∞) | 15 | 0 | |
| − $s^1$ | $\frac{-24\epsilon-144-15\epsilon^2}{2\epsilon+12}$ (−ve) | 0 | 0 | |
| + $s^0$ | 15 | 0 | 0 | |

$$\lim_{\epsilon\to 0} \frac{2\epsilon+12}{\epsilon} = +\infty$$

$$\lim_{\epsilon\to 0} \frac{-24\epsilon-144-15\epsilon^2}{2\epsilon+12} = -12$$

since there are TWO sign changes. TWO poles are in the right half of s-plane.

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Q: 2 $s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0$

sol:

| | | | | |
|---|---|---|---|---|
| $s^6$ | 1 | 8 | 20 | 16 |
| $s^5$ | 2 | 12 | 16 | 0 |
| $s^4$ | 2 | 12 | 16 | 0 → Row of Auxilary equation. |
| $s^3$ | 0 | 0 | 0 | 0 → Row of zero |
| $s^2$ | | | | |
| $s^1$ | | | | |
| $s^0$ | | | | |

Now A.E ⇒ $2s^4+12s^2+16=0$

$\frac{dA(s)}{ds} = 8s^3+24s=0$

| | | | | |
|---|---|---|---|---|
| $s^6$ | 1 | 8 | 20 | 16 |
| $s^5$ | 2 | 12 | 16 | 0 |
| $s^4$ | 2 | 12 | 16 | 0 |
| $s^3$ | 8 | 24 | 0 | 0 |
| $s^2$ | 6 | 16 | 0 | 0 |
| $s^1$ | 2.6 | 0 | 0 | 0 |
| $s^0$ | 16 | | | |

Whenever the Routh array is abruptly becomes zero i.e one row becomes completely zero. Then construct an Auxilary equation $A(s) = 2s^4+12s^2+16 \Rightarrow \frac{dA(s)}{ds} = 8s^3+24s=0$

* the first column elements are positive. then also we can't say the system is marginally stable (or) stable. then also we can get a chance to become system unstable.

i.e $2s^4+12s^2+16$ ⇒ roots of Auxilary equation.

A(s) = closed loop poles lying on jω axis.

If these roots are repeated then the system is unstable.

$$\frac{-12 \pm \sqrt{144-(8\times16)}}{4} = -2, -4$$

$(s^2+2)(s^2+4)$ $s=\pm j2;\ s=\pm j4$

Q:10

| | | | | |
|---|---|---|---|---|
| $s^5$ | 2 | 4 | 2 | |
| $s^4$ | 1 | 2 | 1 | ← Row of A.E |
| $s^3$ | $\cancel{0}$ 4 | $\cancel{0}$ 4 | $\cancel{0}$ 0 | ← Row of zero |
| $s^2$ | 1 | 1 | 0 | ← Row of A.E |
| $s^1$ | $\cancel{0}$ 2 | $\cancel{0}$ 0 | 0 | ← Row of zero |
| $s^0$ | 1 | 0 | 0 | |

Here all the first column elements are +ve. so the system is stable. But this statement is wrong. How it will be we will see.

$$A.E_1 = s^4 + 2s^2 + 1$$

$$(s^2+1)^2 = 0 \Rightarrow s^2 = -1 \; ; \; s^2 = -1$$

$$s = \pm j1 \; ; \; s = \pm j1 \ldots$$

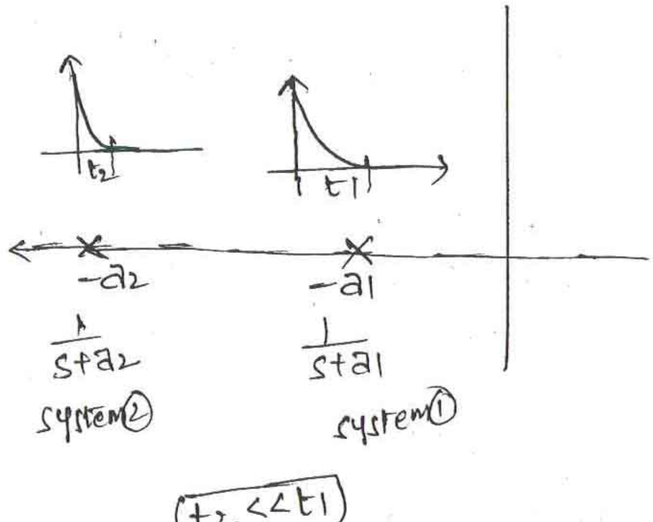
$$\frac{dA.E}{ds} = 4s^3 + 4s \ldots$$

$$A.E_2 = s^2 + 1 = 0 \Rightarrow s = \pm j1 \ldots$$

So that the roots are repeated on the imaginary axis.

The system is unstable :...

Relative stability using Routh Array :-



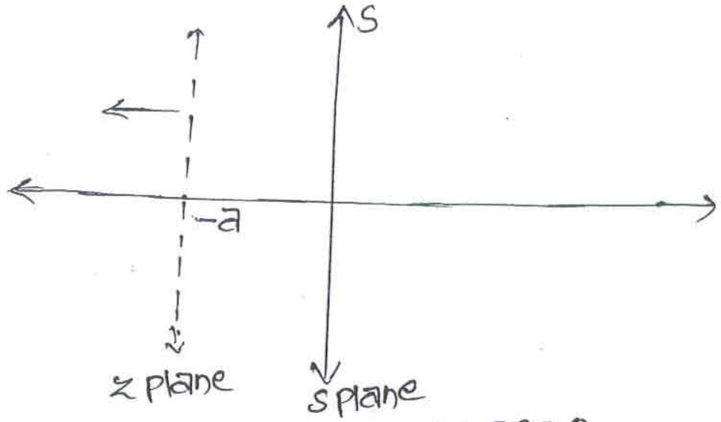

* Both system① & system② are said to be absolutely stable.

But observing both the systems we can conclude that, the system② is recovering fastly from the shock.

∴ the system② is relatively more stable than system① because $\boxed{t_2 << t_1}$ ...

By using Routh Array :-

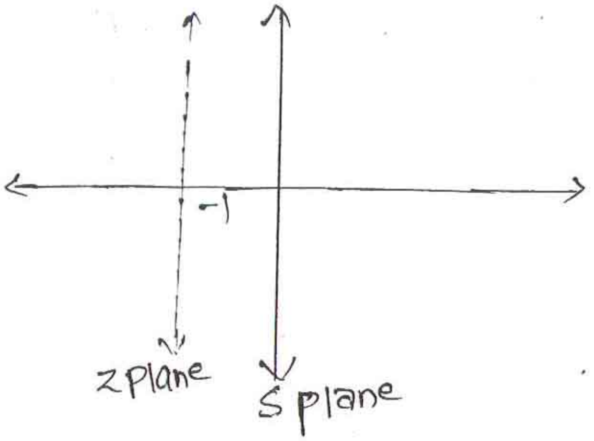
Roots belongs to 'z' plane are 'a' times faraway from 's' plane. ⇒ 's' plane is relatively more stable than 'z' plane.




$s+a=z$

$\Rightarrow s=z-a$

Ex:- $P(s)= s^3+7s^2+25s+39=0$
check whether the roots are lying more Negatively w.r.to "$-1$".




$(s+1)=z$

$s=z-1$

$\therefore P(s)=(z-1)^3+7(z-1)^2+25(z-1)+39=0$

$z^3+4z^2+14z+20=0$

$$\begin{array}{c|cc} z^3 & 1 & 14 \\ z^2 & 4 & 20 \\ z^1 & 9 & 0 \\ z^0 & 20 & \end{array}$$

In the z-plane point of view the system is more stable.

Q:7 $s^5+15s^4+85s^3+225s^2+274s+120=0$

Sol:- $s+1=0$ ↳ z-plane.

$s+1=z$

$s=(z-1)$

$(z-1)^5+15(z-1)^4+85(z-1)^3+22(z-1)^2+274(z-1)+120=0$

** This condition valid only when they provided the original polynomial is stable.

Put $s=-1$ in original equation, if it $=0$, one of the root lie on the left side ⇒ Generalization.

NOTE:- Put $s=-1$, if $P(-1)$ is $>0$ ⇒ all the roots will lie to the left side of $-1$. If $P(-1)=0$ one root will lie on $-1$ and remaining to the left of $-1$.

CONDITION :- Original polynomial should be stable - (then only this statement is True). (or) Then only Generalization is possible.

$P(-1) = -1+15+(-85)+225-274+120$

$= 0 \ldots$

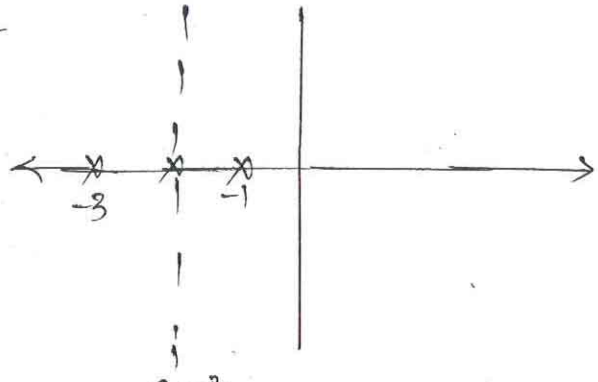
$\therefore P(-1) = 0 \ldots$

Since we get answer zero, so one root lies b/w −1 & origin & the remaining roots will be left lie in left half of $s=-1$.

NOTE:- Now the question is, let us check this statement is valid or not for $P(s) = s^3+6s^2+12s+9=0$

Then the roots are $s = -1, -2, -3$

If we are consider $s=-2$ as the z-plane (or) reference. then




Previously we are discussing that all the roots are lying in the left side of $s=-1$.

But here $s=-2$ is taken as z-plane, then one root is lie in the ~~left~~ right side of z-plane i.e $s=-2$.

so the generalized statement is wrong according to this pblm view.

since, the system polonomial is stable (it will be check before starting the procedure)...

Note :- while doing the Generalization we have to take $s=-1$ plane only. (or) take the pole which is low value i.e which is nearer to imaginary axis so that all the poles are lie in the left side of ~~s-p~~ z-plane...

Conditionally stable system :—

↳ means stability of the system depends on system

page no 66

Q:6 Parameters.

Q:6 :-

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 3 | K |
| $s^3$ | 2 | 2 | 0 |
| $s^2$ | 2 | K | 0 |
| $s^1$ | $\frac{4-2K}{2}$ | 0 | 0 |
| $s^0$ | K | | |

for stability $\frac{4-2K}{2} > 0 \Rightarrow K < 2$

$K > 0$

$\boxed{\therefore 0 < K < 2}$ → stability limit for 'K'.

When $K < 2 \Rightarrow$ All the first column elements are +ve co-eff$^n$. So roots in L.H.S therefore the response is bounded.

Now

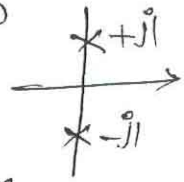
$K = K_{marginal} = 2$

$s^1$ row $= 0$

$\Rightarrow A(s) = 2s^2 + K = 0$

$2s^2 + 2 = 0$

$\Rightarrow s = \pm j1$




$\omega = \omega_{marginal} = 1$ rad/sec

When we design the system with $K=2$, the system will acts as oscillator & oscill freq = 1 rad/sec

Note:- for $s^6$ pblm, we have roots $\pm j1.4, \pm j2$. for that we can find $K_{mar}$ & system response is critically stable i.e oscillatory. But we can't findout the oscill freq$^n$ since two freq$^n$ are occurs. so that the limitation is we have to need only A.E as $s^2$ terms only (or) Higher oder with same roots.

Q:8

C.E = $1 + G(s)H(s) = 0$

$s^3 + 2s^2 + 2s^2 + 4s + 2s + 4 + K = 0$

| | | |
|---|---|---|
| $s^3$ | 1 | 6 |
| $s^2$ | 4 | 4+K |
| $s^1$ | $\frac{24-(4+K)}{4}$ | 0 |
| $s^0$ | 4+K | |

To find oscillary freq$^n$, the A.E should be $s^2$. So $s^1$ row = 0

$\frac{24-(4+K)}{4} = 0 \Rightarrow K = K_{mar} = 20$

$A(s) = 4s^2 + (4+K) = 0$

$4s^2 + 24 = 0$

$s = \pm j\sqrt{6}$

$\omega_n = \omega = \sqrt{6}$

Suppose he is asking the range of stability

$4 + K > 0$

$\frac{24-(4+K)}{4} > 0$

$\Rightarrow \boxed{-4 < K < 20}$

Q.9 :- $C.E = 1 + G(S)H(S) = 0$

$$= 1 + \frac{K(S-2)^2}{(S+2)^2} = 0$$

$$\Rightarrow (S+2)^2 + K(S-2)^2 = 0$$

$$\Rightarrow S^2[1+K] + S[4-4K] + (4+4K) = 0$$

| | | |
|---|---|---|
| $S^2$ | $1+K$ | $4+4K$ |
| $S^1$ | $4-4K$ | $0$ |
| $S^0$ | $4+4K$ | |

condition for stability.

$1+K > 0$

$\Rightarrow \underline{K > -1}$

$4-4K > 0$

$4 > 4K$

$\underline{K < 1}$

$4+4K > 0$

$4K > -4$

$\underline{K > -1}$

$\boxed{-1 < K < 1}$

but in given problem K range is $K \geq 0$. so $\boxed{0 < K < 1}$ is the stability limit

Q.12

Sol:-

$C.E = 1 + G(S)H(S) = 0$

$$\Rightarrow 1 + \frac{K(S+1)}{(S^3 + aS^2 + 2S + 1)} = 0$$

$$\Rightarrow S^3 + aS^2 + 2S + 1 + K(S+1) = 0$$

| | | |
|---|---|---|
| $S^3$ | $1$ | $2+K$ |
| $S^2$ | $a$ | $K+1$ |
| $S^1$ | $\frac{(2+K)a-(K+1)}{a}$ | $0$ |
| $S^0$ | $K+1$ | |

given $\omega = \omega_n = \underline{2 rad/sec}$

so

$A(s) = aS^2 + (K+1) = 0$

$S^2 = -\left(\frac{K+1}{a}\right)$ //...

But $\frac{(2+K)a - (K+1)}{a} = 0$

$\Rightarrow a = \frac{K+1}{K+2}$ //

$$\therefore S^2 = -\frac{\cancel{(K+1)}(K+2)}{\cancel{(K+1)}}$$

$S^2 = -(K+2)$

$S = \pm j\sqrt{K+2}$

$\therefore \omega = \omega_n = 2 = \sqrt{K+2}$

$\Rightarrow \boxed{K = 2}$ //...

$a = \frac{2+1}{2+2} = 0.75$ //...

Q:3: $G(S) = \frac{1}{s^3+1.5s^2+s-1}$ → O.L.T.F

To find stability of this we have to find the poles & then plot the poles plot. (or)

$O.L \text{ poles} = s^3+1.5s^2+s-1=0$

| | | |
|---|---|---|
| $s^3$ | 1 | 1 |
| $s^2$ | 1.5 | -1 |
| $s^1$ | $\frac{2.5}{1.5}$ | 0 |
| $s^0$ | -1 | |

unstable

C.L system :- C.L poles $= 1+G(S)H(S)=0$

$$1+\frac{1}{(s^3+1.5s^2+s-1)}20(s+1)=0$$

$$(s^3+1.5s^2+s-1)+(20(s+1))=0$$

| | | |
|---|---|---|
| $s^3$ | 1 | 21 |
| $s^2$ | 1.5 | 19 |
| $s^1$ | 8.3 | 0 |
| $s^0$ | 19 | |

stable...

Q:4 $G(S) = \frac{K}{(S+2)(S+4)(s^2+6s+25)}$

(a) 590 (b) 790 (c) 990 (d) 1190

sol: $1+G(S)H(S)=0 \Rightarrow 1+\frac{K}{(s^4+12s^3+69s^2+198+200)}=0$

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 69 | (200+K) |
| $s^3$ | 12 | 198 | 0 |
| $s^2$ | 52.5 | (200+K) | 0 |
| $s^1$ | $\left(\frac{10395-12(200+K)}{52.5}\right)$ | 0 | 0 |
| $s^0$ | 200+K | 0 | 0 |

since oscillatory behaviour,

$$\frac{10395-12(200+K)}{52.5}=0$$

$\Rightarrow K = K_{mar} = 666$ //...

suppose range of for stability is $\boxed{200 < K < 666}$

Ans :- a

since if $K>666$ choosen, the system may unstable.

Q:5 step response $= 1-e^{-t}(1+t)$

sol: $I.R = \frac{d}{dt}(\text{step}) = -\left[-e^{-t}(1+t)\right] * e^{-t}(1)$

$= \cancel{e^{-t}} + \cancel{(\ldots)} * e^{-t} + te^{-t} = te^{-t}$

$C(S) = \frac{1}{(S+1)^2}$

unstable...

Conventional

Q.1) Here one pblm may occur. i.e $\frac{C}{R}$ (or) $\frac{C}{D}$ ? what ever the forward path, the $\Delta$ value remains same. So the stability for both the i/p's is same.

C.E → $1 + G(S)H(S) = 0$

$$1 + \frac{(K_p s + K_I)10}{S(s^2+s+20)} = 0$$

$$s^3 + s^2 + s(20 + 10K_p) + 10K_I = 0$$

| | | | |
|---|---|---|---|
| $s^3$ | 1 | $20+10K_p$ | |
| $s^2$ | 1 | $10K_I$ | |
| $s^1$ | $20+10K_p-10K_I$ | 0 | |
| $s^0$ | $10K_I$ | | 0 |

for stability

1) $10K_I > 0 \Rightarrow K_I > 0$

2) $20 + 10K_p - 10K_I > 0$

$$\boxed{K_p > \frac{10K_I - 20}{10}}$$ //...

Q.2)

$$G(s) = \frac{K}{(s+1)^3 (s+4)}$$

$1 + G(S)H(S) = 0$

$\Rightarrow (s+1)^3 (s+4) + K = 0$

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 15 | $(4+K)$ |
| $s^3$ | 7 | 13 | 0 |
| $s^2$ | 13.14 | $(4+K)$ | 0 |
| $s^1$ | $\frac{170.8-7(4+K)}{13.14}$ | 0 | 0 |
| $s^0$ | $(4+K)$ | 0 | 0 |

Range of stability:-

$4 + K > 0$

$\boxed{K > -4}$

$$\frac{170.8 - 7(4+K)}{13.4} > 0$$

$\Rightarrow K < 20.4$

$\boxed{\therefore -4 < K < 20.4}$ //...

$A(S) = 13.14s^2 + (4+K) = 0$

$13.14s^2 + (4+20.4) = 0$

$\Rightarrow s = \pm j1.36$

$\omega = \omega_n = 1.36$ rad/sec ...

Q.3)

2012

$1 + G(S)H(S) = 0$

$S(s^2+1)(s+4) + K(s+2)^2 = 0$

2) Damping co-efficient (or) Damping factor (or) Actual damping

$\boxed{\alpha = \zeta\omega_n}$

3) Damping co-efficient (or) Damping factor — Time constant of underdamped response

$$T = \frac{1}{\alpha} = \frac{1}{\zeta\omega_n}$$

4) Damped Natural freq ($\omega_d$)

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \text{ rad/sec}$$

5) $s^2 + 2\zeta\omega_n s + \omega_n^2 = (s+\zeta\omega_n)^2 + \omega_d^2$

6) Damping ratio $= \dfrac{\text{Actual damping}}{\text{Critical damping}} = \dfrac{\zeta\omega_n}{\omega_n} = \zeta$

Actual damping $= \zeta\omega_n$

At $\zeta = 1$, Actual damping becomes critical damping

Critical damping $= \omega_n$

** Note:-

** If in the problems Damping factor is given you can take $\zeta$ for damping factor is $< 1$.

If $\zeta > 1 \Rightarrow$ The damping factor is greater than 1 then taking it as damping ~~ratio~~ factor.

TRANSIENT ANALYSIS :- (underdamped case)

let $R(s) = \frac{1}{s}$

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$$= \frac{1}{s} - \frac{(s + 2\zeta\omega_n)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$C(s) = \frac{1}{s} - \frac{(s+\zeta\omega_n)}{(s+\zeta\omega_n)^2 + \omega_d^2} - \frac{\omega_d}{(s+\zeta\omega_n)^2 + \omega_d^2} \cdot \frac{\zeta\cancel{\omega_n}}{\cancel{\omega_n}\sqrt{1-\zeta^2}}$$

$$c(t) = 1 - e^{-\zeta\omega_n t} \cdot \cos\omega_d t - e^{-\zeta\omega_n t} \sin\omega_d t \cdot \frac{\zeta}{\sqrt{1-\zeta^2}}$$

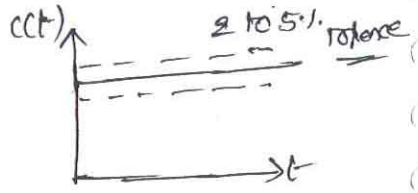
$$= 1 - \frac{e^{-\zeta\omega_n}}{\sqrt{1-\zeta^2}}\left[\sqrt{1-\zeta^2} \cdot \cos(\omega_d t) + \zeta\sin\omega_d t\right]$$

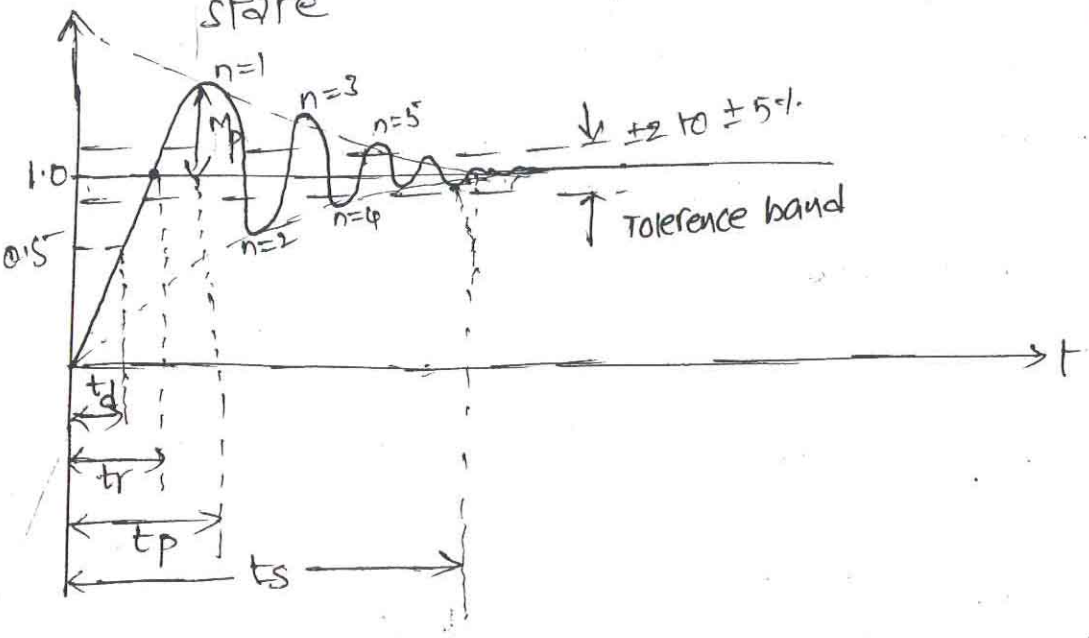
$$\boxed{A \sin\omega t + B\cos\omega t = \sqrt{A^2+B^2}\cdot \sin(\omega t + \tan^{-1} B/A)}$$

$$\boxed{c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin\left(\omega_d t + \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)} \;//\ldots$$

↓ steady state

↓ transient state




1) Delay Time ($t_d$) :- $\boxed{t_d = \frac{1+0.7\zeta}{\omega_n}}$ see [at is an empherical formula. i.e can't be derivative one (or) Imaginary experimental result]

2) Rise Time :- $(t_r) = ?$

At $t=t_r$; $c(t)=1$

$$c(t)\Big|_{t=t_r} = 1 - \frac{e^{-\zeta\omega_n t_r}}{\sqrt{1-\zeta^2}}\sin(\omega_d t_r + \theta) = 1$$

$$\frac{e^{-\zeta\omega_n t_r}}{\sqrt{1-\zeta^2}}\sin(\omega_d t_r + \theta) = 0$$

$\Rightarrow$ Since $\sin(\omega_d t_r + \theta) = 0$

$\Rightarrow (\omega_d t_r + \theta) = \pi$

→ Here may be take $0^\circ$ also. Since $\sin(0^\circ) = 0$ //

$\omega_d t_r + \theta = 0$

$\Rightarrow t_r = -\frac{\theta}{\omega_d}$

but $t_r$ should not be -ve.

$$\omega_d t_r + \theta = \pi$$

$$\boxed{t_r = \frac{\pi - \theta}{\omega_d}} \;//\ldots$$

where $\theta = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$ //...

3. Peak Time $(t_p)$:-

It is the time taken by the response to reach maxm value.

Generally to get maxm or min value, we are differentiate that particular funn & equate to zero.

$$\frac{d}{dt}c(t) = 0$$

$$\Rightarrow \frac{d}{dt}\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \cdot \sin(\omega_d t_p + \theta)\right] = 0$$

$$\left[0 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\cos(\omega_d t_p + \theta)\cdot\omega_d + \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \cdot \zeta\omega_n \sin(\omega_d t_p + \theta)\right] = 0$$

$$\Rightarrow \cancel{\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}}\{\cos(\omega_d t_p + \theta)\}\omega_d = \cancel{\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}} \cdot \zeta\omega_n \sin(\omega_d t_p + \theta)$$

$$\Rightarrow \tan(\omega_d t_p + \theta) = \cancel{\frac{\zeta\omega_n}{\omega_d}}\ \frac{\omega_d}{\zeta\omega_n}$$

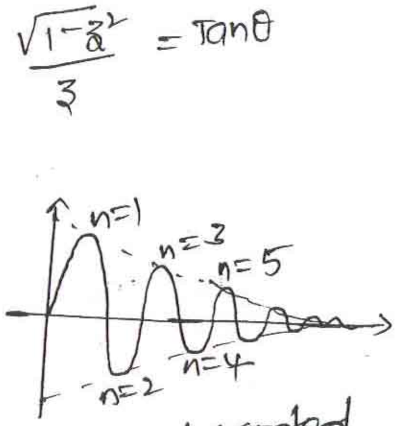
$$\Rightarrow \tan(\omega_d t_p + \theta) = \frac{\zeta\omega_n\sqrt{1-\zeta^2}}{\zeta\omega_n\cancel{\sqrt{1-\zeta^2}}}$$

$$\Rightarrow \tan(\omega_d t_p + \theta) = \frac{\cancel{\omega_n}\sqrt{1-\zeta^2}}{\zeta\cancel{\omega_n}} = \frac{\sqrt{1-\zeta^2}}{\zeta} = \tan\theta$$

$$\Rightarrow \tan(\omega_d t_p + \theta) = \tan\theta$$

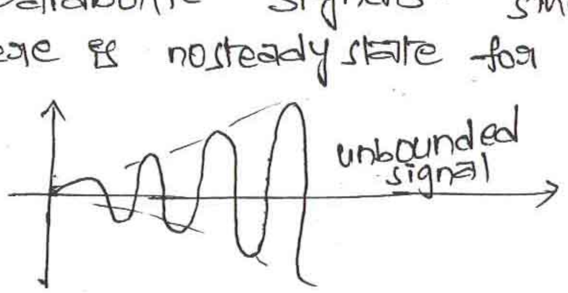
(since $t = t_p$) $\therefore$ $\Rightarrow \omega_d t_p = n\pi$

$$\boxed{t_p = \frac{n\pi}{\omega_d}}$$




* We don't evaluate the transient response for unbounded ramp & parabolic signals. since they are unbounded signals. There is no steady state for unbounded signals.

Ex:



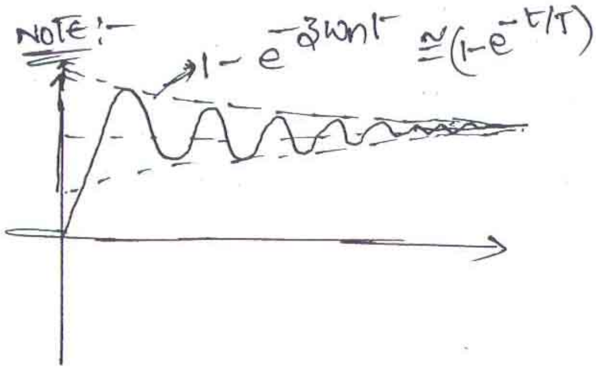

4) Settling Time :- $(t_s)$

It is the time taken by the response to reach and settle with in the limits of tolerence band ($\pm 2\%$ (or) $\pm 5\%$)

2% Tolerence band $\rightarrow$ $4T = 4/\zeta\omega_n$

5% Tolerence band $\rightarrow$ $3T = 3/\zeta\omega_n$

NOTE :-




$c(t) = 1-e^{-t/T}$

If $t = 4T \Rightarrow e^{-t/T} = e^{-4T/T} = 0.02$

$1-0.02 = \underline{0.98}$

$c(t) = 98\%$ of O/P ....

If $t = 5T$ $e^{-t/T} = 0.05$

$\Rightarrow 1-0.05 = 0.95$

$c(t) = \underline{95\%}$ of O/P

5) <u>Maximum Peak overshoot</u> :— ($M_p$)

$$c(t)\Big|_{t=t_p=\frac{\pi}{\omega_d}} = 1-\frac{e^{-\zeta\omega_n t_p}}{\sqrt{1-\zeta^2}}\sin(\omega_d t_p+\theta)$$

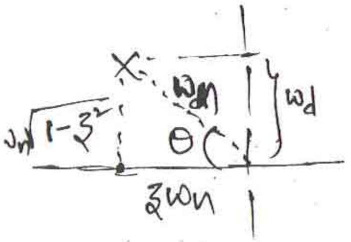
$t_p = \frac{n\pi}{\omega_d}$

($n=1$)

$$= 1-\frac{e^{-\zeta\omega_n \times \frac{\pi}{\omega_d}}}{\sqrt{1-\zeta^2}}\sin\left(\cancel{\omega_d}\frac{\pi}{\cancel{\omega_d}}+\theta\right)$$

$$= 1-\frac{e^{-\zeta\cancel{\omega_n}\cdot\left(\frac{\pi}{\cancel{\omega_n}\sqrt{1-\zeta^2}}\right)}}{\sqrt{1-\zeta^2}}\sin(\pi+\theta)$$

$$= 1+\frac{e^{-\zeta\pi/\sqrt{1-\zeta^2}}}{\sqrt{1-\zeta^2}}\sin\theta$$




$\sin\theta = \frac{\cancel{\omega_n}\sqrt{1-\zeta^2}}{\cancel{\omega_n}}$

$= \underline{\sqrt{1-\zeta^2}}$ .....

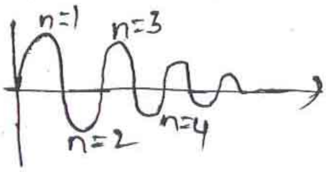
$$= 1+\frac{e^{-\zeta\pi/\sqrt{1-\zeta^2}}}{\cancel{\sqrt{1-\zeta^2}}}\left(\cancel{\sqrt{1-\zeta^2}}\right)$$

$$c(t)\Big|_{t=t_p} = 1+e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

$$\therefore \%M_p = c(t)\Big|_{t=t_p} - 1 = \underline{e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100} \; ....$$

$$\boxed{\therefore \%M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}}} \times 100 \; \% ....$$

Similarly you can find $M_p$ for $2^{nd}$, $3^{rd}$, & $4^{th}$ overshoots by putting 'n' value to its corresponding.



Let 'K' = Gain = $\frac{b_0}{a_0}$

$\Rightarrow \frac{X_o(S)}{X_i(S)} = K$ $\qquad X_i(S) \longrightarrow \boxed{K} \longrightarrow X_o(S)$

Ex:- Amplifiers, sensors/Transeducers

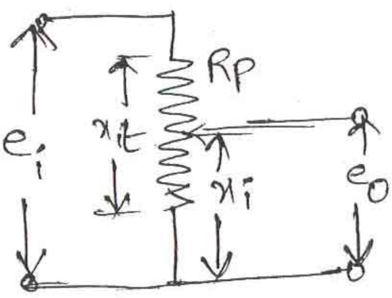
An Amplifiers the input is amplified with a gain.

$\frac{\text{Amplifier O/P}}{\text{Amplifier i/p}} = K \rightarrow$ some value.

Another Example is:-

Potentiometer :- [variable resistive display-Traseducer]

The potentiometer is also an Exple of zero order system. The application of a pair of potentiometers is "Error detector". Here the input is displacement of probe wiper and the output is electrical energy.



Total resistance of Potentiometer = $R_p$

Resistance per unit length = $\frac{R_p}{x_t}$

Resistance of wiper displacement of '$x_i$' units $= \frac{R_p \times x_i}{x_t}$

Apply voltage division Rule

$$e_o = \frac{\left(\frac{\cancel{R_p} \times x_i}{x_t}\right) \times e_i}{\cancel{R_p}}$$

$$e_o = \frac{x_i}{x_t} \times e_i$$

input = $x_i$

output = $e_o$

$$\therefore \boxed{e_o = \frac{e_i}{x_t} \times x_i}$$

$$e_o = k_p x_i$$

where $k_p$ = potentiometer Gain $= \frac{e_i}{x_t}$

units $\rightarrow \left(\frac{\text{volt}}{\text{mm}}\right)$

$$E_o(S) = k_p X_i(S)$$

$$X_i(S) \longrightarrow \boxed{k_p} \longrightarrow E_o(S)$$

Actually we are saying gain is increased (or) decreased means we are varying either $e_i$ (or) $x_r$. So practically we are varying $e_i$ (or) $x_r$ (if $x_r$ = const $e_i$ only vary). Theoritically we are gain (k) in changes means thats only.

NOTE:- 1. Potentiometers in control system are used as "Error detector".

2. A pair of Potentiometers acts as error detectors.

3. there is no time response for zero order system because their input and output characteristics are linear dependent.

ii. first order System :-

$$\frac{X_o(S)}{X_i(S)} = \frac{b_0}{a_1 S + a_0} = \frac{b_0/a_0}{(a_1/a_0)S+1}$$

When $s^1$ terms & constant terms are taken & remaining all 's' terms are taken as zeros then it is called as "first order system".

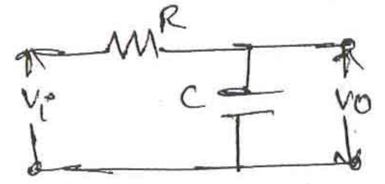
let $k$ = gain = $\frac{b_0}{a_0}$

T = Time constant element = $\frac{a_1}{a_0}$

$$\frac{X_o(S)}{X_i(S)} = \frac{k}{1+ST}$$

$$X_i(S) \longrightarrow \boxed{\frac{k}{1+ST}} \longrightarrow X_o(S)$$

Example:- RC Network



1) RC Network is an example of first order system.

2) Boiling of milk (or) water is also another ex.

3) The Temparature measurement by using Thermometer is another ex.

$$\frac{V_o}{V_i} = \frac{1}{RCS+1}$$

T = RC

4) Water storage in a Tank is also another Example.

Explanation:-

When we consider the Boiling of water (or) milk; first we are giving flame as input; and the water heating will depends upon the base of container and also volume (or) Area of the water. The area or volume of water will indicates that capacitor & Base of container will indicates that resistor. since base is con' regulating (or) varying element.

Lightning of flame → step input
base of container → Resistance
volume (or) area of container → capacitor

so Any system which is combination of resistor and capacitor will be under the category of first order system.

## Transient Analysis :-

$$X_i(s) = \frac{1}{s} \quad \text{(unit step)}$$

$$\Rightarrow X_o(s) = \frac{K}{s(1+sT)}$$

$$X_o(s) = K\left(\frac{1}{s} - \frac{T}{(1+sT)}\right)$$

$$= K\left(\frac{1}{s} - \frac{1}{s+\frac{1}{T}}\right)$$

$$x_o(t) = K\left[1-e^{-t/T}\right)$$

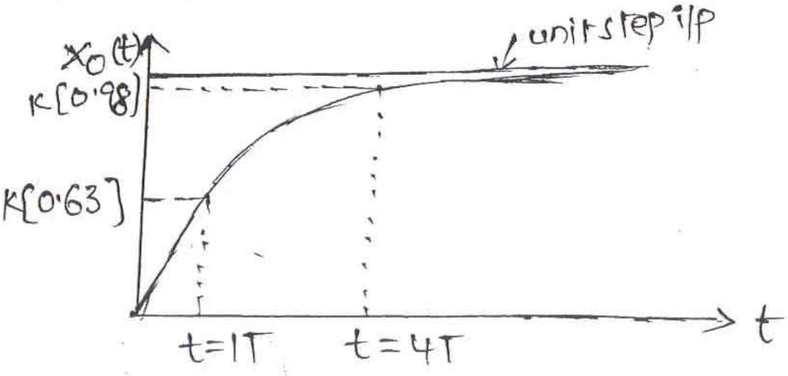
$$\boxed{\therefore\ x_o(t) = K\left(1-e^{-t/T}\right)}$$

At $t=0$ ; $x_o(t)=0$

At $t=T$ ; $x_o(t) = K\left[1-e^{-1}\right] = K\left[1-0.37\right] = K\left[0.63\right]$

⋮

At $t=4T$ ; $x_o(t) = K\left[1-e^{-4}\right] = K\left[1-0.02\right] = K\left[0.98\right]$




so for reaching the final value it will takes $\simeq 4T$ time.

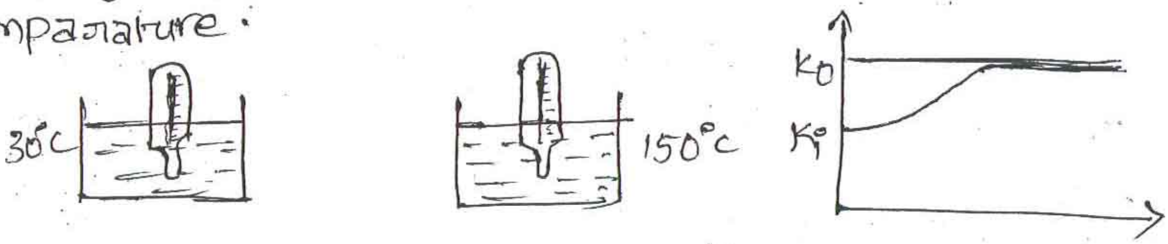
so time taken for boiling of water is 4T time.

→ consider another example thermometer. when we put thermometer on mouth, that amplies we are giving step input. After 4T time it will give steady state temparature of our body.

→ consider water tank in that valve will be acts as a resistor and water tank represents capacitor. so it will form a first order control system.

→ Pnematic system is also another example for first order constrol system. It is similar to water tank. It is related to Air instead of water, thats only change for water tank & pnematic system.

Note:- let us consider ~~two~~ one thermometers & take two containers which is having water with a temparature of 30° & 150°. respectively. first place the thermometer in 30° container, After 4T time it will shows 30° temparature. Now take that thermometer place it another container. so it will starts from 30° temp. not from 0° temparature. After 4T time it will shows 150° temparature.




for this the expression will be

$$\boxed{x_o(t) = K_o + (K_i - K_o)\,e^{-t/T}} \quad \ldots$$

if $K_i = 0$

$$x_o(t) = K_o\,(1 - e^{-t/T}) \ldots$$

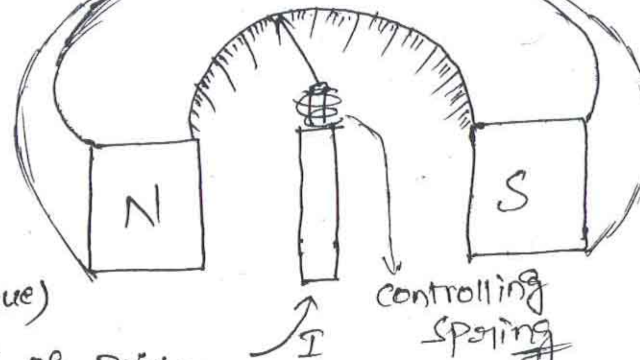
so we get the actual expression.

Def:— the Time constant is defined as Time taken by the response to reach 63 % of the final value.

## Second order system :—

Example for second order system is "PMMC" instrument

Any moving system having Mass, friction & spring constn.




Here the input = $T_d$ (defl. Torque)

Output = Angular displacement of Pointer

Here the moving system is there so its own mass & for any moving system friction will be there. We are using spring for controlling Torque. So spring constant 'k' is also exist.

$$\therefore \quad T_d = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + k\theta$$

$$T_d(s) = (Js^2 + Bs + k)\,\theta(s)$$

$$T.F = \frac{\theta(s)}{T_d(s)} = \frac{1}{Js^2 + Bs + k}$$

$$= \frac{(1/J)}{s^2 + \frac{B}{J}s + \frac{k}{J}} \quad \rightarrow ①$$

Which is a second order system.

By the result of deflection Torque the pointer will be moves, but it does not stops. when $T_d = T_c$ the pointer will stops and make some oscillations. This oscillations are called as "undamped natural frequency of oscillations".

The main reason for the oscillations is less friction (B). to keep the friction as higher value we are adding some extra friction to this. This extra friction is called "Damping Torque".

so that the pointer will stops & makes no oscillations.

The additional damping provided ~~provided~~ to the pointer can be represented by damping ratio '$\zeta$'.

In T.F eq ① if friction $B=0$, then

$$T.F = \frac{1/J}{s^2+\frac{K}{J}}$$ ; The response i.e inverse laplace transform gives $\sin(\frac{K}{J}t)$ → which is sinusoidal. That gives constant frequency undamped natural freq of oscillation.

Now compare Eq ① with standard second order system

$$T.F \text{ i.e } \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}, \qquad \omega_n^2 = \frac{K}{J} \Rightarrow \boxed{\omega_n = \sqrt{\frac{K}{J}}}$$

$$2\zeta\omega_n = \frac{B}{J} \Rightarrow \boxed{\zeta = \frac{B}{2}\sqrt{\frac{1}{JK}}} = \frac{B}{2}\sqrt{\frac{1}{JK}} \;//\ldots$$

* In Actual discussion we are changing '$\zeta$' values means we are adding the friction to the system.

SECOND ORDER SYSTEM:-

The Response of second order (or) higher order systems exhibits continuous & sustained oscillations about the steady state values of the input with the freq$^n$ is known as "undamped Natural freq$^n$" ($\omega_n$ rad/sec).

These oscillations in the Response are damped to the steady state value of input using approximate damping methods, where the damping is mathematically represented as "Damping ratio" ($\zeta$).

Standard second order system Transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \;//\ldots$$

$\zeta \rightarrow$ Damping ratio (zeta)

$\omega_n \rightarrow$ undamped natural freq$^n$

## Effect of Damping on Nature of Response :-

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

char Eqn = $1 + G(s)H(s) = 0$

$\Rightarrow s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

$$\text{roots} = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= -\zeta\omega_n \pm \left[\omega_n \underbrace{\sqrt{\zeta^2 - 1}}_{D}\right]$$

where $\zeta^2 - 1$ = Discriminent = D

when

$D = \zeta^2 - 1 = 0 \Rightarrow \zeta = 1$

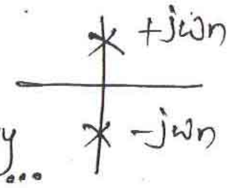
$D = \zeta^2 - 1 > 0 \Rightarrow \zeta > 1$

$D = \zeta^2 - 1 < 0 \Rightarrow \zeta < 1$

**case (i) :- undamped case :- ($\zeta = 0$)**

$$T.F = \frac{\omega_n^2}{s^2 + \omega_n^2} \quad ; \text{ roots} = \pm j\omega_n$$

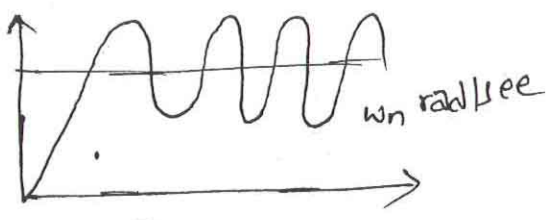
roots are imaginary...




$= \omega_n \sin(\omega_n t)$

Here No additional damping is given i.e $\zeta = 0$. Here No additional damping is given i.e $\zeta = 0$ in the above T.F as $\zeta = 0$.

$\Rightarrow$ constant undamped oscillations

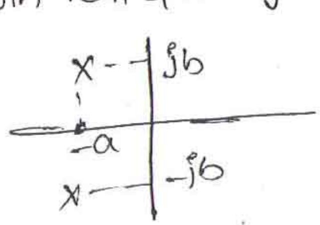



**case (ii) :- underdamped case ($0 < \zeta < 1$)**

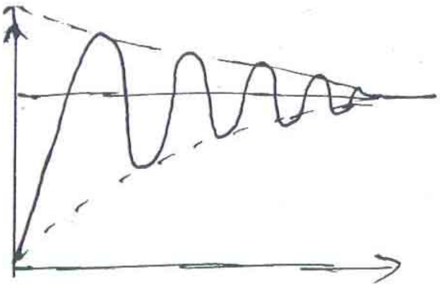
$$T.F = \frac{\omega_d}{(s + \zeta\omega_n)^2 + \omega_d^2}$$

$$= e^{-at}\sin(\omega_d t)$$

Roots are having both real & imaginary parts i.e complex conjugate.



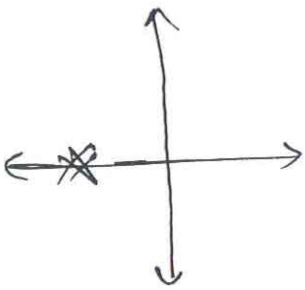

$(-a \pm jb)$ roots



case (iii) :- Critically Damped case ($\zeta = 1$)

$T \cdot F = \dfrac{\omega_n^2}{(s+\omega_n)^2}$

roots are real & equal. $-a, -a$

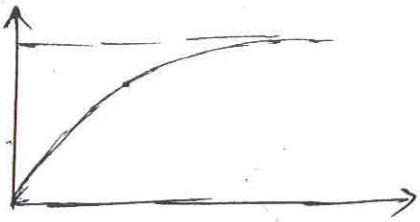


$(s+\omega_n)^2 = 0$

$\Rightarrow s_1, s_2 = -\omega_n, -\omega_n$

$L^{-1}(T \cdot F) = t e^{-at}$

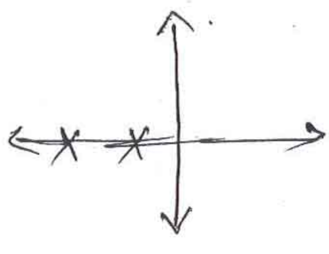
So No oscillations since there is no sine Term.



And also response is fast. but practically it doesnot exist.

case (iv) :- over damped case ($\zeta > 1$)

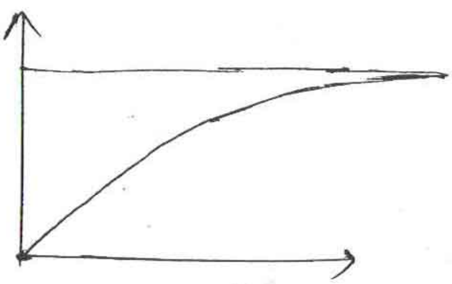
$T \cdot F = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$



The roots are real & unequal.

$L^{-1}(T \cdot F) = e^{-at} \pm e^{-bt}$

The response is very slow since Two exponential Terms are exist.



Conclusion :

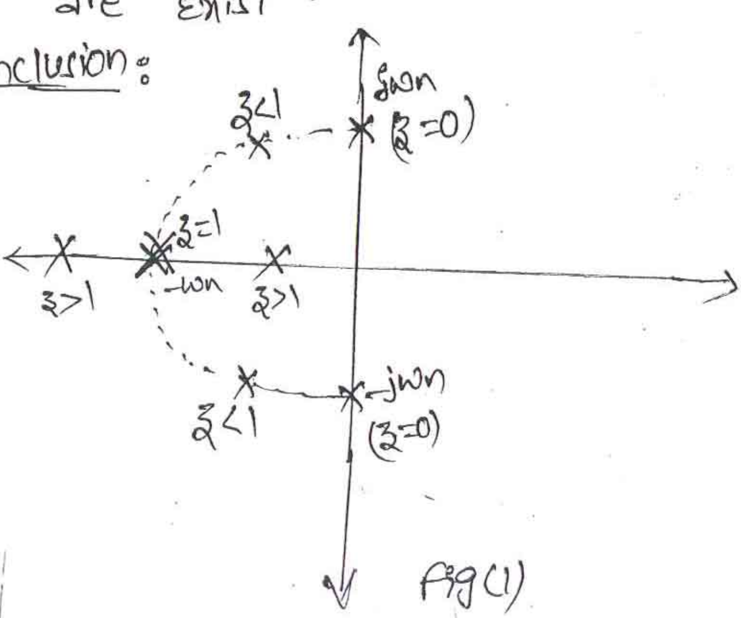



fig (1)

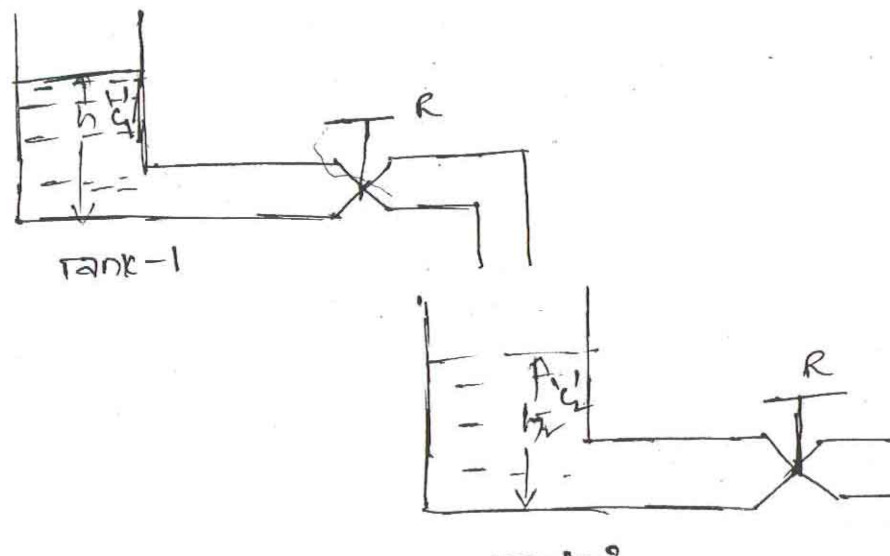
NOTE:- Among all the systems which one is preferable.
underdamped system is more preferable.

Explanation:- In ($\xi>1$) case, the response becomes very slow. so we never take that case (or) system.

In ($\xi=0$) case, there is no steady state condition, the system will makes continuous oscillations. so we can't take.

$\xi=1$ case, practically Never Exist. so we can't take. since $\xi=1$ case having the roots equal, that means with Equal Time constants. we ~~Neve~~ are not design Two systems with Equal Time constants. Since for designing two systems with equal Time constants the two systems have to be design with Equal (or) Identical system. This is not possible practically, Just on paper we can show.

Non interacting system is an example for critically damped system.




The Transfer function of Non-Interacting system $= \frac{k}{(1+sT)^2}$

That means roots are real & equal. so practically we are not design the Non-Interacting system. for designing of Non interacting system the Two Tanks Time constants must be equal that means $R_1C_1 = R_2C_2$. but practically we can't design the Two Tanks with ~~eq~~ Identical values. so critically stable systems are Not Exist practically on paper we can show $R_1C_1 = R_2C_2$. Practically $R_1C_1 \neq R_2C_2$.

Note:- for all above practical cases case(ii) is preferable. But it is not practically. $\zeta=1$ means friction is more and here the response also is fast from response (case(iii)). ∴ so it is impractical. Generally friction more means response will be slow. so case(iii) is undesirable.

case(ii) is preferable. (In this we are also having some oscillations. explain below)

Note :-

Two tanks designing with equal inlet & outlet then only $R_1C_1 = R_2C_2$ ∴ equal values of $C_1$ & $C_2$, $R_1$ & $R_2$ is not possible practically. so '$\zeta$' value should be slightly less than (or) greater than 1. which results is in less oscillations (in underdamped case) and practically possible case (critically damped case is overcomed).

case(i) → Having oscillations ✗

case(ii) → Taking more time to reach final value. ✗

case (iv) → is preferable. (slightly $\zeta<1$ or $\zeta>1$)

case(iii) → Not practically exist. ✗

→ Most of the control systems are designed for $\zeta<1$ because the response can be analysed using more No:of performance specifications.

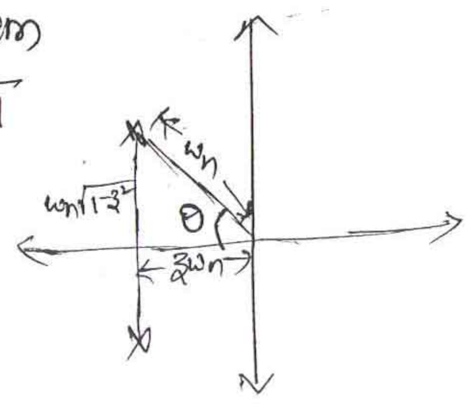
→ $\zeta=1$ practically doesn't exist. Hence control system having higher order dynamics are designed for $\zeta<1$ (or) $\zeta$ slightly greater than 1.

→ Fig ① represents Root locus of the second order system obtained by varying the damping ratio '$\zeta$' from 0 to $\infty$. It is a semi circular path with radious of $\omega_n$.

Characteristics of underdamped system :-

Roots of the second order system

$$s_1, s_2 = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1}$$




1) $\cos\theta = \frac{\zeta\omega_n}{\omega_n}$

$$\boxed{\theta = \cos^{-1}(\zeta)}$$

Def:- When we define the rules for OLTF, If ... points)
If be given OLTF. Then there is no find out CLTF. We talk interms OLTF poles. But what ever the root locus is obtained it is a closed loop poles moment. How it will be? we will see:

$1+G(S)H(S)=0$

$\Rightarrow G(S)H(S)=-1$ } → All the rules of frame for this only

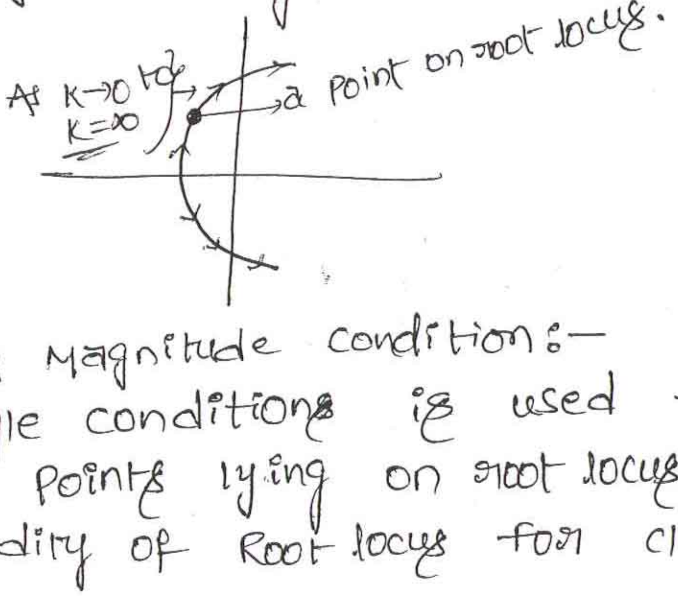
note:-
Every shape of root locus must & should satisfy one cond^n i.e Angle condition.

we choose any point in Root locus. If the angle is at the point is satisfy the root locus then the point is in the Root locus & the root locus is correct.

* correctness of Root locus
* To check whether a point is on the Root locus.
* To check whether they are lying or not on root locus
* To check the poles whether they are lying or not on root locus

Note:- Another cond^n is magnitude cond^n.

When we draw a root locus diagram, we don't know the value of 'k' at any point. for that we will go for magnitude cond^n.




Angle & Magnitude condition:-
The Angle conditions is used for checking whether certain points lying on root locus or not and also the validity of Root locus for closed loop poles.

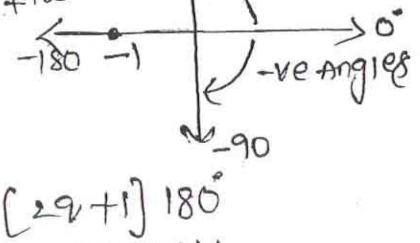
$1+G(S)H(S)=0$

$G(S)H(S)=-1$

$\angle G(S)H(S) = -1+j0$

$= +180-\tan^{-1}\left(\frac{0}{1}\right)$

$= 180 \approx \pm 180^\circ \approx \pm[2q+1]\,180^\circ \ldots$



Angle condition may be stated as for a point lie on root locus the angle evaluated at that point must be an odd multiple of $\pm 180°$. (since $\pm(2q+1)180$).

The Magnitude condition is used for finding the value of system gain 'K' at any point on root locus.

$1+G(S)H(S)=0$

$\Rightarrow G(S)H(S) = -1+j0$

$|G(S)H(S)| = \sqrt{(-1)^2+(0)^2} = 1$

age-70
napter-5
9:9

$\boxed{\therefore \; |G(S)H(S)| = 1}$ //...

$G(s) = \dfrac{K}{S(S^2+7S+12)}$ ; $K=?\,|\, s=-1+j1$

for finding gain $K \Rightarrow$ we have to use Angle magnitude condⁿ. first of all we have to check whether $s=-1+j1$ is on root locus or not.

$$G(S)\Big|_{S=-1+j1} = \frac{K+j0}{(-1+j1)\,[(-1+j1)^2+7(-1+j1)+12]}$$

$$= \frac{K}{[-1+j1]\,[5+j5]}$$

NOW

$$\angle G(S)\Big|_{S=-1+j1} = \frac{0°}{[(180)-\tan^{-1}(\frac{1}{1})]\,(\tan^{-1}(\frac{5}{5}))} = \frac{0}{(180-45)\,(45)}$$

$$= -180 \;...$$

NOW

$$|G(S)|_{S=-1+j1} = 1$$

$$\Rightarrow \frac{\sqrt{K^2+0^2}}{\sqrt{(-1)^2+1^2}\cdot\sqrt{5^2+5^2}} = 1 \Rightarrow K = \sqrt{100}$$

$$= 10$$

Adv:2 :- If you equate (differentiate) the char'' eq" => $1+G(S)H(S)=0$ => then magnitude

$|1+G(S)H(S)| = 0$

If we equate any thing to zero, finite value can't be get.

If we equate to 1, then we can easily findout the value of K.

Q:6

$$G(S)H(S) = \frac{K}{(S+1)^4}$$

sol:-

$S_1 = -3+j4$

Now

$$\angle G(S)H(S)\Big|_{s=-3+j4} = \frac{K}{(-3+j4+1)^4} = \frac{K}{(-2+j4)^4} = \frac{0}{4[180-\tan^{-1}(\frac{4}{2})]}$$

$$= -464^\circ \ldots$$

$$\angle G(S)H(S)\Big|_{s=-3\pm j2} = \frac{K}{(-3\pm j2+1)^4} = \frac{K}{(-2\pm j2)^4} = \frac{0}{4[180+\tan^{-1}(-2/2)]}$$

$$= \pm 540^\circ \ldots \quad \frac{0^\circ}{4\times(-135)}$$

$$= +540^\circ \ldots$$

$S_1$ is not on Root locus since Angle is not multiple of 180°, $S_2$ is on root locus since Angle is multiple of 180°.

## Construction Rules of Root locus :-

Rule NO:1 :- Root locus is symmetrical about real axis. $[G(S)H(S) = -1]$

Rule no:2: Let P = no:of open loop poles
Z = NO:of open loop zeros

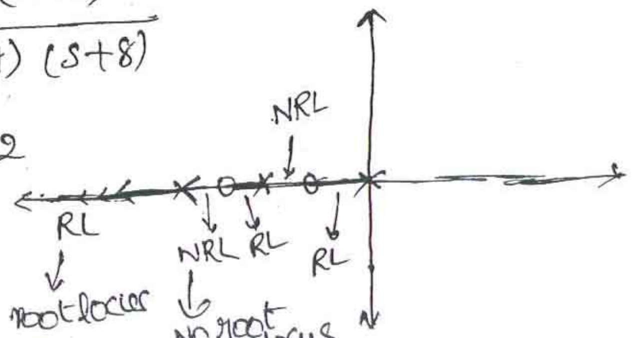
& $P>Z$ ; then
The NO:of Branches of Root locus = P ; $P>Z$
; $Z>P$
The no:of branches terminating at zeros = Z ;
*** → " " terminating at infinity = P-Z

Rule NO:3:- Point on real axis. is said to be on real axis. Root locus, if to the right side of this point the sum of openloop Poles & zeros is ODD.

ex:-
$$G(S)H(S) = \frac{K(S+2)(S+6)}{S(S+4)(S+8)}$$

ruleno:2 → $P=3$, $Z=2$
$P-Z=1$

ruleno:3 →



Root locus means As gain is varies from k = 0 to ∞, the Root exist at the origin will moves to zero at s = -2 so that here root locus exist.

Next the pole at s = -4, will reaches to s = -6 zero in left side only since root locus exist. It will not moving in rightside since there is no root locus.

Here No: of Branches = no: of root locus Paths = 3

[b/w 0 & -2
-4 & -6
-8 & -∞].

note
Here we are drawing Pole zero Plot for openloop Transfer function. But the root locus is closed loop poles Path. How it will be?

Explanation :-

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s+2)(s+8)}{s(s+4)(s+8)} = 0$$

$$\text{C.L Poles} = s(s+4)(s+8) + K(s+2)(s+6) = 0$$

when K = 0

$$\Rightarrow s = 0;\ s = -4,\ s = -8$$

Though we apply for OLTF, & we talk about the open loop Poles & openloop zeros & the Root locus is the path of closed loop poles as shown in above.

Rule No: 4 :-

Angle of Asymptotes = ?

$\boxed{\text{How many branches terminating at } \infty = P - Z.}$

[these branches will terminating at ∞ are on the Asymptotes or Parallel to the Asymptotes.]

The (P-Z) Branches terminating at ∞ will go along certain straight lines known as "Asymptotes of root locus".

∴ No: of Asymptotes will be equal to (P - Z).

$$\text{Angle of Asymptotes} = \frac{(2q+1)180}{(P-Z)}\ ;\ q = 0, 1, 2, \ldots (P-Z)-1$$

Eg:- Suppose $P-Z=2$

$$\theta_1 = \frac{(2q+1)\,180}{P-Z} = 90^\circ \longrightarrow q=0$$

$$\theta_2 = \frac{(2q+1)180}{P-Z} = +270 = -90 \rightarrow q=1$$

(Q:12)

Char Equ $\Rightarrow$ $s(s+4)(s^2+2s+5)+K(s+1)=0$

Now OLTF $\Rightarrow$ ?

Procedure for OLTF from char Equ:-

$$\underbrace{s(s+4)(s^2+2s+5)}+K(s+1)=0$$

$$1+\frac{K(s+1)}{s(s+4)(s^2+2s+5)}=0$$

$$1+\frac{K(s+1)}{s^2(s+4)(s+3)}=0$$

$$1+G(s)H(s)=0$$

$$G(s)H(s)=\frac{K(s+1)}{s^2(s+4)(s+3)}$$

Now $P=4,\ Z=1,\ P-Z=3$

$$\theta_1 = \frac{2(0)+1}{3}180 = 60^\circ\ ;\quad \theta_2 = \frac{2(1)+1}{3}180 = 180^\circ$$

$$\theta_3 = \frac{2(2)+1}{3}180 = -60^\circ = 300$$

Note:-

$$\boxed{\text{Angle Between the Asymptotes } = \frac{2\times180}{(P-Z)}} = \frac{2\times180}{3} = 120^\circ$$

Rule No:5 :-

Centroid :-

It is the intersection point of Asymptotes on the real axis. It may or may not be a part of root locus.

$$\text{centroid} = \frac{\sum \text{real part of open loop poles} - \sum \text{open loop zeros}}{P-Z}$$

Q:10

Char Equ $= s^3+5s^2+(K+6)s+K=0$

Sol:-

NOW OLTF = ?

$$s^3+5s^2+6s+Ks+K=0.$$

$$1+\frac{K(s+1)}{s^3+5s^2+6s}=0$$

compare it with $1+G(s)H(s)=0$

$$\Rightarrow G(s)H(s)=\frac{K(s+1)}{s^3+5s^2+6s}=\frac{K(s+1)}{s(s+2)(s+3)}$$

$P=3;\ Z=1;\ P-Z=2$      Poles $=0,-2,-3$
zeros $=-1$

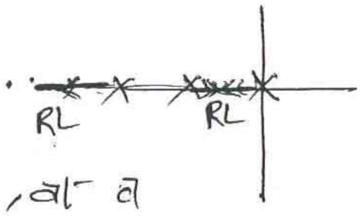
$$\text{centroid }(\sigma)=\frac{(0-2-3)-(-1)}{(3-1)}=\frac{-5+1}{2}=(-2,0)$$

Rule NO: 6:-

Breakaway Points:-

They are those points where multiple roots of the char. eqn occur.

Suppose we have these locations of poles.




* when poles are moving towards eachother, at a perticular point they are meet & breaks. There is a seperate procedure will be usefull for finding break away point.

Procedure:-

1) $1+G(s)H(s)=0$
2) Write 'K' interms of 's'.
3) find $\frac{dK}{ds}=0$
4) the roots of $\frac{dK}{ds}=0$ will give Break away Points.
5) To Test valid Break away Point substitute in step② ; if $K=+ve \Rightarrow$ valid Break away Point.

General Predictions about breakaway Point:-

1) the Branches of root locus either approach (or) leave Break away Point at an angle of

$$\left(\frac{\pm 180}{n}\right) \longrightarrow n=2$$

$$\frac{360}{n} \longrightarrow n>2$$

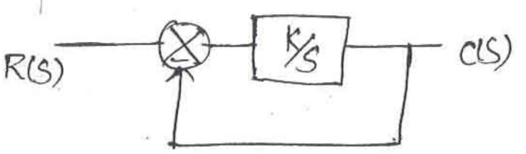
Where n = NO: of branches approaching (or) leaving the Break away Point.

2) The complex conjugate part for the branches of root locus approaching (or) leaving the break away point is a circle.

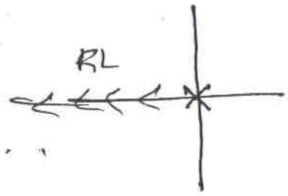
3) when ever there are two adjusently placed poles on the real axis with the section of real axis b/w them as the part g root locus then there exist a breakaway point between the addiusently placed poles.

chapter⑤ conventional Q:4

sol:- let us consider first order system:—

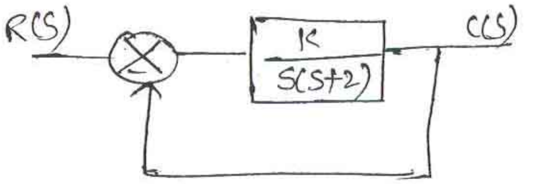



$$\frac{C(S)}{R(S)} = \frac{K}{S+K}$$



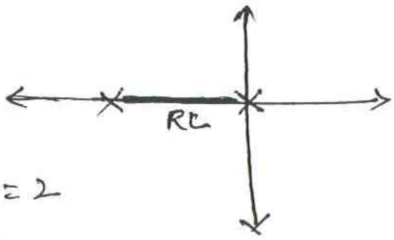

Now for root locus we need $G(S)H(S) = \frac{K}{S}$ //...

Let us consider second order system:-




$$\frac{C(S)}{R(S)} = \frac{K}{S^2+2S+K}$$

$$G(S)H(S) = \frac{K}{S(S+2)}$$




rule no:2 :- $P=2;\ Z=0;\ P-Z=2$

rule no.3:- $\theta_1 = 90;\ \theta_2 = -90$

rule no:5:- $\frac{0+(-2)-0}{2} = -1 = \sigma$

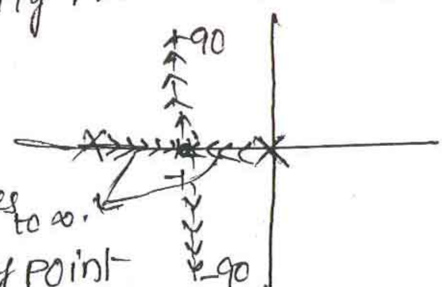
rule no:6:- when ever two poles are adjusently placed, definately there will be a Break away point.

$S^2+2S+K=0$

$K = -S^2-2S$

$\frac{dK}{dS} = -2S-2=0$

$\Rightarrow \underline{S=-1} \rightarrow$ Break away point

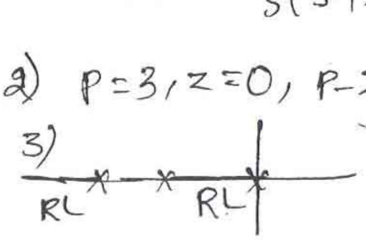



As the pole is added to first order system, the (left) poles will move towards imaginary axes. So stability will be reduces.

Ex:- $G(S) = \frac{K}{S(S+2)(S+4)}$ → by adding another pole.

2) $P=3, Z=0, P-Z=3$

3)




4) $\theta_1 = 60$
$\theta_2 = 180$
$\theta_3 = 300 = -60$

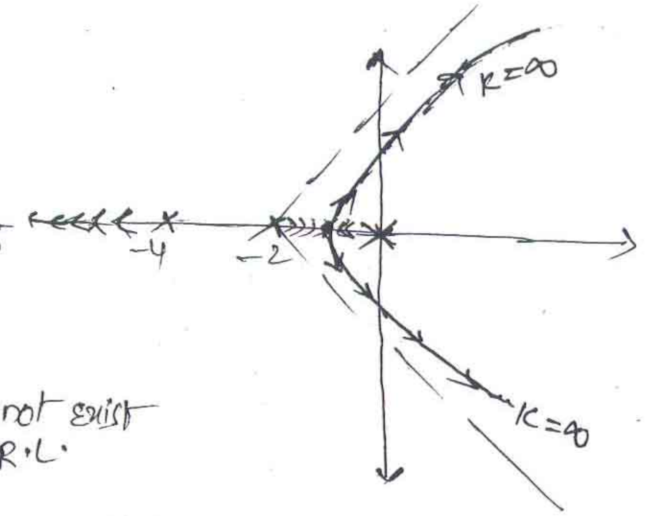
5) $\sigma = \frac{0+(-2)+(-4)}{3} = \underline{-2}$ ...

6) B.A.P: $s^3+6s^2+8s+K=0$

$K=-(s^3+6s^2+8s)$

$\frac{dK}{ds}=0 \Rightarrow -3s^2-12s-8=0$

$\Rightarrow s=-0.8,-3.15$



$s=-0.8$ ✓

$s=-3.15$ ✗ → since not exist on R.L.

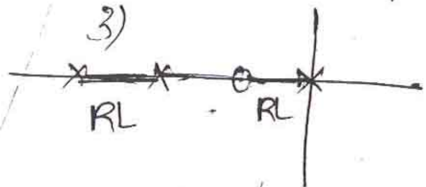
so by adding another pole the root locus will be shifted towards Imaginary axis. so stability will reduces ↓↓.

Ex:- $G(s)H(s)=\frac{K(s+1)}{s(s+2)(s+4)}$ → By adding a zero to the T.F

sol:-

2) $P=3, Z=1, P-Z=2$

3)



4) $\theta_1=90;\ \theta_2=-90$

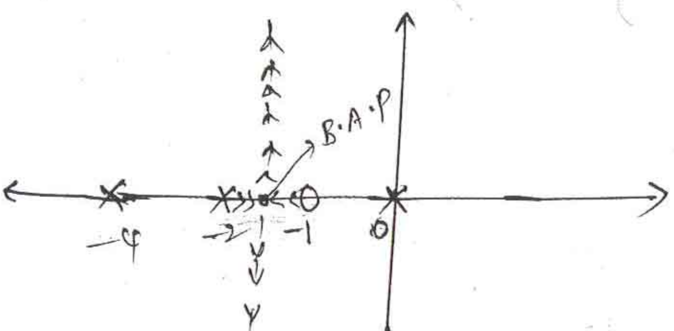
5) $\sigma=\frac{0+(-2)+(-4)-(-1)}{2}=-2.5$

6) B.A.P:- $s(s+2)(s+4)+K(s+1)=0$

$\Rightarrow K=-\frac{(s^3+6s^2+8s)}{s+1}$

$\frac{dK}{ds}=-\left(\frac{(s+1)(3s^2+12s+8)-(s^3+6s^2+8s)(1)}{(s+1)^2}\right)=0$

$\Rightarrow s=-2.8$



so By adding a zero to an (unstable T.F) T.F then its stability will be increases ↑↑.

note:- If $Z>P$ highely stables means 100% Efficiency. But 100% $\eta$ Not possible. so always $P>Z$ & we are discussing all the roots for $P>Z$ only.

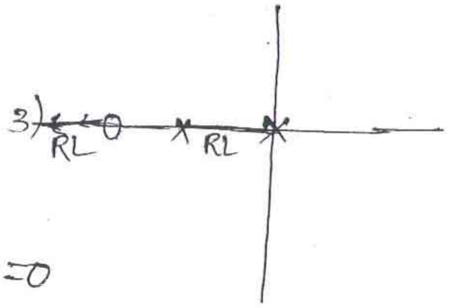
In otherwords mathematically If $Z>P \Rightarrow$ {Numerator Power > Dinominator Power} It is an Improper fraction. so always we have to intrested to take $P>Z$ only.

Note:- When ever there is a zero on real axis and to the left side of that zero, if there are no poles & zeros on the real axis, if the entire section of the real axis to the left side of zero as a part of root locus then there exists a break away point to the left side of that zero.

Ex:- $G(s) = \dfrac{K(s+2)}{s(s+1)}$

Sol:

2) $P=2,\ Z=1,\ P-Z=1$

3) 


6) B.A.P:-

Char eqn:-

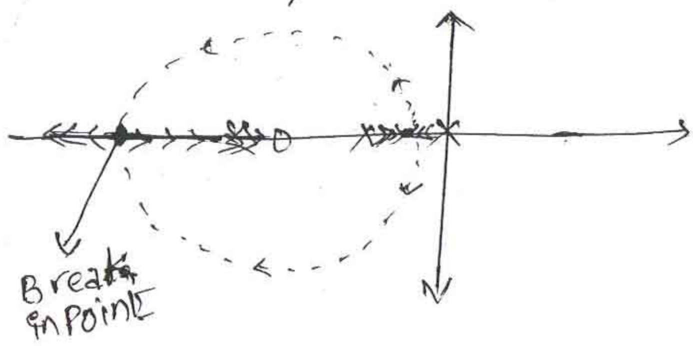
$$s(s+1) + K(s+2) = 0$$

$$\frac{dK}{ds} = \frac{d}{ds}\left(\frac{-s^2-s}{s+2}\right) = \frac{(s+2)(-2s-1) - [-s^2-s](1)}{(s+2)} = 0$$

$$\Rightarrow s = -2 \pm \sqrt{2}$$

$$s = -0.6,\ -3.4$$

Now let us see what is the shape of the root locus.




If the poles is unable to reach the zeros in real axis path, then they are choosing complex conjugate path. It will form a circle.

Note:-

→ when ever root locus is a circle, it will have appropriate & centre & appropriate radious.

→ If you choose one point at complex conjugate path that must be satisfy the Angle condition.

→ The Appropriate centre & radious are finding from Angle condi & the B.A.P, B.I.P points are on the circle.

General approach :- → (How we will get circle)

$$G(s) = \frac{K(s+b)}{s(s+a)}$$

let $s = x + jy$

$$G(x+jy) = \frac{K(x+jy+b)}{(x+jy)(x+jy+a)} = \frac{K[(x+b)+jy]}{[x^2+jxy+ax+jxy-y^2+jay]}$$

$$= \frac{K\left([x+b] + jY\right)}{x^2+ax-Y^2+j(2xY+aY)}$$

Now the Angle is

$$= \tan^{-1}\left(\frac{Y}{x+b}\right) - \tan^{-1}\left(\frac{2xY+aY}{x^2+ax-Y^2}\right) = 180$$

now Apply both sides Tan, we get $\quad \tan(A-B) = \frac{\tan(A)-\tan B}{1+\tan A \cdot \tan B}$

$\tan(180)=0$

$$\therefore \left(\frac{Y}{x+b} - \frac{2xY+aY}{x^2+ax-Y^2}\right) = 0 \times \text{dinominator}$$

$$\Rightarrow \frac{\cancel{Y}}{x+b} - \frac{2x\cancel{Y}+a\cancel{Y}}{x^2+ax-Y^2} = 0 \Rightarrow x^2+ax-Y^2-\left[(2x+a)(x+b)\right]=0$$

$$x^2+ax-Y^2-\left[2x^2+2xb+ax+ab\right]=0$$

$$-x^2-Y^2-2xb=ab$$

$$x^2+Y^2+2xb=-ab$$

$$x^2+Y^2+b^2+2xb+Y^2=-ab+b^2$$

$$(x+b)^2+Y^2=b(b-a) \quad \text{....circle eqn}$$

centre $= (-b,0) = (-2,0)$

radious $= \sqrt{b(b-a)} = \sqrt{2(2-1)}$

$= \sqrt{2}$

Short cut method:- for centre, radious

Break away Point $= -2 \pm \sqrt{2}$

$= (\text{centre} \pm \text{radious})$

$\therefore$ centre $= (-2,0)$

radious $= \sqrt{2}$

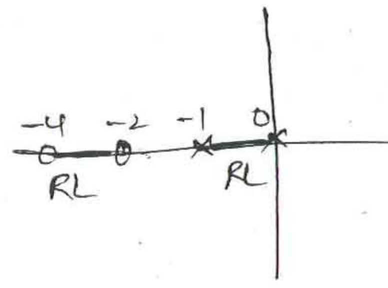
Prediction for rule no: 5 :-

when ever there are Two adjusently placed zeros on real axis with the section of real axis as a part of root locus then there Exist a break away point between adjucently placed zeros.

Ex:- $G(s) = \frac{K(s+2)(s+4)}{s(s+1)}$

Sol:

2) $P=2,\ Z=2,\ P-Z=0$

3) 

6) B.A.P:-

$$s^2+s+K(s^2+6s+8)=0$$

$$K = \frac{-s^2-s}{s^2+6s+8}$$

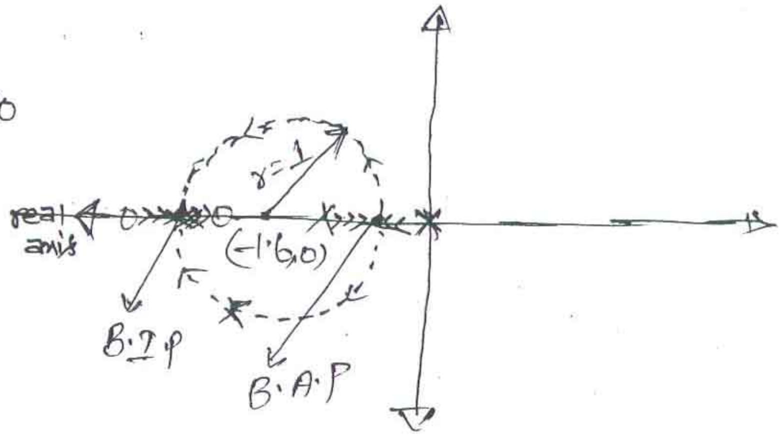
$$\frac{dK}{ds} = 0 \Rightarrow \frac{(s^2+6s+8)(-2s-1)-\left[(-s^2-s)(2s+6)\right]}{(s^2+6s+8)^2} = 0$$

$\Rightarrow 5s^2+16s+8=0$

$s = \dfrac{-16 \pm \sqrt{256-160}}{10}$

$= -1.6 \pm 1$

$s = -0.6, -2.6$




$s = -1.6 \pm 1$
$=$ centre $\pm$ radious

$\therefore$ centre $= (-1.6, 0)$
radious $= 1$

Note:- B.A.P may be complex Number. this will be understand by knowing the rule ⑦ & rule ⑧.

Rule NO: 7 :-
Intersection of root locus with Imaginary axis :-
The roots of A.E A(s) at $K = K_{mar}$ from Routh array gives the intersection of root locus with $j\omega$ axis.

Ex:- $G(s) = \dfrac{K}{s(s+2)(s+4)}$

rule no: ④ :

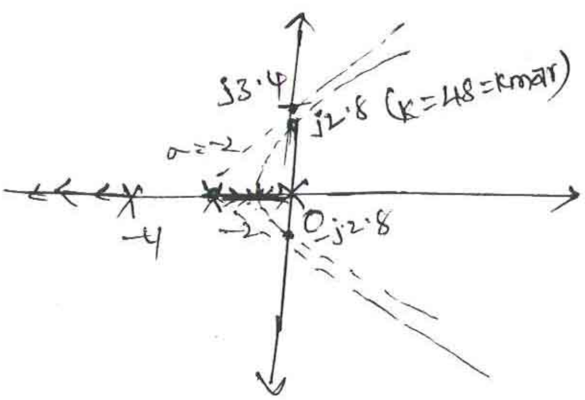
| | | |
|---|---|---|
| $s^3$ | 1 | 8 |
| $s^2$ | 6 | K |
| $s^1$ | $\frac{48-K}{6}$ | |
| $s^0$ | K | |

$\frac{48-K}{6} > 0 \Rightarrow K < 48$

$K > 0$

$\boxed{\therefore 0 < K < 48}$ stability limit

At $K = K_{mar}$
$A(s) = 6s^2 + K = 0$
$6s^2 + 48 = 0 \Rightarrow s = \pm j2.8$




Here the Root locus is may be Above 200 Asym... or below Asym...
for that we will go for -
Intersection of Asymptotes with $j\omega$ axis :-

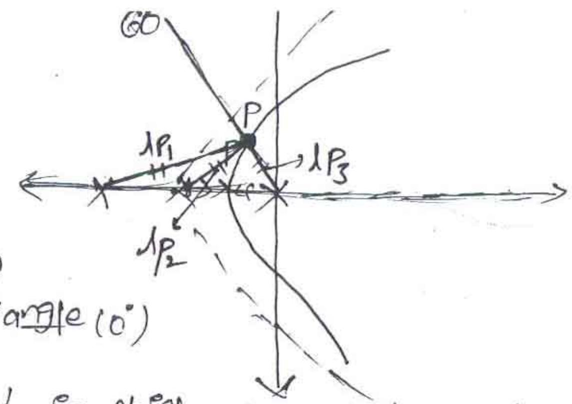
$\tan\theta = \dfrac{y}{x}$

$\tan 60^\circ = \dfrac{y}{2} \Rightarrow y = 2\sqrt{3}$
$= 3.4$
$\sim j3.4$

So Root locus cuts the $j\omega$ axis below Asymptote.

Q:- Find K when ζ = 0.5.

Sol: $\cos\theta = \zeta \Rightarrow \theta = 60^\circ$




when 'ζ' is given & find the value of $k = ?$ for that first we will find θ & then draw a line with that angle from ref angle (0°) in +ve direction (ccw).

Now make a point 'P' in which the root locus touches that angle line. Now gain (k) at P = ? for that short cut method is

$$\boxed{k = \frac{\text{Product of vector lengths of Poles}}{\text{Product of vector lengths of zeros}}}$$

$$k = \frac{l_{P_1} \times l_{P_2} \times l_{P_3}}{1}$$

note:- while drawing this analysis make x-axis & Y-axis with equal exact scale on paper.
(or) appropriate ......

Angle of departure & arrival :-

* the angle of departure is obtained when complex Poles terminate at ∞.

* the Angle of arrival is obtained when poles Terminate at zero (0).

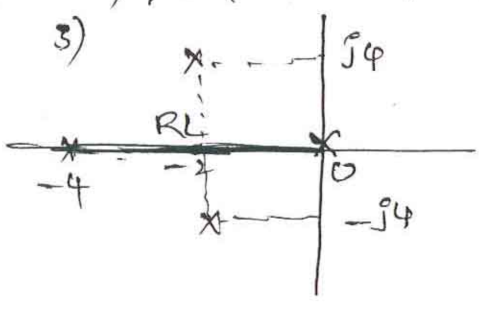
$$\boxed{\begin{matrix}\phi_D = 180 + \phi \\ \phi_A = 180 - \phi\end{matrix}} \quad \text{where} \quad \boxed{\phi = \Sigma\phi_z - \Sigma\phi_p}$$

PageNo: (70)

Q: (11):- $G(S)H(S) = \dfrac{K}{S(S+4)(S^2+4S+20)}$

Sol: 2) $P=4, Z=0, P-Z=4$

3)




4) $\theta_1 = 45^\circ, \theta_2 = 135^\circ, \theta_3 = 225^\circ$

5) $\sigma = \dfrac{0+(-4)+(-2)+(-2)-0}{4}$

$= -2$

6) B.A.P:- C.E ⇒ $S^4 + 8S^3 + 36S^2 + 80S + K = 0$

$\Rightarrow K = -S^4 - 8S^3 - 36S^2 - 80S$

(P6) The OLTF of unity feedback system is $G(s) = \frac{K}{s(1+sT)}$

The ratio $\frac{TK_1 - 1}{TK_2 - 1}$ when $\zeta$ changes from 0.16 to 0.45 will be

(a) $\cong 20$  (b) $\cong 30$  (c) $\cong 40$  (d) $\cong 50$

sol:

$1 + G(s) = 0$

$1 + \frac{K}{s(Ts+1)} = 0$

$Ts^2 + s + K = 0$

$s^2 + \frac{s}{T} + \frac{K}{T} = 0$

$\Rightarrow \omega_n = \sqrt{\frac{K}{T}}$

$\zeta = \frac{1}{2\sqrt{KT}}$ //....

(i) $\zeta = \zeta_1 = 0.16$

let $K = K_1$

$0.16 = \frac{1}{2\sqrt{K_1 T}}$

$K_1 T = 9.76$

$\Rightarrow K_1 T - 1 = 9.76 - 1$

$K_1 T - 1 = 8.76$ ....

(ii) $\zeta = \zeta_2 = 0.45$

let $K = K_2$

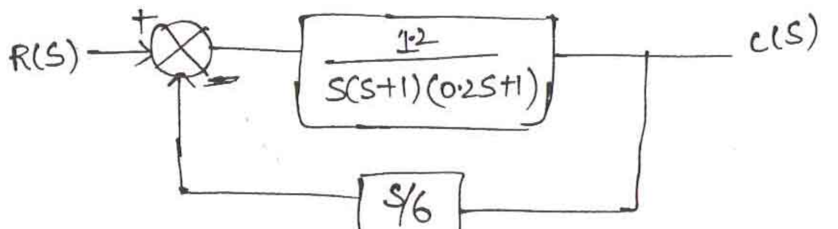
$0.45 = \frac{1}{2\sqrt{K_2 T}}$

$K_2 T = 1.23$

$K_2 T - 1 = 0.23$ ...

$\therefore \frac{K_1 T - 1}{K_2 T - 1} = \frac{8.76}{0.23} \cong 38 \cong 40$ ....

Q:1 65 page




sol:

Char" Eq" $1 + G(s)H(s) = 0$

$1 + \frac{20 \times 1.2}{s(s+1)(0.2s+1)} \times \frac{s}{6} = 0$

$\Rightarrow 0.2s^2 + 1.2s + 1 + 4 = 0$

$s^2 + 6s + \frac{5}{0.2} = 0 \Rightarrow s^2 + 6s + 25 = 0$

i) $\omega_n = 5$ ; $\zeta = 0.6$

ii) $\omega_d = \omega_n\sqrt{1-\zeta^2} = 5\sqrt{1-(0.6)^2} = 4$ rad/sec

iii) $\zeta = 0.6 \Rightarrow$ damping factor $(\alpha) = \zeta\omega_n = 5 \times 0.5 = 3$

iv) Maximum overshoot % $M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} = e^{-0.6\pi/\sqrt{1-(0.6)^2}} \times 100$

$= 9.46\%$

v) $T_s$ = settling time

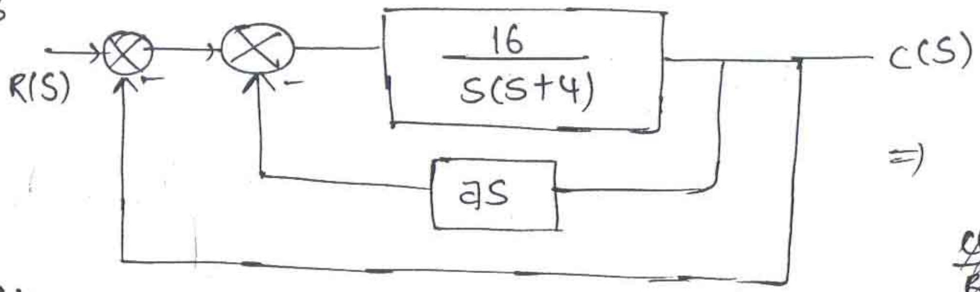
$2\% \rightarrow \frac{4}{\zeta\omega_n} = \frac{4}{3} = 1.33$ sec

$\omega_d = 2\pi f_d \Rightarrow f_d = \frac{2\pi}{4} = \frac{\pi}{2}$ z. $\frac{\omega_d}{2\pi} = 0.63$ cycle/sec

vi) No. of cycles 2% T.B $= t_s \times f_d = 1.33 \times 0.63 = 0.84$ cycles

vii) $T = 1/f_d = \frac{1}{0.63} = 1.58$ sec

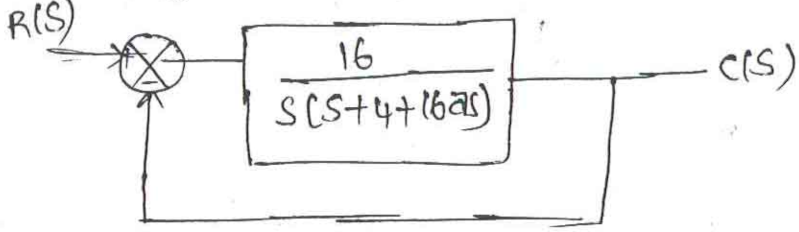

Q:3




$$\Rightarrow \frac{16/s(s+4)}{1+\frac{16}{s(s+4)}\cdot(as)}$$

$$\frac{C(S)}{R(S)} = \frac{16}{s(s+4+16a)}$$

Sol:-




i) % $M_p = 1.5$ % then $a = ?$

$$e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.015$$

$$\Rightarrow \zeta = 0.8$$

C.E :-

$$1 + G(S)H(S) = 0$$

$$\Rightarrow 1 + \frac{16}{S(S+4+16as)} = 0$$

$$\Rightarrow s^2 + (4+16a)s + 16 = 0$$

$$\omega_n = 4;\ \cancel{2}\omega_n\zeta = \cancel{2}(2+8a) \Rightarrow a = 0.15$$

ii) $e_{ss}$ for unit ramp input

ⓐ without a :-

$a = 0$

$$G(S) = \frac{16}{S(S+4)}$$

$$e_{ss} = \lim_{S\to 0} \frac{S \cdot R(S)}{1+G(S)H(S)}$$

$$= \lim_{S\to 0} \frac{S \cdot (1/S^2)}{1+\frac{16}{S(S+4)}}$$

$$= \frac{1}{4} = 0.25 \text{ units}$$

ⓑ with 'a' :-

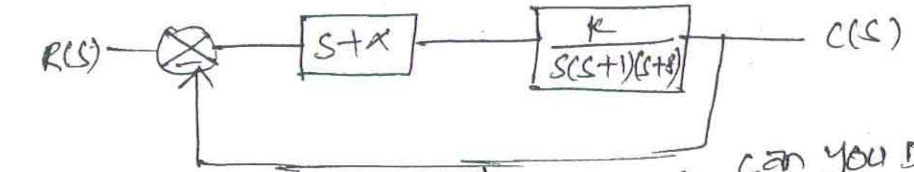
$$G(S) = \frac{16}{S(S+4)+16as}$$

$$e_{ss} = \lim_{S\to 0} \frac{S \cdot (1/S^2)}{1+\frac{16}{S(S+4+16a)}}$$

$$= 0.4 \text{ units}$$

for $K>0$ the system is stable. So the RL is exist in left half of s-plane & there is no point of intersection. So Ans: C.

Q:- Draw the Root locus for a given control system.




can you Draw the root locus? Ans: No

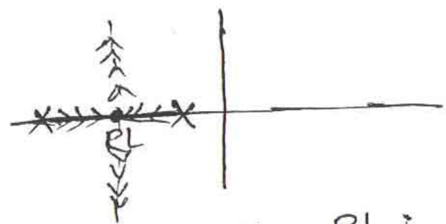
Sol:- $G(S)H(S) = \frac{K(S+\alpha)}{S(S+1)(S+8)}$

We can draw a RL for any control system, It is necessary that all the poles & zeros must be specified. If any poles or zeros are not specified we draw the root contour.

ROOT CONTOURS:-

Root contours are multiple root locus diagrams obtained by varying multiple parameters in a Transfer funⁿ drawn on same s-plane.

Case 1:- let 'K' only consider, & $\alpha = 0$.




so $G(S) = \frac{K\cancel{S}}{\cancel{S}(S+1)(S+8)} = \frac{K}{(S+1)(S+8)}$

for such type of T.F we have already drawn the RL.

Case 2:-

To draw w.r.to $\alpha$. ⇒ That means $K=0$ ✗

Actually we are drawn the root locus by varying 'K' from 0 to $\infty$, & Not by varying the poles locations ($\alpha$).

In that situations we are taking the C.E.

$$C.E = 1 + \frac{K(S+\alpha)}{S(S+1)(S+8)} = 0 \Rightarrow S(S+1)(S+8) + K(S+\alpha) = 0$$

$$\Rightarrow S(1+S)(S+8) + KS + K\alpha = 0$$

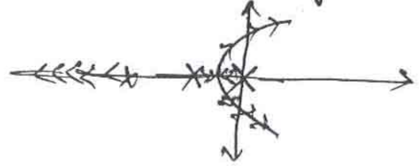
$$\Rightarrow 1 + \frac{K\alpha}{S(S+1)(S+8)+KS} = 0$$

$$\Rightarrow G(S)H(S) = \frac{K\alpha}{S(S+1)(S+8)+KS}$$

If he doesn't given 'K' value put K as min. value. If he given then you can take that corresponding K value.

Put $K=1$;

$$G(S)H(S) = \frac{\alpha}{S(S^2+9S+9)}$$



Q:- find B.A.P for K=10 above problem;

Sol:- for finding the B.A.P we are taking case(ii) instead of case(i), it is easily understand by seeing that.

In case(i) $\alpha=0$; but in case(ii) we are taking the usual T.F with $K$ & $\alpha$.

$$\therefore G(s)H(s) = \frac{10\alpha}{s(s+1)(s+8)+10s}$$

let $10\alpha = K'$

$$\Rightarrow G(s)H(s) = \frac{K'}{s(s+1)(s+8)+10s}$$

$$= \frac{K'}{s(s^2+9s+18)}$$

$$C.E \Rightarrow s(s^2+9s+18)+K'=0$$

$$\Rightarrow \frac{dK'}{ds}=0 \Rightarrow 3s^2+18s+18=0$$

$$s^2+6s+6=0$$

$$s = \frac{-6 \pm \sqrt{36-24}}{2}$$

$$= -3 \pm \sqrt{3}$$

$$= -1.26, -4.73$$

(Pb) If in the Pblm he is asking Draw the RL for K=10?

Sol: i) first we have to take $\alpha=0$ & then draw the RL as 'K' alone.

ii) Now place $K=10$; $10\alpha=K'$ $\Rightarrow$ Draw the Root locus.

NOTE:- sensitivity is defined as the ability of the system to detect smallest change in it.

$$1 + \left[-\frac{K(S-1)}{S(S+3)}\right] = 0$$

$$\Rightarrow S(S+3) - K(S-1) = 0$$

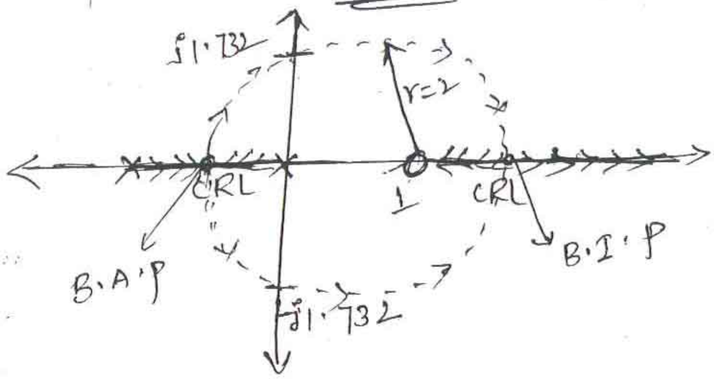
$$K = \frac{(S+3)S}{S-1} = \frac{-S^2-3S}{1-S} \;//\ldots$$

$$\frac{dK}{dS} = 0 \Rightarrow S^2 - 2S - 3 = 0$$

$S = 1 \pm 2$ = centre ± radices

(+1, 0)    $2 = r$

$\underline{S = -1, 3}$



7) C.E. $S^2 + 3S - KS + K = 0$

| | | |
|---|---|---|
| $S^2$ | 1 | K |
| $S^1$ | 3-K | 0 |
| $S^0$ | K | |

$K > 0$; $3 - K = 0$
$\boxed{K = 3} = K_{mar}$

$S^2 + K = 0$
$S = \pm j1.732 \ldots$

stability limit $\boxed{0 < K < 3}$ //..

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Q:(8) Ans: C

Q:(7): Ans: E

Q:(5): Ans: C

Q:(4): Ans: D

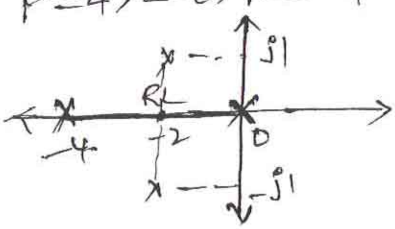
Q:(3): ***

Sol:-

$$G(S) = \frac{K}{S(S+4)(S^2+4S+5)}$$

2) $P = 4, Z = 0$; $P - Z = 4$

3)



5) $\sigma = \frac{(0) + (-4) + (-2) + (-2)}{4} = -2$

4) $\theta_1 = 45°$, $\theta_2 = 135°$; $\theta_3 = 225°$; $\theta_3 = 315°$

6) B.A.P :- C.E ⇒ $S(S+4)(S^2+4S+5) + K = 0$

$S^4 + 8S^3 + 21S^2 + 20S + K = 0$

$\Rightarrow K = -S^4 - 8S^3 - 21S^2 - 20$

$\frac{dK}{dS} = 0 \Rightarrow 4S^3 + 24S^2 + 42S + 20 = 0$

$\Rightarrow S = -2, -0.78, -3.22 \ldots$

All 3 B.A.P are valid ...

Ans: - 3 real B.A.P

7)

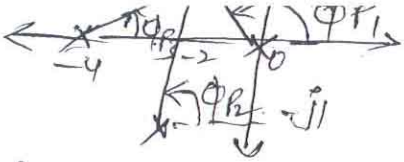
| | | | |
|---|---|---|---|
| $S^4$ | 1 | 21 | K |
| $S^3$ | 8 | 20 | 0 |
| $S^2$ | 18.5 | K | 0 |
| $S^1$ | $\frac{370-8K}{18.5}$ | 0 | 0 |
| $S^0$ | K | 0 | 0 |

$\frac{370-8K}{18.5} = 0 \Rightarrow K = K_{mar} = 46.25$

$A(S) = 18.5S^2 + 46.25 = 0$

$\Rightarrow S = \pm j1.58 \ldots$

$= \underline{\underline{2}}$



(8) Angle of Departure:—

$\phi_{P_1} = 180 - \tan^{-1}(1/2) = 153.5^\circ$

$\phi_{P_2} = 90^\circ$

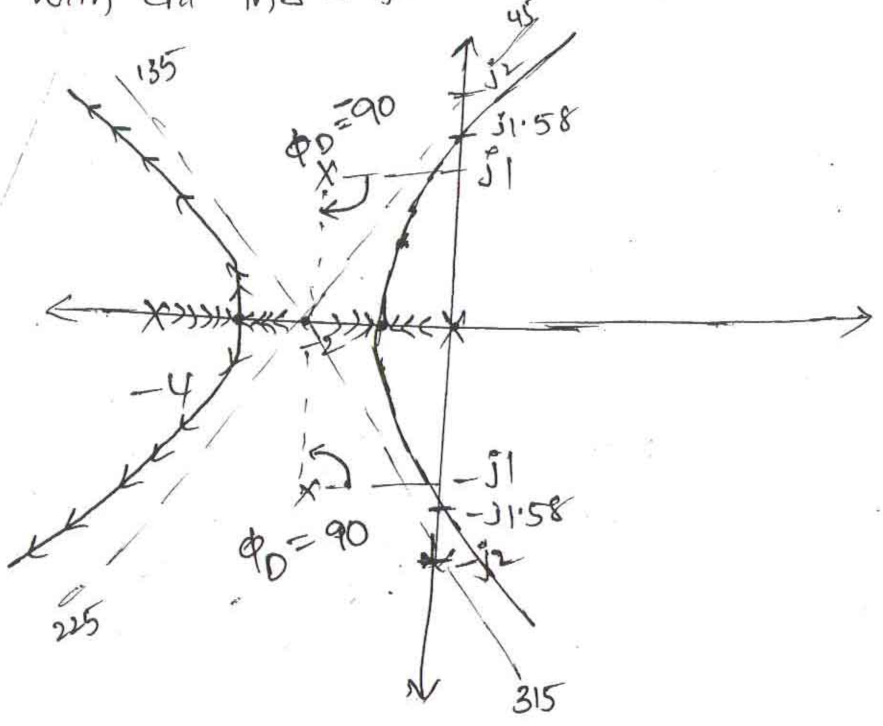
$\phi_{P_3} = \tan^{-1}(1/2) = 26.5^\circ$

$\Sigma\phi_P = \phi_{P_1} + \phi_{P_2} + \phi_{P_3} = 270^\circ$

$\therefore \phi = \Sigma\phi_Z - \Sigma\phi_P = 0 - 270^\circ = \underline{\underline{-270^\circ}}$

$\therefore \phi_D = 180 + \phi = \underline{\underline{-90^\circ}}$

with all this information now we will draw the root locus.

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Q:1:—

check for Root locus is always Routh-Hurwitz array.

$$G(s) = \frac{K(s+a)}{s^2(s+b)}$$

$$\Rightarrow s^3 + bs^2 + Ks + a = 0$$

| | | |
|---|---|---|
| $s^3$ | 1 | K |
| $s^2$ | b | aK |
| $s^1$ | $\frac{bK-aK}{b}$ | |
| $s^0$ | aK | |

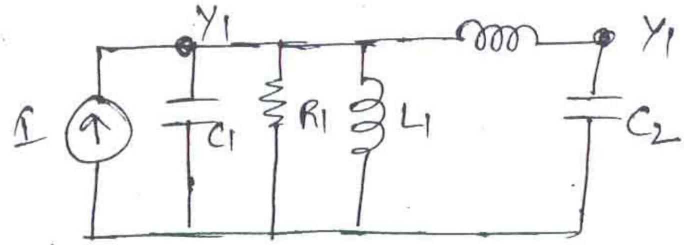
$\frac{bK-aK}{b} > 0$

$\Rightarrow bK > aK$ ; $aK > 0$

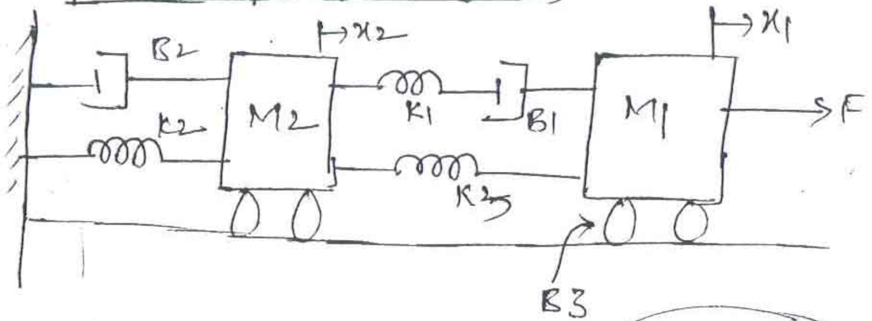
$\underline{b > a}$ $\boxed{K > 0}$

$\therefore K_{mar} = K = 0$ ; A.E $\Rightarrow$ $A(s) = \frac{bK-aK}{b} \cdot s = 0$

$\Rightarrow \underline{s = 0}$



(Pb)



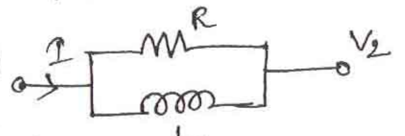
Sol:-

$$F = M_1\frac{d^2x_1}{dt} + B_3\frac{dx_1}{dt} + B_1\frac{d(x_1-x_2)}{dt} + K_1(x_1-x_2) + K_3(x_1-x_2)$$

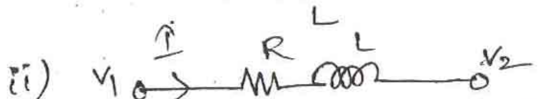
$$0 = M_2\frac{d^2x_2}{dt^2} + B_2\frac{dx_2}{dt} + K_2x_2 + B_1\frac{d(x_2-x_1)}{dt} + (x_2-x_1)(K_1+K_3)$$

But we can't write like this.

note:-

i) 

$$I = \frac{V_1-V_2}{R} + \frac{1}{L}\int(V_1-V_2)\,dt$$

ii) 

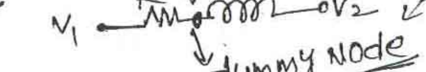
$I = ?$

$$I = \frac{V_1-V_2}{R+jX_L} = \frac{V_1-V_2}{Z} ✓.$$

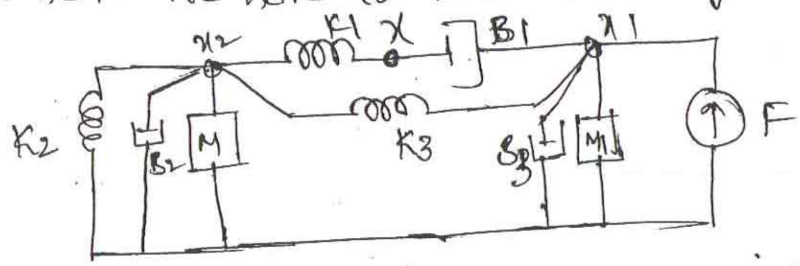
$\left(\cancel{I = \frac{V_1-V_2}{R} + \frac{1}{L}\int(V_1-V_2)dt}\right)$ you should not write like that so. both. But

where 'Z' is the total impedence i.e combination of $R$, $L$.

so here we have to take a dummy node.



dummy Node like that here also



Note:- Assume a dummy node b/w the series elements.

At Node $x_1$:-

$$F = M\frac{d^2x_1}{dt^2} + B_3\frac{dx_1}{dt} + B_1\frac{d}{dt}(x_1-x) + K_3(x_1-x_2)$$

At dummy Node '$x$':-

$$0 = B_1\frac{d(x-x_1)}{dt} + K_1(x-x_1)$$

At node $x_2$:-

$$0 = M_2\frac{d^2x_2}{dt^2} + B_2\frac{dx_2}{dt} + K_2x_2 + K_1(x_2-x_1) + K_3(x_2-x_1)$$

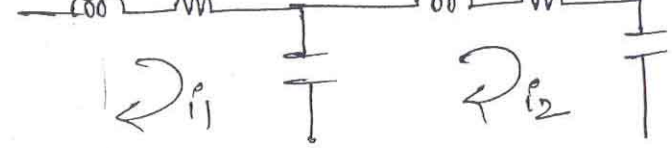
Force - voltage Analogy :-

$$V = L_1\frac{di_1}{dt} + R_3 i_1 + R_1(i_1 - i) + \frac{1}{C_3}\left(\int (i_1 - i_2)\,dt\right)$$

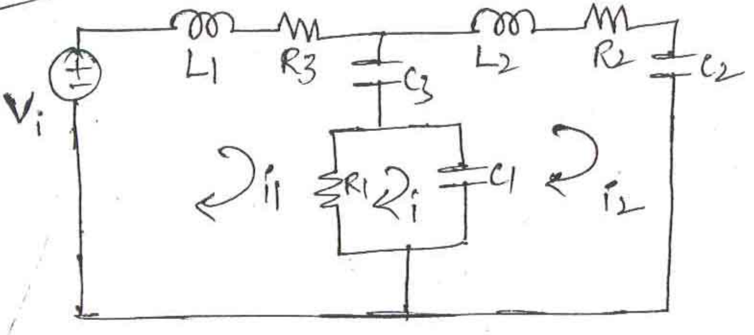
$$0 = R(i - i_1) + \frac{1}{C_1}(i - i_1)$$

$$0 = L_2\frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt + \frac{1}{C_1}\int (i_2 - i)\,dt + \frac{1}{C_3}\int (i_2 - i_1)\,dt$$

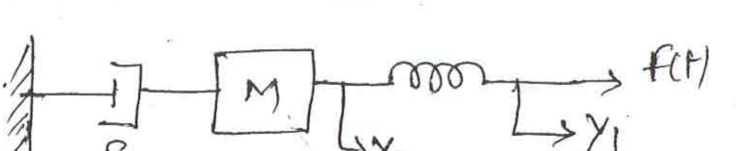
step(1) :-




step(2) :-




(Pb)



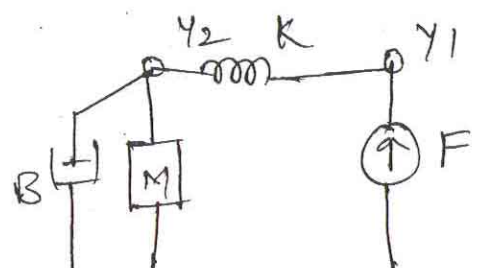

(a) $M\frac{d^2y_1}{dt^2} + B\frac{dy_1}{dt} = K(y_1 - y_2)$

(b) $M\frac{d^2y_1}{dt^2} + B\left(\frac{dy_1}{dt}\right) = K(y_2 - y_1)$

(c) $M\frac{d^2y_2}{dt^2} + B\frac{dy_2}{dt} = K(y_2 - y_1)$

(d) $M\frac{d^2y_2}{dt^2} + B\frac{dy_2}{dt} = K(y_1 - y_2)$

Sol :-

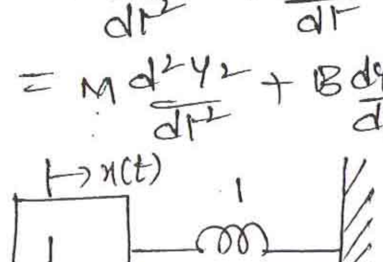



$$F = K(y_1 - y_2) \Rightarrow y_1 = \frac{F + Ky_2}{K}$$

$$0 = M\frac{d^2y_2}{dt^2} + B\frac{dy_2}{dt} + K(y_2 - y_1)$$

$$= M\frac{d^2y_2}{dt^2} + B\frac{dy_2}{dt} + Ky_2 - K[y_1]$$

(Pb)




(a) $\sin t$ (b) $\sin\sqrt{t}$ (c) $\sin\frac{t}{\sqrt{2}}$ (d) $\sin 2t$

for ~~The Pole of mechns~~ unit impulse force The resulting oscillation will be?

Sol :-

$$F = M\frac{d^2x}{dt^2} + Kx$$

Given $M = K = 1$

$$F = \frac{d^2x}{dt^2} + x \Rightarrow F(S) = (S^2+1)\,X(S)$$

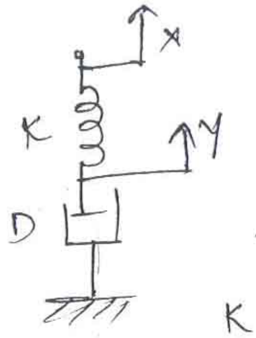
$$X(S) = \frac{1}{\ } F(S) \Rightarrow X(S) = \frac{1}{S^2+1} \quad (1)$$

Since $F(S)$ = unit impulse response force ...

$$X(S) = \frac{1}{S^2+1}$$

$$\Rightarrow x(t) = \underline{\sin t}$$

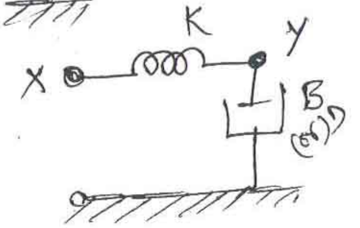
(Pb)




The pole of the mechanical system will be

ⓐ $0, -\frac{K}{D}$ ⓑ $0, -D/K$ ⓒ $-\frac{K}{D}$ ⓓ $-D/K$

Sol:-




There is no force & No mass: Analyse the Diagram as it is.

At node Y :-

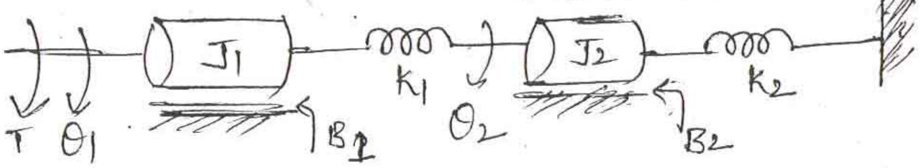
$$0 = D\frac{dy}{dt} + K(Y-X)$$

$$0 = (DS+K)Y(S) - KX(S)$$

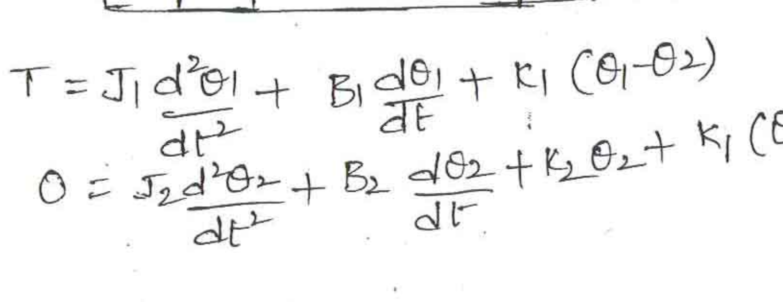
$$\Rightarrow \frac{Y(S)}{X(S)} = \frac{K}{DS+K}$$

$$\boxed{\text{Pole } S = \frac{-K}{D}}$$

(Pb)




Sol:- Nodal diagram for the Rotational system :-




$$T = J_1\frac{d^2\theta_1}{dt^2} + B_1\frac{d\theta_1}{dt} + K_1(\theta_1-\theta_2)$$

$$0 = J_2\frac{d^2\theta_2}{dt^2} + B_2\frac{d\theta_2}{dt} + K_2\theta_2 + K_1(\theta_2-\theta_1)$$

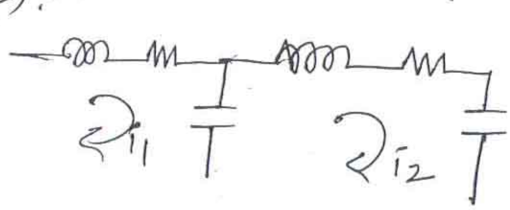
T-V Analogy :-

$$V = L_1\frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int(i_1-i_2)\,dt$$

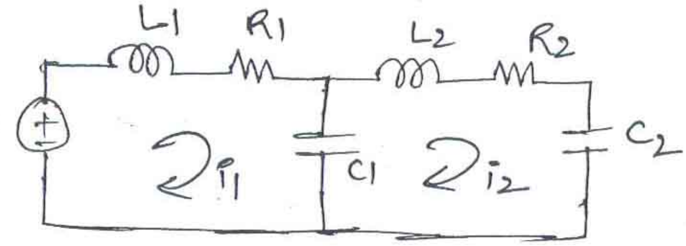
$$0 = L_2\frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int(i_2)dt + \frac{1}{C_1}\int(i_2-i_1)dt$$

step(1):-


step(2):-




step(3):-



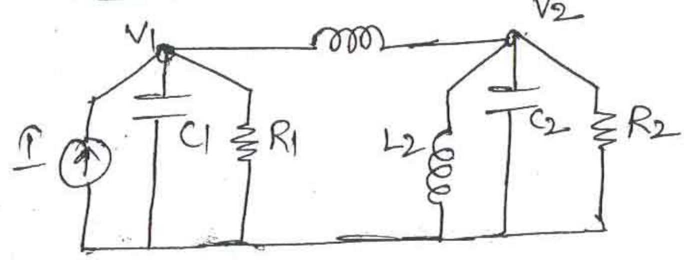

T-I Analogy:-

$$I = C_1\frac{dV_1}{dt} + \frac{V_1}{R_1} + \frac{1}{L_1}\int (V_1 - V_2)\,dt$$

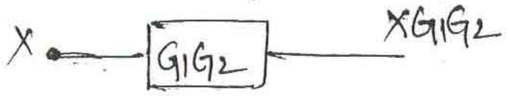
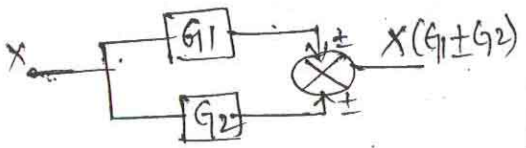
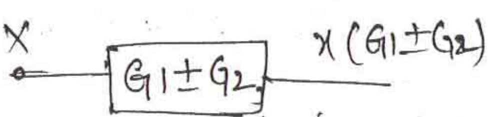
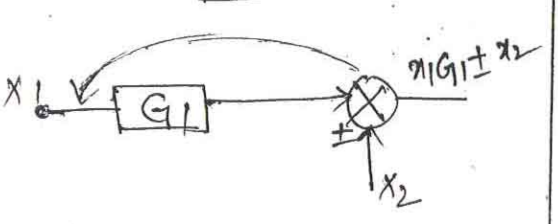
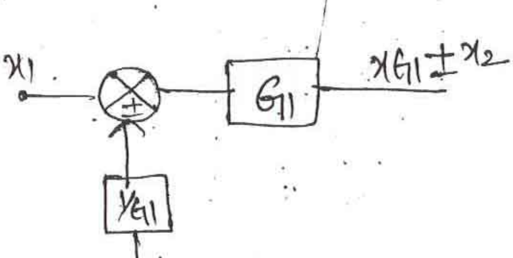
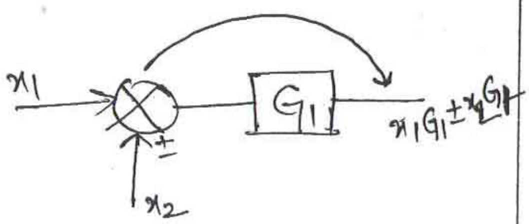
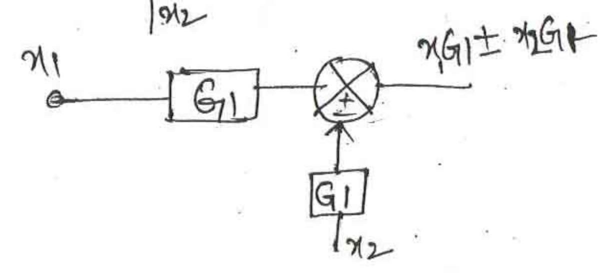
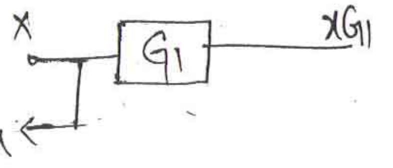
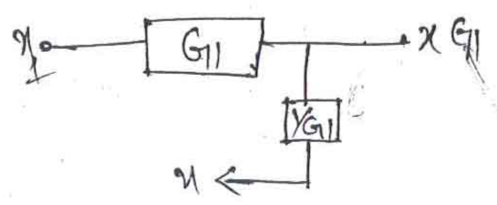
$$0 = C_2\frac{dV_2}{dt} + \frac{V_2}{R_2} + \frac{1}{L_2}\int V_2\,dt + \frac{1}{L_1}\int (V_2 - V_1)\,dt$$

step(1):-

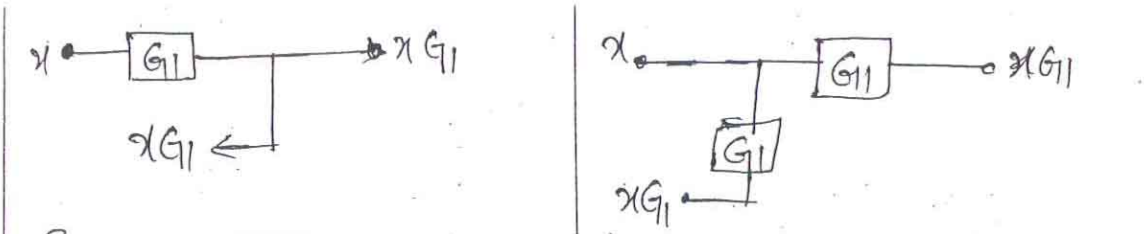

step(2):-



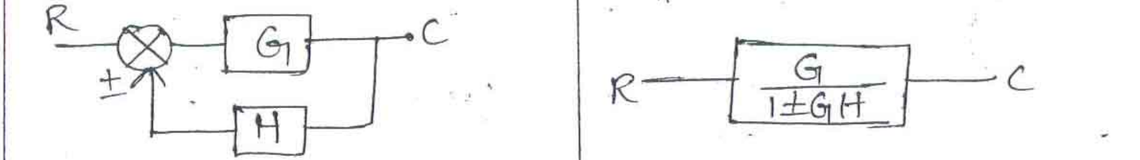

## Block Diagram Algebra:-

| Rules | original diagram | equivalent diagram. |
| --- | --- | --- |
| 1) combining blocks in series |  |  |
| 2) combining blocks in parallel |  |  |
| 3) shifting the summing point before the block |  |  |
| 4) shifting the summing point after the block |  |  |
| 5) shifting the take off point after the block |  |  |

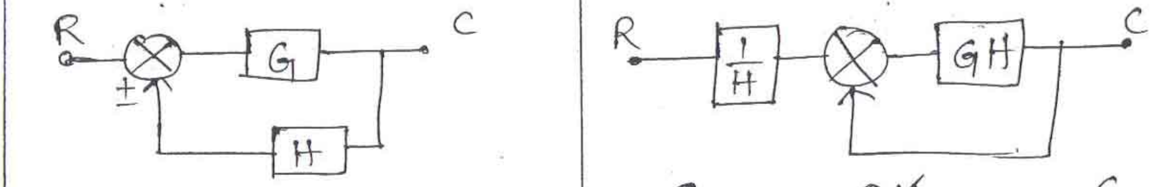
6. shifting the takeoff point before the block.



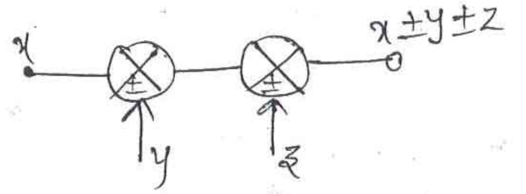

7. Transfer func of closed loop c.s



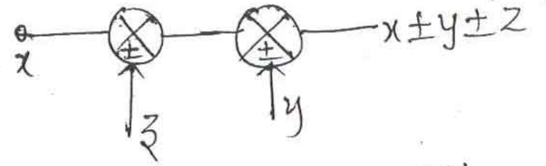

8. Block diagram transformation




9. Interchanging the summing elements

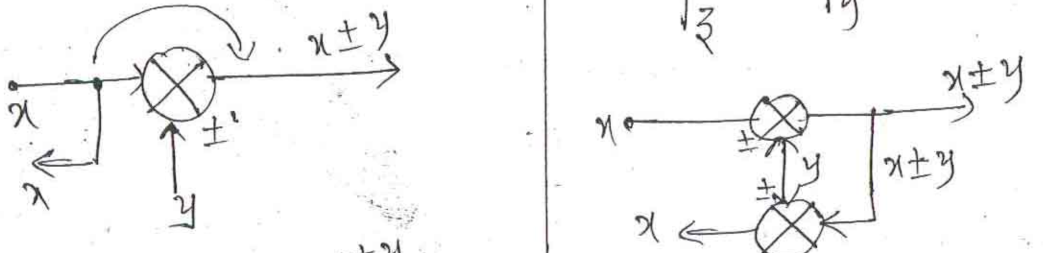



$$\frac{C}{R} = \frac{1}{\cancel{H}} \times \frac{G\cancel{H}}{1 \pm GH} = \frac{G}{1 \pm GH}$$



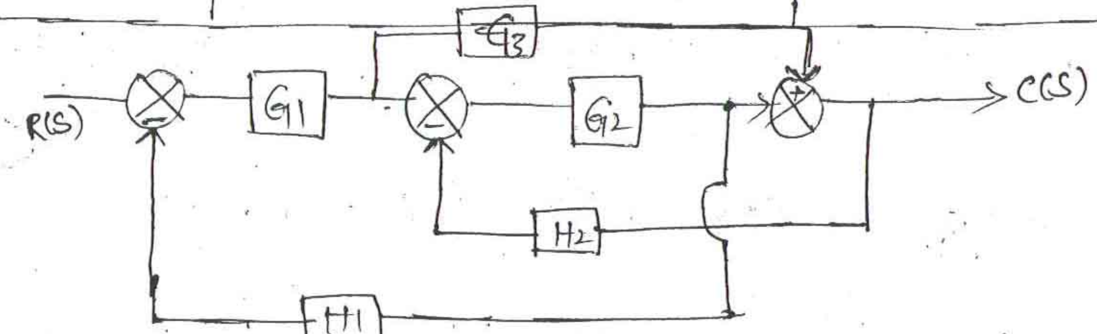

CRITICAL Rules

10. shifting the take off point after the summing point

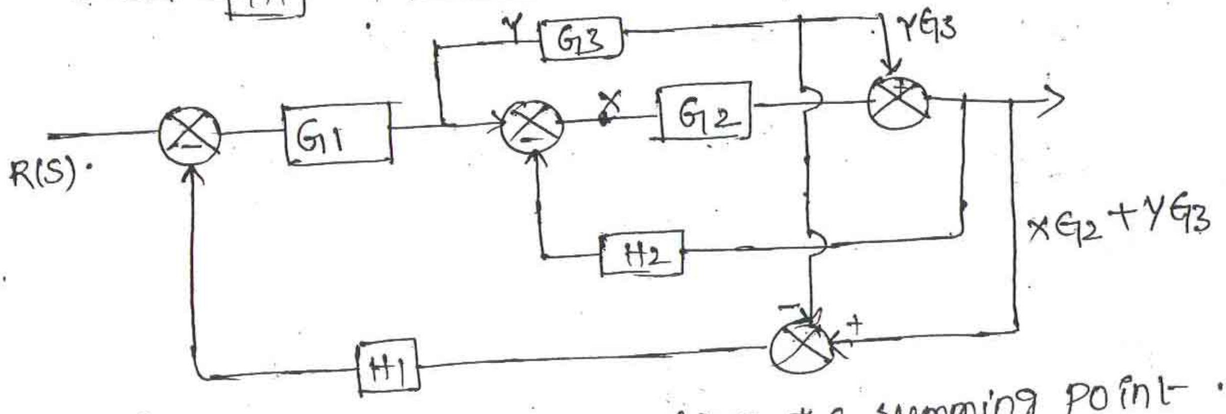



11. shifting the takeoff point before the summing point


Pb

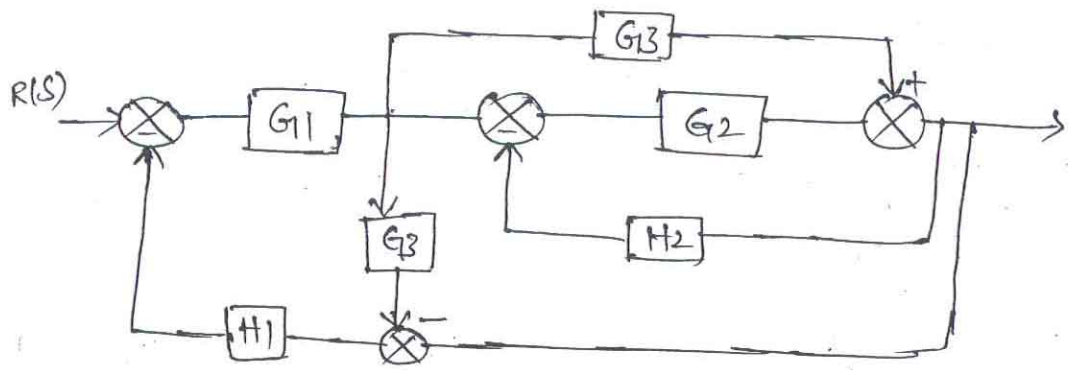


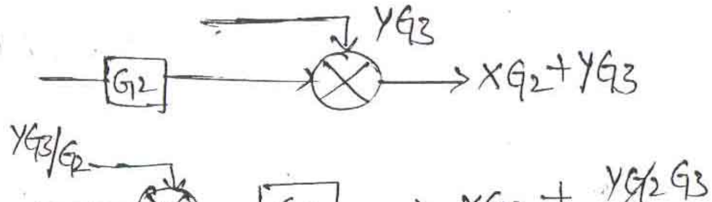

Sol:-




step(1) :- shift takeoff point after the summing point.

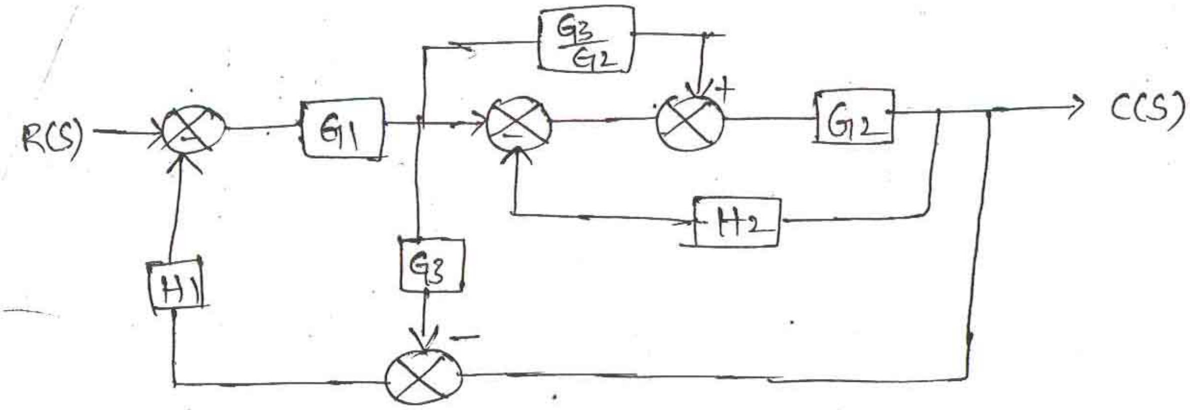
step(2) :- shift take off point before G3 block.



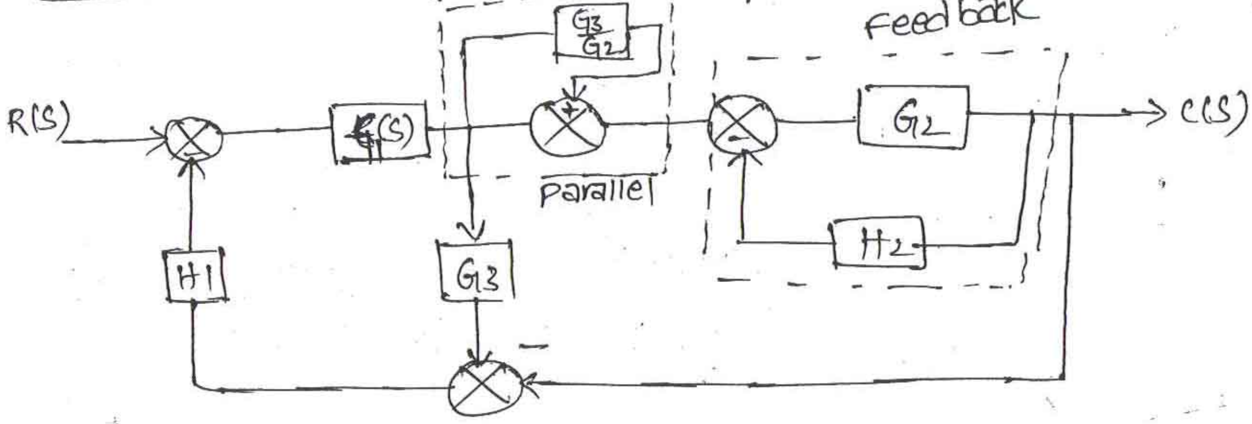




$$XG_2 + \frac{Y\cancel{G_2}G_3}{\cancel{G_2}}$$

step(3):- shifting the summing point before G2.




step(4):- Interchange the Summing Point

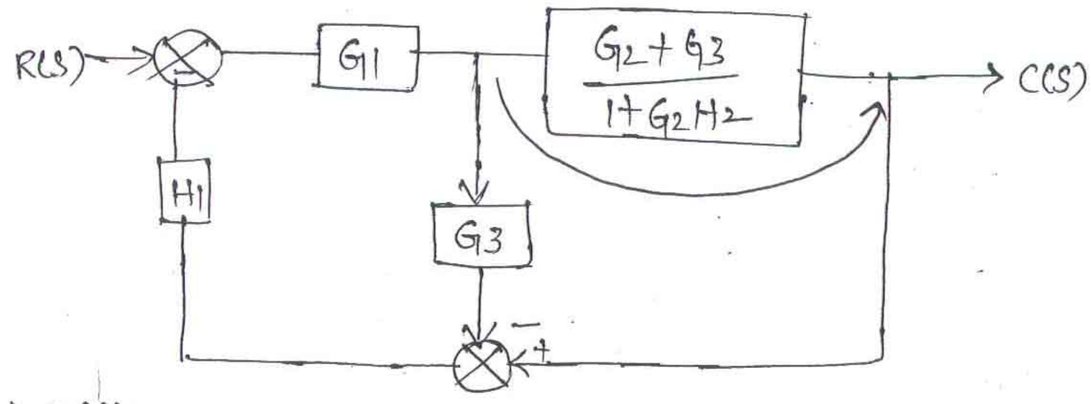



parallel :- $1 + \frac{G_3}{G_2} = \frac{G_2 + G_3}{G_2}$

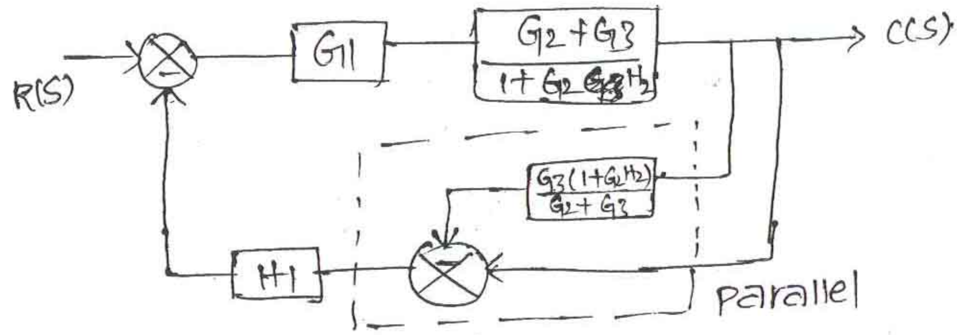
feedback :- $F.B = \frac{G_2}{1 + G_2H_2}$

series : $\frac{G_2 + G_3}{1 + G_2H_2}$

step(5):-




step(6):- shift the summing point.



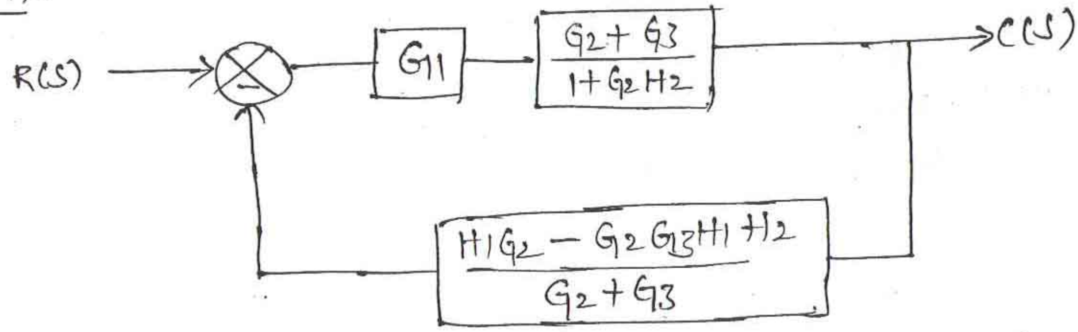

parallel:-

$$1-\frac{G_3(1+G_2H_2)}{G_2+G_3} = \frac{G_2+\cancel{G_3}-\cancel{G_3}-G_2H_2G_3}{G_2+G_3}$$

In series with H1.

$$\Rightarrow \frac{H_1G_2 - G_2G_3H_2H_1}{G_2+G_3} \quad //$$

step(7):-

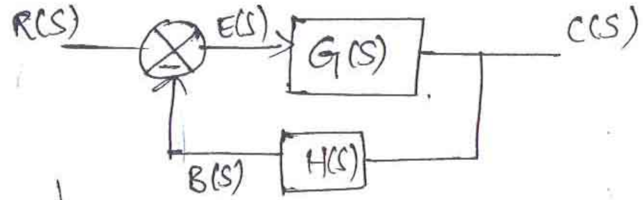



$$\frac{C(S)}{R(S)} = \frac{G_1G_2+G_1G_3}{1+G_2H_2+G_1G_2H_1-G_1G_2G_3H_1H_2}$$

# SIGNAL FLOW GRAPH :-

It is the Graphycal Representation of control system in which nodes representing each of the system variables are connected by direct branches.

## S.F.G. for C.L.C.S :-



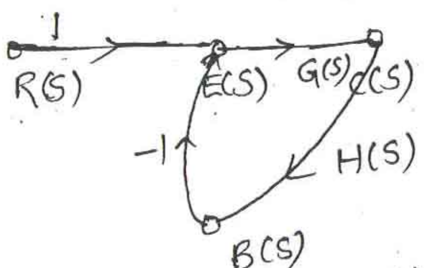

Block diagram

Now we have to draw the S.F.G for the given block diagram.

step(1):- Identify the No: of nodes from input to output.

1 O R(S) 2 O E(S) 3 O C(S) → nodes
4 O B(S)

Step(2):-
Now Join the nodes with corresponding gains.



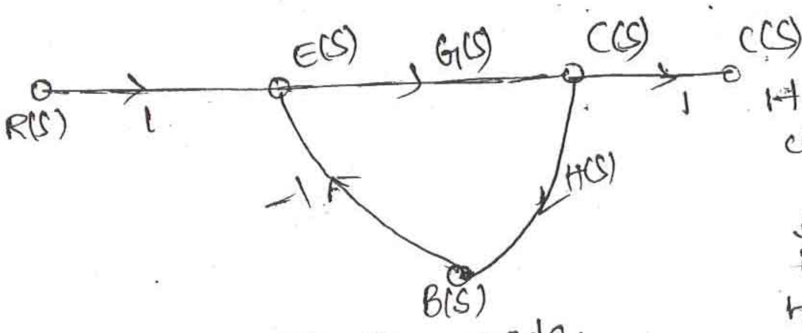

This is the signal flow graph for above Block dgm. where R(S) is the i/p node & C(S) is the o/p node.

## TERMINOLOGY OF S.F.G's :-

1) Node :- It represents the system variables and is equal to the sum of all incoming signals at it.

2) Input Node :- (or) Source Node :-
It is the node having only outgoing branches.

3) Output Node (or) Sink Node :-
It is the node, which is having only incoming branches

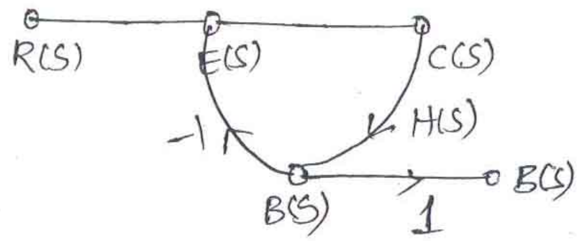



Where R(S) = Source node.
C(S) = Sink node

Here we are taking C(S) as sink node. but as per the defination sink node only have incoming branches, not having outgoing branches. so we have to extend C(S) by gain 1.

∴ C(S) = i/p × TF = C(S) × 1
= C(S) //

we can take any Node as sink node. If we are taking B(S) as o/p node then we have to extend that with a gain of unity.




4) Mixed Node/ chain Node :-
It is a node having both incoming & outgoing branches.

5) Path :- It is a Node the Traversal of the connected branches in the direction of branch arrow, such that No node is traversed more than once.

6) Forward path :- It is a Path ~~which~~ originates & ~~Ter.~~ from Input node to output node.

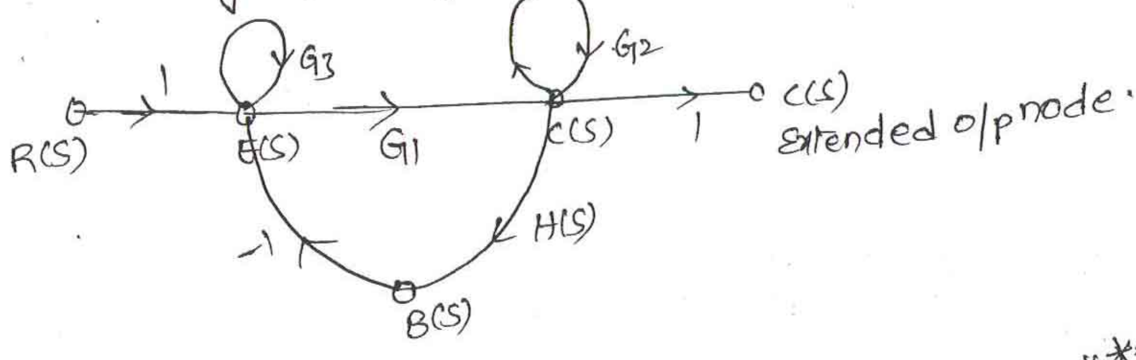
7) LOOP :- It is a path which originates & Terminates at the same Node.

Note :- self loops on Input node are not valid loops and should not be considered when writing the T.F.

loops on o/p node are valid loops.

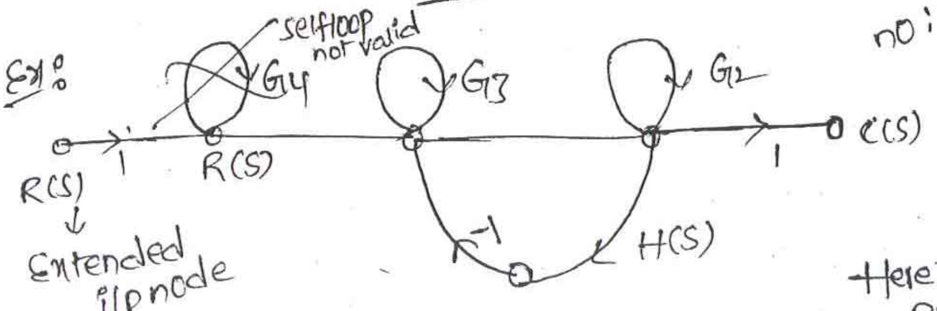
8) Non-Touching Loops :- Two (or) more loops are said to be Non-touching if they don't have a common Node.

Ex :-




No: of loops = 3

Ex:




no: of loops = 3 ***

Here one Incoming branch will be Present at R(S). so we are extending i/p node.

when we extending the i/p node ⇒ $R(s) = R(s) + G_{2p} R(s)$

so it is not actual $R(s)$.

Thats why the self loops at the i/p node are not consider.

## Mason's Gain Formula :-

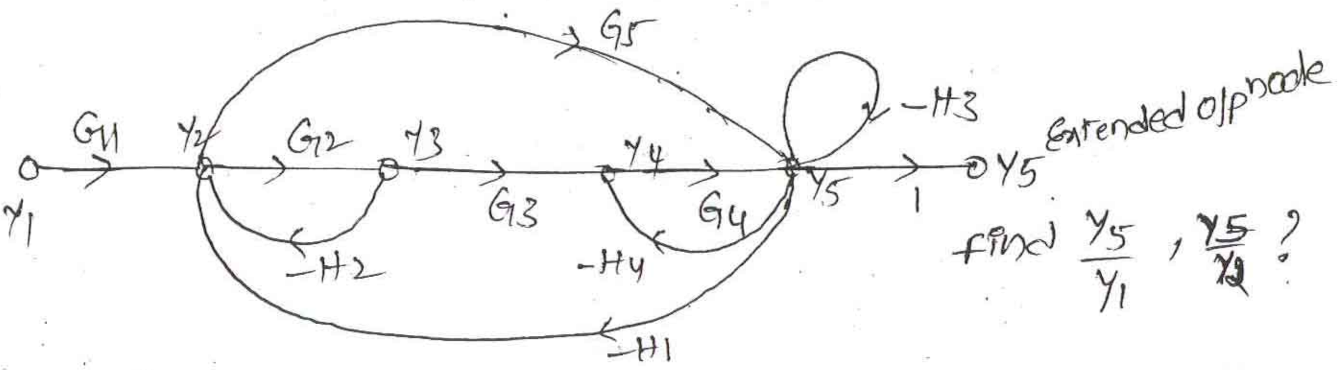
$$\left.\begin{matrix}\text{Overall Gain} \\ \text{(or)} \\ \text{Transfer function}\end{matrix}\right\} = \frac{\sum P_k \Delta_k}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2 + \dots + P_k\Delta_k}{\Delta}$$

where $P_k$ = Gain of 'k'th forward path.

$$\Delta = 1 - \begin{bmatrix}\text{sum of all loop gains} \\ \text{of all individual loops}\end{bmatrix} + \begin{bmatrix}\text{sum of gain products of} \\ \text{all two non-touching loops}\end{bmatrix} - \begin{bmatrix}\text{sum of gain products} \\ \text{of three non touching loops}\end{bmatrix} + \dots$$

$\Delta_k$ = it is the value of '$\Delta$' obtained by removing all loops touching $k^{th}$ forward path.

(Pb)




find $\frac{Y_5}{Y_1}$, $\frac{Y_5}{Y_2}$ ?

sol:- Case i :- $\frac{Y_5}{Y_1}$ ?

1) forward path :- $P_1 = G_1 G_2 G_3 G_4$

$P_2 = G_1 G_5$

2) To find $\Delta$ :-

i) individual loops ⇒ $L_1 = -G_2 H_2$, $L_3 = -H_3$

$L_2 = -G_4 H_4$, $L_4 = -G_2 G_3 G_4 H_1$

$L_5 = -G_5 H_1$

ii) Two Non touching loops :-

$L_1' = L_1 L_2 = G_2 G_4 H_2 H_4$

$L_2 = L_1 L_3 = G_2 H_2 H_3$

$\Delta = 1 - [-G_2H_2 - G_4H_4 - H_3 - G_2G_3H_1G_4 - G_3H_1 + G_5H_1] + [G_2H_2G_4H_4 + G_2H_2H_3]$

3) To find $\Delta_1, \Delta_2$ :-

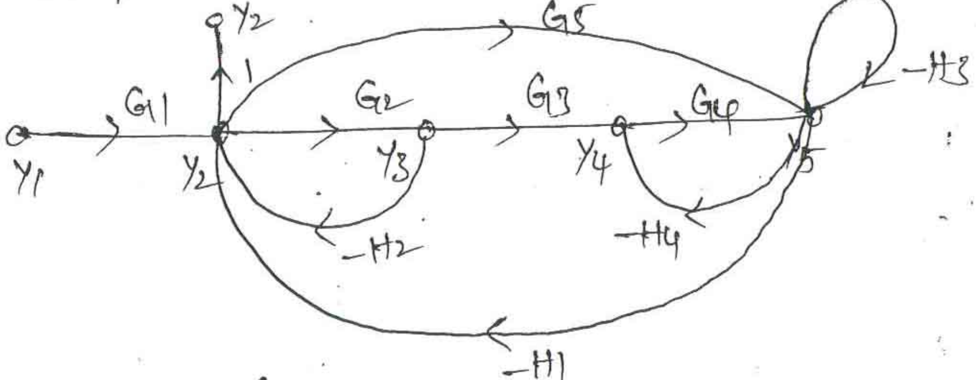
$\Delta_1 = 1$

$\Delta_2 = 1$

$$\frac{Y_5}{Y_1} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3G_4 + G_1G_5}{1 + (G_2H_2 + G_4H_4 + H_3 + G_2G_3H_1G_4 + G_3H_1 + G_5H_1) + [G_2H_2G_4H_4 + G_2H_2H_3]}$$

case (ii) :- $\frac{Y_2}{Y_1}$ ?

NOW $Y_2$ is the o/p node. so we have to extend the $Y_2$ node.




To find $\frac{Y_5}{Y_2} = \frac{(Y_5/Y_1)}{Y_2/Y_1} = \frac{Y_5}{Y_1} \times \frac{Y_1}{Y_2}$

1) forward path $P_1 = G_1$

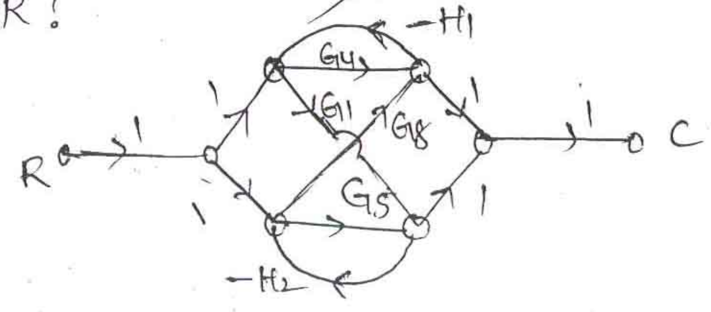
2) (i) $\Delta$ is same for both. since $\Delta$ does not depends on forward path.

(ii) to find $\Delta_1$ : $\Delta_1 = 1 - [-G_4H_4 - H_3] = 1 + G_4H_4 + H_3$

$$\frac{Y_2}{Y_1} = \frac{P_1\Delta_1}{\Delta}$$

$$\frac{Y_5/Y_1}{Y_2/Y_1} = \frac{\left[\frac{(G_1G_2G_3G_4 + G_1G_5)}{\cancel{\Delta}}\right]}{\frac{G_1(1 + G_4H_4 + H_3)}{\cancel{\Delta}}} = \frac{G_2G_3G_4 + G_5}{1 + G_4H_4 + H_3}$$ //....

(Pb) find C/R ?

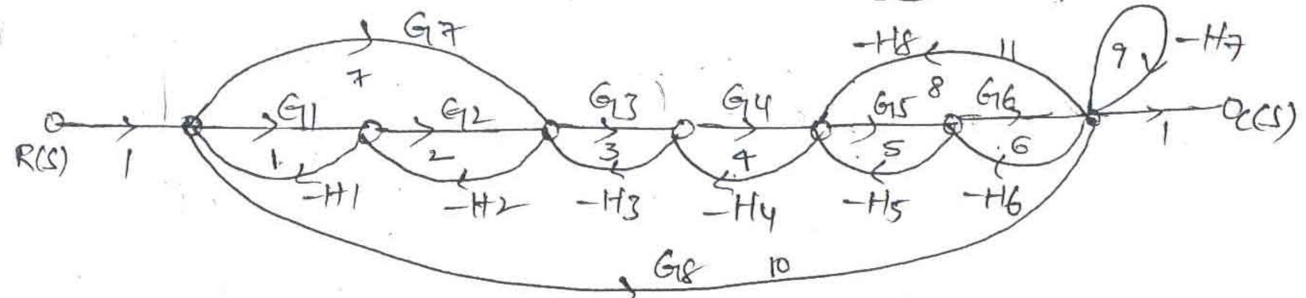


Sol:-

$$\frac{C}{R} = \frac{G_1G_2 + G_3G_5 + G_2G_1 + G_3G_8 + G_2G_1C}{1 - [-G_4H_1 - G_5H_2 + G_1G_8H_1H_2] + [G_4H_1G_5H_2]}$$

(P5) For the signal flow graph find
(i) No: of forward paths
(ii) No: of feed back paths

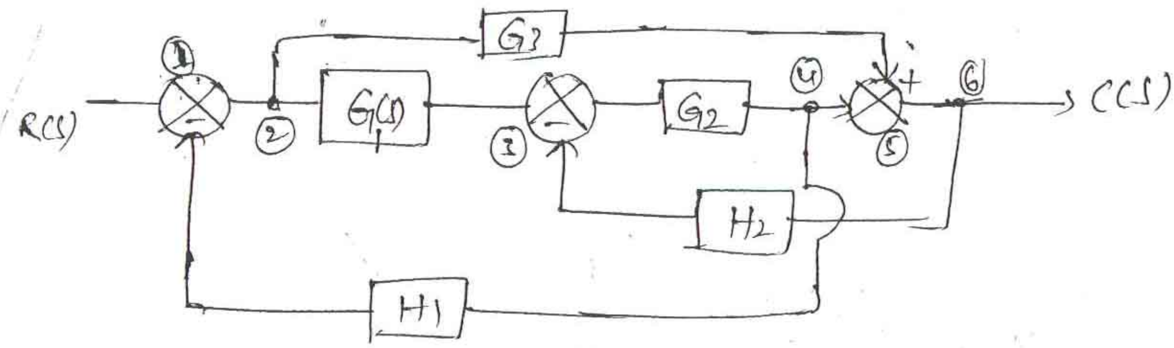
(a) 3, 8
(b) 3, 9
(c) 3, 10
(d) 3, 11

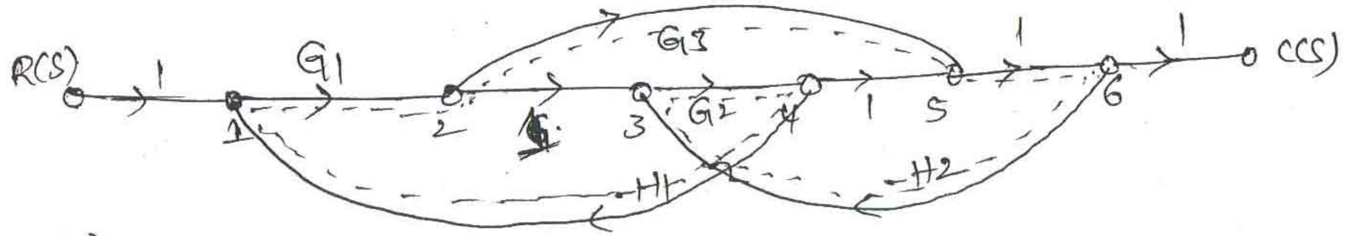



$L_{10} = G_8(-H_8)(-H_4)(-H_3)(-H_2)(-H_1)$

$L_{11} = G_8(-H_6)(-H_5)(-H_4)(-H_3)(-H_2)(-H_1)$

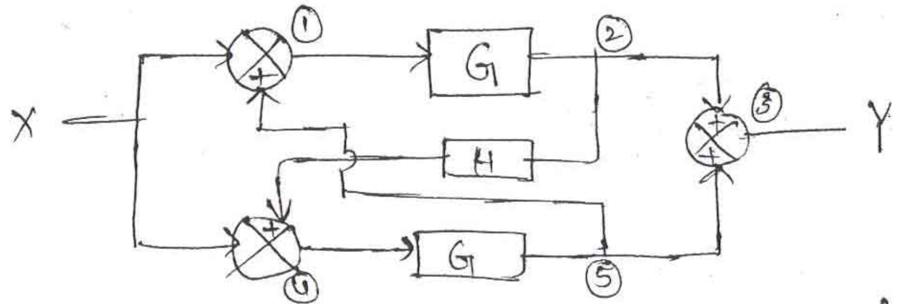
(P3)




$$\frac{C(S)}{R(S)} = \frac{G_1G_2 + G_1G_3}{1 + G_1G_2H_1 + G_2H_2 - G_1G_3G_2H_1H_2}$$ //...

(P6) $\frac{Y}{X}$ equals to - (a) $\frac{G}{1-GH}$ (b) $\frac{2G}{1-GH}$ (c) $\frac{GH}{1-GH}$ (d) $\frac{2GH}{1-GH}$

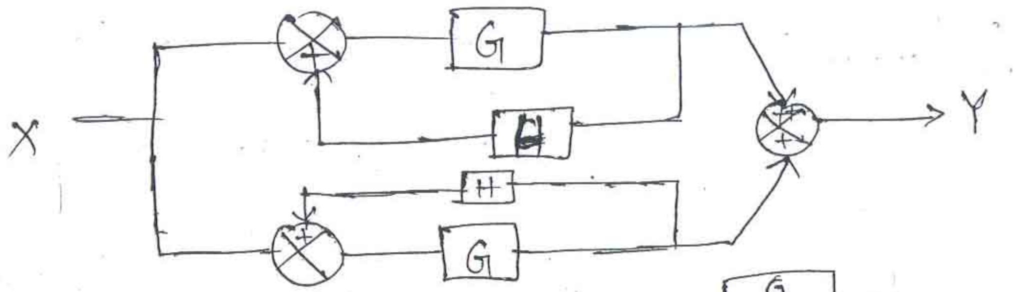


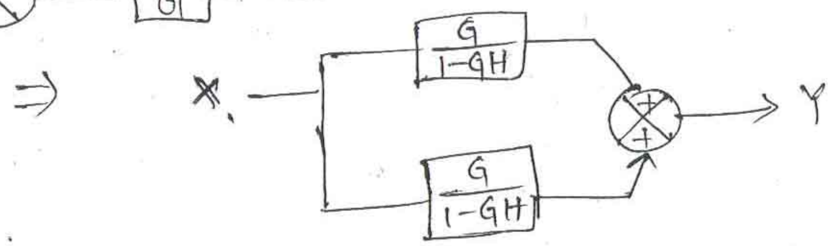

Sol:- $$\frac{Y}{X} = \frac{G + G + G^2H + G^2H}{1 - (GH)^2} = \frac{2(G + G^2H)}{(1+GH)(1-GH)}$$

$$= \frac{4G(1+GH)}{(1+GH)(1-GH)}$$

$$= \frac{2G}{1-GH} \; //...$$

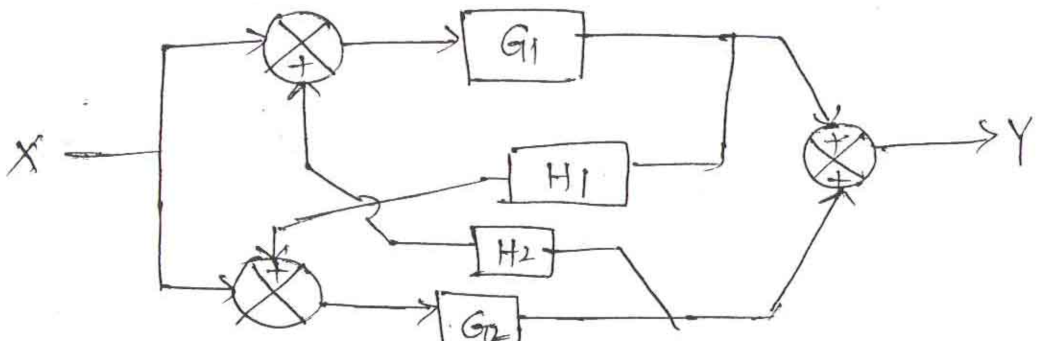
If we want to solve in block dgm reduction Technique.





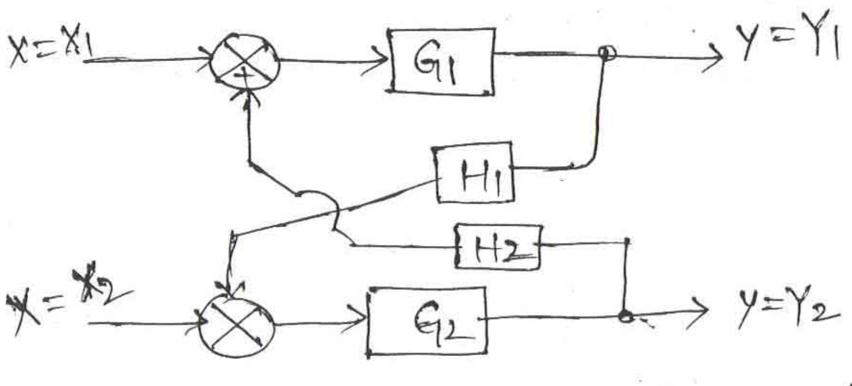
(Pb)




Sol:-

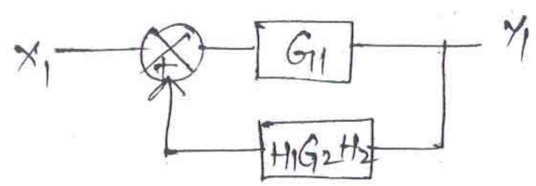
$$\frac{Y}{X} = \frac{G_1 + G_2 + G_1G_2H_1 + G_1G_2H_2}{1 - G_2H_1G_2H_2} \; //...$$

Block Diagram Reduction Method :-



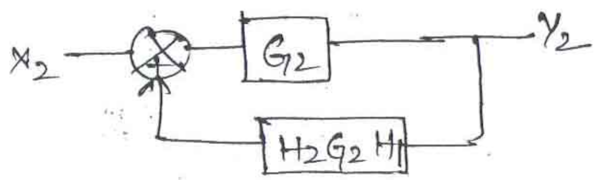

Note :- When ever Multiple inputs & multiple o/ps comes go for superposition theorem.

Case(i):- $\frac{Y_1}{X_1}\Big|_{X_2=Y_2=0}$



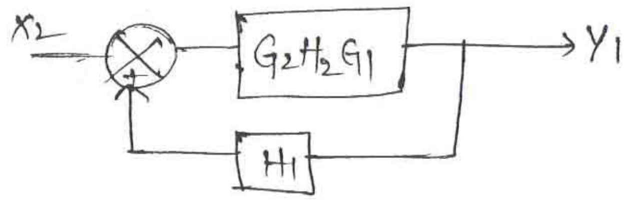

$$\frac{Y_1}{X_1} = \frac{G_1}{1-G_1H_1G_2H_2} \;//\ldots$$

Case(ii):- $\frac{Y_2}{X_2}\Big|_{Y_1=X_1=0}$



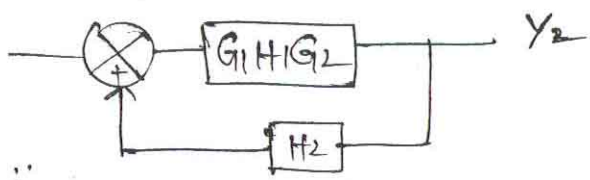

$$\frac{Y_2}{X_2} = \frac{G_2}{1-G_1H_1G_2H_2} \;//\ldots$$

Case(iii):- $\frac{Y_1}{X_2}\Big|_{Y_2=X_1=0}$




$$\frac{Y_1}{X_2} = \frac{G_2H_2G_1}{1-G_2GH_2H_1} \;//$$

Case(iv):- $\frac{Y_2}{X_1}\Big|_{X_1=Y_1=0}$



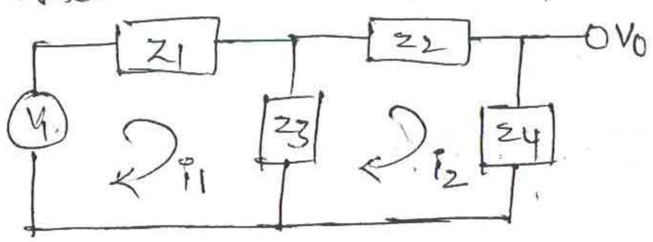

$$\frac{Y_2}{X_1} = \frac{G_1H_1G_2}{1-G_1H_1G_2H_2} \;//\ldots$$

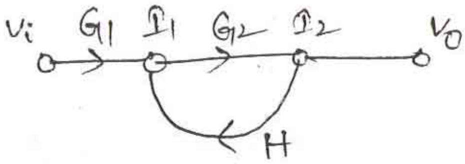
$$\therefore \frac{Y}{X} = \frac{Y_1}{X_1} + \frac{Y_1}{X_2} + \frac{Y_2}{X_2} + \frac{Y_2}{X_1} \;//\ldots$$

$$= \underline{\hspace{2em}} \text{ Ans}$$

Page no (55) (Pb)

What are the values of $G_2$ & $H$.




Sol:- step(1):- first identify the Nodes:

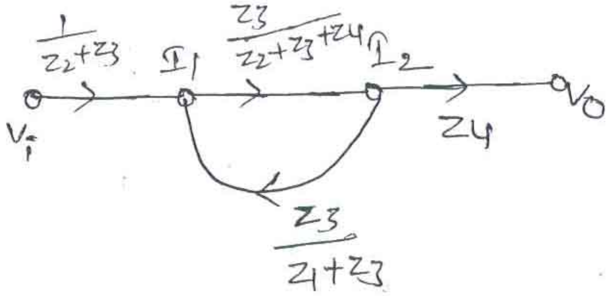
$V_i$ o    o $I_1$    o $I_2$    o $V_0$

step(2):- find the Gains of each forward path. for that apply KVL eqn's to the ckt.

$$V_i = I_1Z_1 + (I_1 - I_2)Z_3$$

$$V_i = I_1[Z_1+Z_3] - I_2Z_3$$

$$I_1 = \frac{V_i}{Z_1+Z_3} + I_2\left(\frac{Z_3}{Z_2+Z_3}\right) \quad \ldots$$




$$0 = I_2 Z_2 + I_2 Z_4 + (I_2 - I_1) Z_3$$

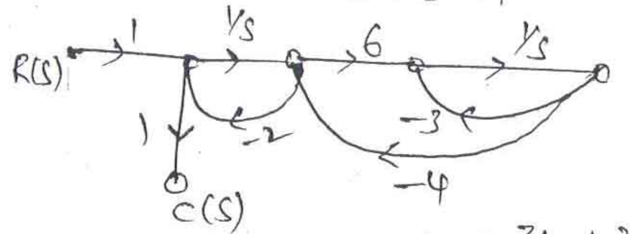
$$I_2 = \frac{Z_3}{Z_2+Z_3+Z_4} I_1 \quad \ldots$$

$$V_o = I_2 Z_4 \neq \frac{Z_3}{Z_2+Z_3+Z_4}$$

So Ans is: $G_2 = \frac{Z_3}{Z_2+Z_3+Z_4}$ ...

$H = \frac{Z_3}{Z_1+Z_3}$ ...

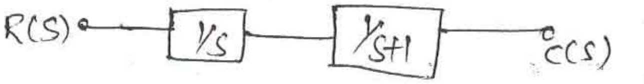
Page 56
Q 5




sol:-

$$\frac{C(S)}{R(S)} = \frac{1\left(1 + 3/s + \frac{24}{s}\right)}{1 - \left[-\frac{3}{s} - \frac{24}{s} - \frac{2}{s}\right] + \frac{6}{s^2}} \quad \ldots$$

Q 6




what is the impulse response?

sol:-

$$\frac{C(S)}{R(S)} = \frac{1}{S(S+1)} = \frac{1}{S} - \frac{1}{S+1} = 1 - e^{-t} \ldots$$

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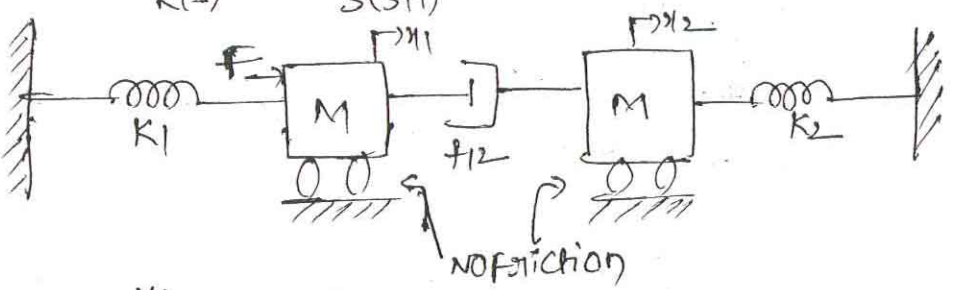
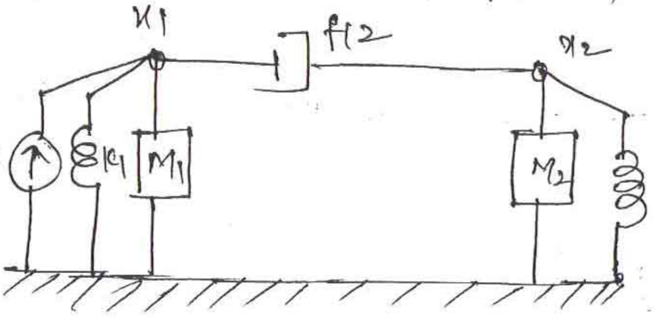
P6
Q: 2




sol:-




$$F = M_1\frac{d^2x_1}{dt^2} + K_1x_1 + f_{12}\frac{d(x_1-x_2)}{dt}$$

$$0 = M_2\frac{d^2x_2}{dt} + K_2x_2 + f_{12}\frac{d(x_2-x_1)}{dt}$$

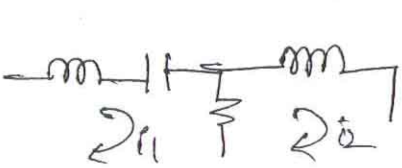
F-V Analogy:-

$$V = L_1\frac{di_1}{dt} + R_{12}(i_1 - i_2) + \frac{1}{C_1}\int i_1\,dt$$

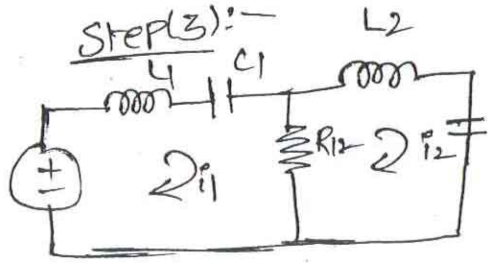
$$0 = L_2\frac{di_2}{dt} + R_{12}(i_2 - i_1) + \frac{1}{C_2}\int i_2\,dt$$

Step(1):-


Step(2):-




Step(3):-



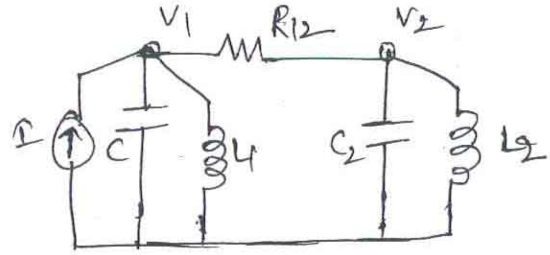
F–I Analogy :–

$I = C\frac{dv}{dt} + \frac{(v_1-v_2)}{R_{12}} + \frac{1}{L_1}\int v_1 dt$

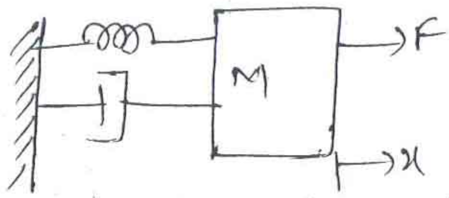
$0 = C_2\frac{dv_2}{dt} + \frac{(v_2-v_1)}{R_{12}} + \frac{1}{C_2}\int v_2 dt$

step(1):– $v_1$ $v_2$

step(2):–



Q.③:–



Sol:–

$F = M\frac{d^2x}{dt^2} + f\frac{dx}{dt} + Kx$

$F(s) = [Ms^2 + fs + K]\,X(s)$

$\Rightarrow \frac{X(s)}{F(s)} = \frac{1/M}{s^2 + \frac{f}{M}s + \frac{K}{M}}$ //...

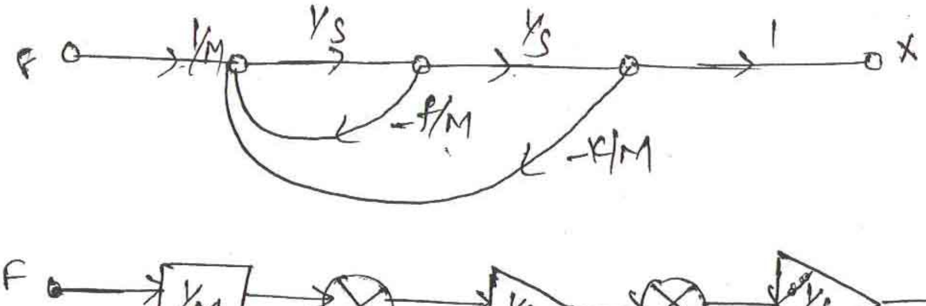
State diagram is nothing but integral based electronic ckt.

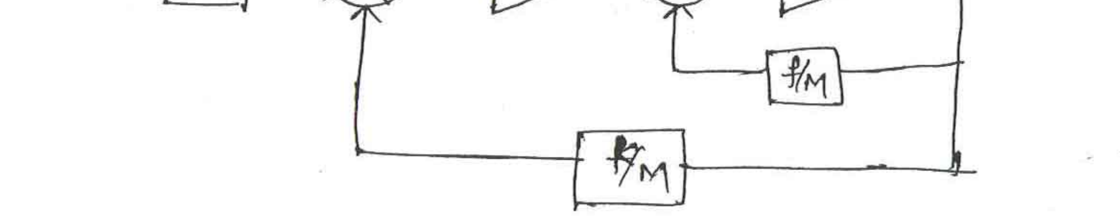
first signal flow graph have to be drawn.

$\frac{X(s)}{F(s)} = \frac{(1/M)}{s^2\left(1 + \frac{f}{M}\cdot\frac{s}{s^2} + \frac{K}{M}\cdot\frac{1}{s^2}\right)}$

$= \frac{(1/M)\cdot(1/s^2)}{1 - \left(-\frac{f}{M}\cdot\frac{1}{s} - \frac{K}{M}\cdot\frac{1}{s^2}\right)}$

$= \frac{(1/M)/s^2}{1 - \left(-\frac{f/M}{s} - \frac{K/M}{s^2}\right)}$ //...





(Fb) suppose if be asked draw the integral based ckt for given T.F $\frac{C(s)}{R(s)} = \frac{s^2+4s+8}{s^3+6s^2+12s+20}$.

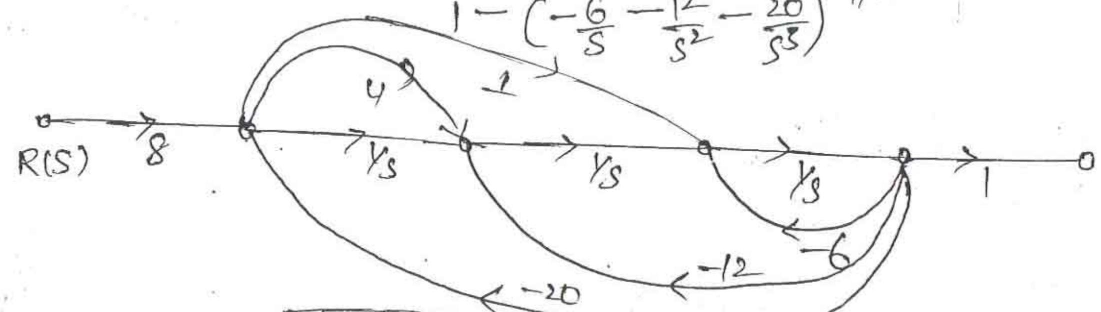
Sol: So first we have to draw signal flow graph.

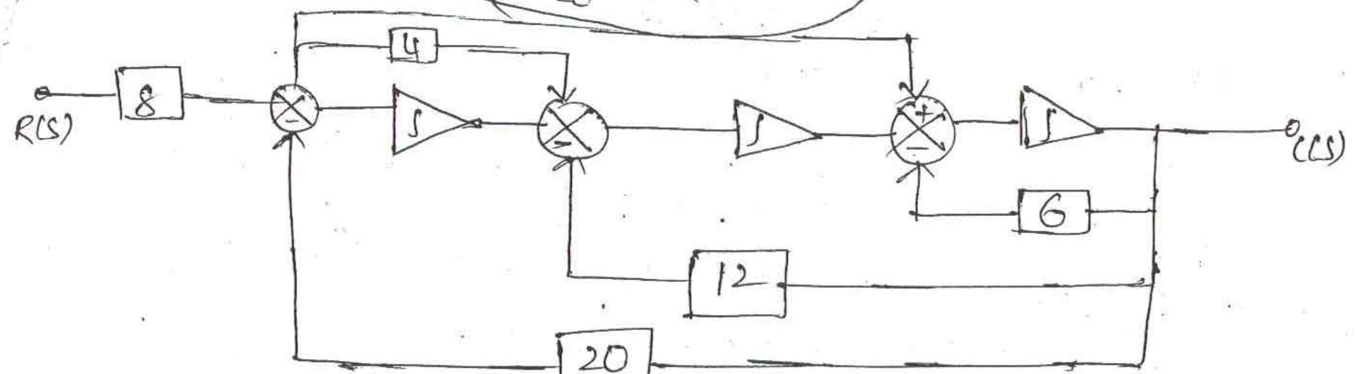
$$\frac{C(s)}{R(s)} = \frac{s^2+4s+8}{s^3+6s^2+12s+20}$$

→ Take the highest power in Numerator & dinominator

$$\frac{C(s)}{R(s)} = \frac{\left(1+\frac{4}{s}+8/s^2\right)}{s\left[1+\frac{6}{s}+\frac{12}{s^2}+\frac{20}{s^3}\right]}$$

$$= \frac{1/s + 4/s^2 + 8/s^3}{1-\left(-\frac{6}{s}-\frac{12}{s^2}-\frac{20}{s^3}\right)} \quad //...$$

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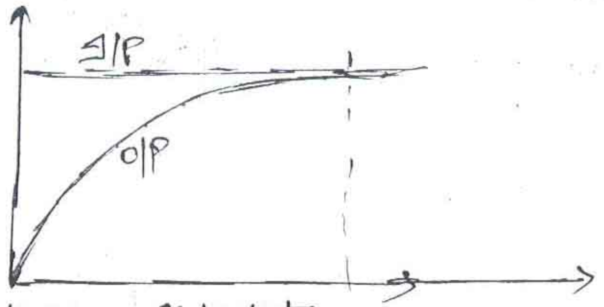
# PART-II

## TIME DOMINE ANALYSIS




* The system Performance can be Analysed by the mathematical functions like T.F etc.

* So To analyse the system Performance the system has to be subjected to disturbance i.e standard Test signals. Those are –

### STANDARD TEST SIGNALS :–

1) sudden input ⇒ step signal
2) velocity type I/p ⇒ Ramp signal
3) Acceleration type I/p ⇒ Parabollic signal
4) sudden shocks ⇒ Impuls signal

Note :– signals ①, ② & ③ are used in Time domine Analysis.
signal ④ is used only in stability Analysis.

signals ① & ④ are bounded signals. (area under the impulse signal = 1)
signals ② & ③ are unbounded signals.

* To evaluate (or) Analyse the Transient response we are using step & Impulse signals since bounded signal.

### TYPE & ORDER :–

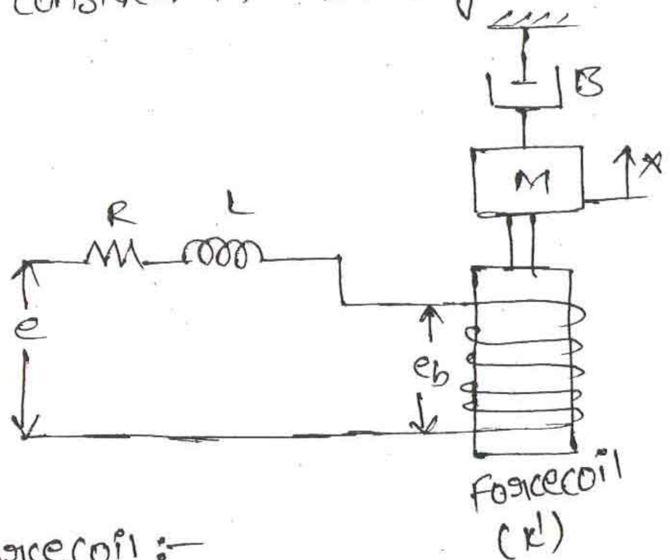
1. Every Transfer function representing a control system has Certain & Order.
2. The steady state analysis depends on Type of the control system.
3. The Type of the system is obtained from openloop T.F G(s)H(s)
4. The Number of openloop poles occuring at origin determines the type of the control system.

let $G(S)H(S) = \frac{K(1+T_a S)}{S^P(1+T_1 S)}$

if
$P=0 \Rightarrow$ Type-0 system
$P=1 \Rightarrow$ Type-1 system
$\vdots$
$P=n \Rightarrow$ Type-n System.

5. The Transient state Analysis depends on order of the control-system.
6. order of the control system is obtained from closed loop T.F $\frac{G(S)}{1+G(S)H(S)}$.
7. The Highest Power of the characteristic equation i.e $1+G(S)H(S)=0$. determines the order of the control system

Let us consider the following example:




Input $\rightarrow$ e
o/p $\rightarrow$ x

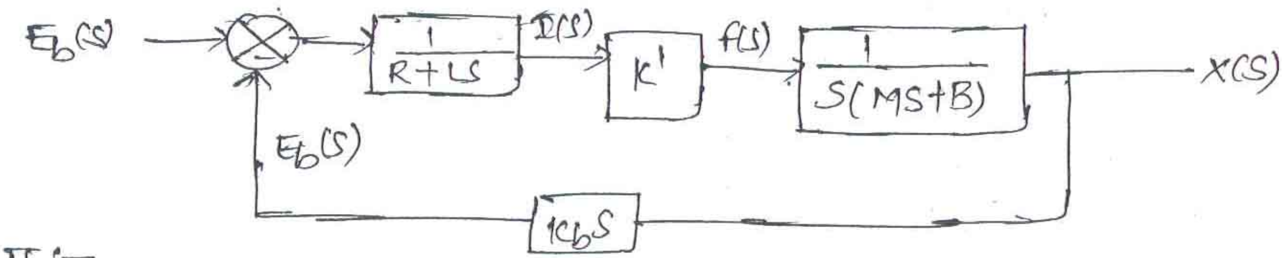
$E(S) = I(S)[R+SL] + E_b(S)$
$E(S) - E_b(S) = I(S)(R+SL) \rightarrow (1)$

At forcecoil :-
$F \propto i$
$F = k'i \Rightarrow F(S) = k' I(S) \rightarrow (2)$

At Mechanical system :-
$$F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt}$$
$$F(S) = [MS^2 + BS]\,X(S) \rightarrow (3)$$

Back EMF $(e_b) \propto$ speed
$$e_b \propto \frac{dx}{dt} \Rightarrow e_b = k_b\frac{dx}{dt}$$
$$E_b(S) = k_b S \cdot X(S) \rightarrow (4)$$

Out of all Eqns which eqn having the overall i/p should taken first and then continue the Block diagram based on corresponding block o/p.




**OLTF:-**

$$G(s)H(s) = \frac{K' K_b \cancel{S}}{(R+SL)\cancel{S}(MS+B)} \rightarrow \text{Type 0 system}$$

**CLTF :-**

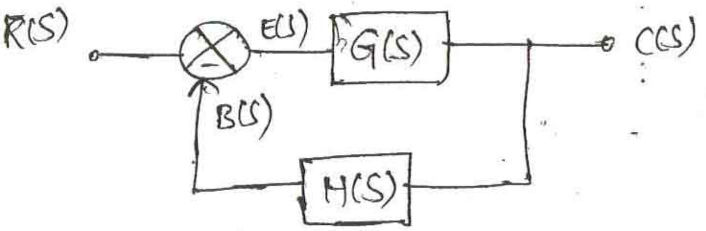
$$1+G(s)H(s) = 1 + \frac{K' K_b S}{(R+SL)S(MS+B)}$$

$$\Rightarrow (R+Ls)(MS+b) + K' K_b S = 0 \longrightarrow \text{order-2}$$

## Steady state Response Analysis :-

It deals with estimation of magnitude of steadystate error b/w input and output, and depends on type of the control system.

To obtain an expression for error :-



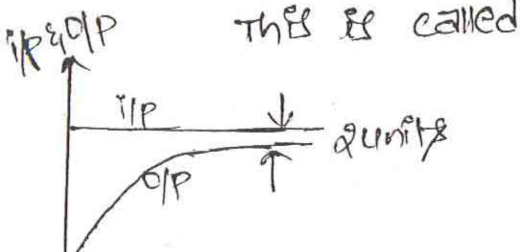

$E(S) = R(S) - B(S)$

$E(S) = R(S) - C(S)H(S) \Rightarrow E(s) = R(s) - G(s)H(s)\cdot E(s)$

$E(S)\left[1+G(s)H(s)\right] = R(s)$

$$\boxed{E(S) = \frac{R(S)}{1+G(S)H(S)}}$$

This is called "Error Ratio".




$$\lim_{t \to \infty} e(t) = 2 \text{ units} = e_{ss}.$$

$$\therefore e_{ss} = \lim_{t \to \infty} e(t)$$

By applying the Final value Theorem:

$$\boxed{e_{ss} = \lim_{s \to 0} s \cdot E(s)}$$ //...

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1+G(s)H(s)}$$

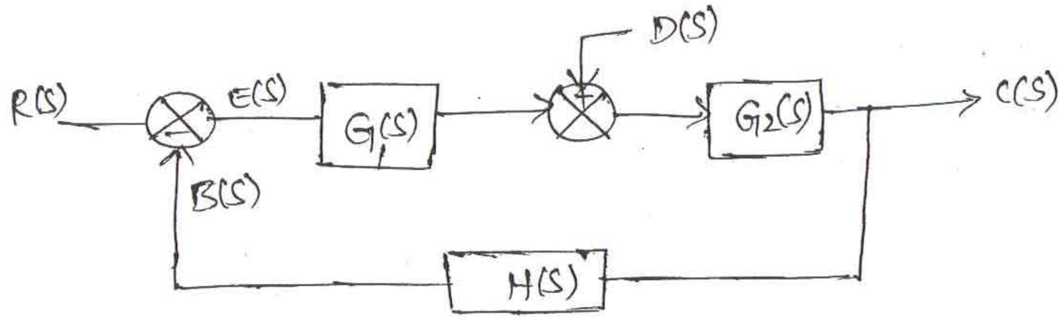
$\therefore$ therefore steadystate error depends on type of the i/p & type of the system.

$$\boxed{e_{ss} = \frac{\lim_{s \to 0} s R(s)}{1 + \lim_{s \to 0} G(s) H(s)}}$$

→ Type of i/p

→ Type of system

To obtain an Expression For error with disturbance :-




$$E(s) = R(s) - B(s)$$

$$E(s) = R(s) - C(s)H(s)$$

where $C(s) = [E(s)G_1(s) + D(s)]\, G_2(s)$

$$C(s) = E(s)\,G_1(s)\,G_2(s) + D(s)\,G_2(s)$$

$$\therefore E(s) = R(s) - [E(s)G_1(s)G_2(s) + D(s)G_2(s)]\,H(s)$$

$$E(s)[1 + G_1(s)G_2(s)H(s)] = R(s) - D(s)G_2(s)H(s)$$

$$E(s) = \frac{R(s) - D(s)G_2(s)H(s)}{1 + G_1(s)G_2(s)H(s)}$$ //...

$$\boxed{E(s) = \frac{R(s)}{1+G_1(s)G_2(s)H(s)} - \frac{D(s)G_2(s)H(s)}{1+G_1(s)G_2(s)H(s)}}$$ //...

$$e_{ss} = \lim_{s \to 0} s \cdot E(s)$$

$$\boxed{e_{ss} = \lim_{s\to 0} \frac{s \cdot R(s)}{1+G_1(s)G_2(s)H(s)} - \lim_{s\to 0} \frac{s \cdot D(s)G_2(s)H(s)}{1+G_1(s)G_2(s)H(s)}}$$ **

In the above diagram $G_2$ represents the plant. How means in water level control if any disturbance is occuring at plant, then error will occur. Similarly here also error is introduces to $G_2$. So $G_2$ is plant.

where

$\lim_{s\to 0} \frac{s \cdot R(s)}{1+G_1(s)G_2(s)H(s)}$ → Error due to step input.

$\lim_{s\to 0} \frac{s \cdot D(s)G_2(s)H(s)}{1+G_1(s)G_2(s)H(s)}$ → Error due to disturbance.

Ex:-

$e_{ss} = 1.5 - 1.5$

When ever comparision is asked b/w the error due to step & error due to Disturbance. Take mod values i.e $|1.5|, |-1.5|$ $= 1.5, 1.5$

If the options are given like this

ⓐ $e_{ss}$ due to step > $e_{ss}$ due to Disturbance

ⓑ " < "

ⓒ " = "

ⓓ None

Steady State Error due to different Type of inputs :-

$$\boxed{e_{ss} = \lim_{s\to 0} \frac{s \cdot R(s)}{1+G(s)H(s)}}$$

1) STEP I/p :- $R(s) = A/s$

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s} \cdot (A/\cancel{s})}{1+G(s)H(s)}$$

$$= \lim_{s\to 0} \frac{A}{1+G(s)H(s)}$$

$$= \frac{A}{1+\lim_{s\to 0} G(s)H(s)}$$

$$\boxed{e_{ss} = \frac{A}{1+K_p}}$$ //...

where $K_p$ = Positional constant

$K_p = \lim_{s\to 0} G(s)H(s)$ ...

$R(S) = A/S^2$

$$e_{ss} = \lim_{s\to 0} \frac{s\cdot (A/s^2)}{1+G(s)H(s)}$$

$$= \frac{A}{\lim_{s\to 0} s + \lim_{s\to 0} sG(s)H(s)}$$

$$= \frac{A}{\lim_{s\to 0} sG(s)H(s)}$$

$$\boxed{e_{ss} = \frac{A}{K_v}}$$

where $K_v$ = velocity error constant

$$K_v = \lim_{s\to 0} s\cdot G(s)H(s)$$

3) Parabolic i/p:-

$R(S) = A/S^3$

$$e_{ss} = \lim_{s\to 0} \frac{s\cdot (A/s^3)}{1+G(s)H(s)}$$

$$= \frac{A}{\lim_{s\to 0} s^2 G(s)H(s)}$$

$$\boxed{e_{ss} = \frac{A}{K_a}}$$

where $K_a$ = Acceleration Error constant

$$K_a = \lim_{s\to 0} s^2 G(s)H(s)$$

$\boxed{K_p, K_v, K_a \rightarrow \text{are called as Static Error Constants}}$

Steady state error for different types of system:-

In this analysis we are observe two things. i.e first one is what is the relation b/w $e_{ss}$ & gain(K). And the second one is all the practical systems are restricted to Type-2 only. How it will be we will discuss.

TYPE-0 system:- $G(S)H(S) = \frac{K(1+T_a S)}{(1+T_1 S)}$

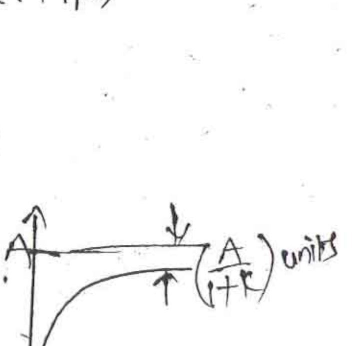
(a) step i/p

$R(S) = A/S$

$$e_{ss} = \lim_{s\to 0} \frac{s\cdot (A/s)}{1+\frac{K(1+T_a s)}{(1+T_1 s)}}$$

$$= \frac{1}{1+K\frac{(1+0)}{(1+0)}}$$

$$\boxed{e_{ss} = \frac{1}{1+K}}$$

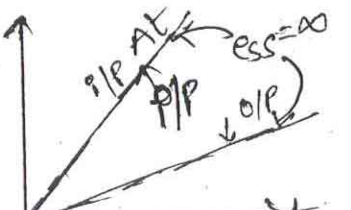



(b) Ramp i/p:-

$R(S) = A/S^2$

$$e_{ss} = \frac{A}{\lim_{s\to 0} s\cdot\frac{K(1+T_a s)}{(1+T_1 s)}}$$

$$= \frac{A}{0} = \infty$$



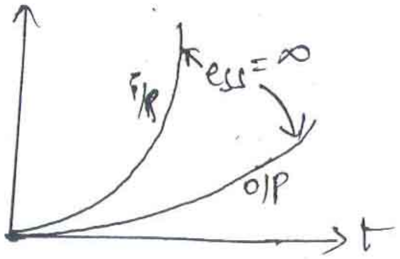
c) Parabollic i/p :-

$R(s) = A/s^3$

$$e_{ss} = \frac{A}{\lim_{s\to 0} s^2 K \frac{(1+T_a s)}{(1+T_1 s)}}$$

$= A/0$

$= \underline{\infty}$




** **Note :-**

| | Step i/p | Ramp i/p | Parabollic i/p |
|---|---|---|---|
| Type 0 | $\frac{A}{1+K_p} = \boxed{\frac{A}{1+K}}$ <br> $K_p = K$ | $\infty$ <br> $K_v = 0$ | $\infty$ <br> $K_a = 0$ |
| Type 1 | $0$ <br> $K_p = \infty$ | $\frac{A}{K_v} = \boxed{\frac{A}{K}}$ <br> $K_v = K$ | $\infty$ <br> $K_a = 0$ |
| Type 2 | $0$ <br> $K_p = \infty$ | $0$ <br> $K_v = \infty$ | $\frac{A}{K_a} = \boxed{\frac{A}{K}}$ <br> $K_a = K$ |

Observations :- 1) $\boxed{e_{ss} \propto \frac{1}{K}}$

**) As gain $K \uparrow \Rightarrow e_{ss} \downarrow$

2) The Main Aim of any control system design is Error = 0. As the type of the system increases ⇒ error will reduces and it is zero value. for Type-3 systems Error ($e_{ss}$) for any input (step, ramp...) $e_{ss} = 0$. But we don't go for Type-3 system eventhough this Advantage also having.

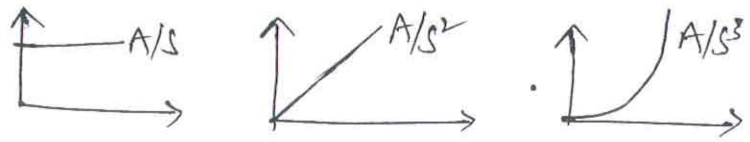
Reason is : As it { ~~complexity~~, ~~cost~~ } still now we don't see any Type-3 system. Then how do you know these two factors.

both are not the reason.

stability → still now we don't study about stability. then how can you say stability. ✗

So the Reason is Non-linearities of the systems.

Explanation:—
let us take → Standard Test signals




Type-1 Type-2 Type-3

✓ $mx+c$ ✓ $mx+c$ ✗ Nonlinear

(of course it not like $mx+c$ but just take $0(x)+c$ since slope = 0)

1) Hysteresis is the one of the form of nonlinearity.
2) Dead Time " " .
3) Saturation " " .
↳ class (6:30) to (12:30) no pblm. If it extended to 5:30 then body present mind absent.

* As going higher Type, the Nonlinearities in the system becomes more predominant. The Basic charn of the system will changes from linear to Non linear. Any c.s first of all it is a L.T.I system we are taking initially.

The Maximum Type of Any linear system is '2'. Beyond Type '2' the system exhibits nonlinear charn more dominents.

ERROR Series:—

$$E(S) = \frac{R(S)}{1+G(S)H(S)}$$

let $F(S) = \frac{1}{1+G(S)H(S)}$

$\therefore E(S) = R(S)\,F(S)$

Applying inverse laplace Transform.

$$e(t) = L^{-1}(R(S)H(S))$$

$$e(t) = \int_{-\infty}^{\infty} f(T)\, r(t-T)\, dT$$ (∵ By the property of convolution)

$$e(t) = \int_{0}^{\infty} f(T)\, r(t-T)\, dT$$ (∵ We don't have any Test signal which starts at $-\infty$ to $0$)

Expanding $r(t-T)$ using Taylor series

$$r(t-T) = r(t) - T\dot{r}(t) + \frac{T^2}{2!}\ddot{r}(t) - \frac{T^3}{3!}\dddot{r}(t) + \cdots$$

$$\therefore e(t) = r(t)\int_0^{\infty} f(T)\,dT - \dot{r}(t)\int_0^{\infty} T f(T)\,dT + \frac{\ddot{r}(t)}{2!}\int_0^{\infty} T^2 f(T)\,dT + \cdots$$

Define the "Dynamic error constants"

$$k_0 = \int_0^{\infty} f(T)\, dT$$

$$k_1 = -\int_0^{T} f(T)\cdot T\, dT$$

$$k_2 = +\int_0^{T} T^2 f(T)\, dT$$

$$k_3 = -\int_0^{T} T^3 f(T)\, dT$$

$$\boxed{\therefore C(t) = k_0 r(t) + k_1 \dot{r}(t) + \frac{k_2 \ddot{r}(t)}{2!} + \frac{k_3 \dddot{r}(t)}{3!} + \cdots}$$

$$\therefore e_{ss} = \lim_{s\to 0} sE(s) = \lim_{t\to\infty} e(t) \ldots$$

To find $k_0, k_1, k_2 \ldots$ we have to find $f(T)$, which can be obtained by inverse laplace of $F(s)$.

**To find Dynamic Error constants :-**

$$L\{f(T)\} = F(s) = \underline{\int_0^{\infty} f(T) e^{-sT}\, dT} \rightarrow \text{basic L.T eqn}$$

$$\lim_{s\to 0} F(s) = \lim_{s\to 0} \int_0^{\infty} f(T) e^{-sT}\cdot dT$$

$$= \int_0^{\infty} f(T)\cdot dT$$

$$= \underline{\underline{k_0}} \ldots$$

$$\frac{d}{ds}F(s) = \frac{d}{ds}\left(\int_0^{\infty} f(T) e^{-sT} dT\right) = -T\int_0^{\infty} f(T) e^{-sT} dT$$

$$\lim_{s\to 0} F(s) = \lim_{s\to 0}\left(-\int_0^{\infty} F(T)\cdot T\, dT\right) = -\int_0^{\infty} T f(t)\, dT = \underline{\underline{k_1}} \ldots$$

Similarly

$$\boxed{\begin{aligned} k_0 &= \lim_{s\to 0} F(s) \\ k_1 &= \lim_{s\to 0} \frac{d}{ds}F(s) \\ k_2 &= \lim_{s\to 0} \frac{d^2 F(s)}{ds^2} \end{aligned}}$$

&c...

where $F(s) = \underline{\underline{\frac{1}{1+G(s)H(s)}}} \ldots$

$$k_n = \lim_{s\to 0} \frac{d^n F(s)}{ds^n}$$

## Relation b/w static & Dynamic Error constants :-

Let us take an example $G(S)H(S) = \frac{100}{S(S+2)}$

I. Static Error constants :-

$K_p = \lim_{S\to 0} \frac{100}{S(S+2)} = \infty$

$K_v = \lim_{S\to 0} \cancel{S} \cdot \frac{100}{\cancel{S}(S+2)} = 50$

$K_a = \lim_{S\to 0} S^2 \cdot \frac{100}{S(S+2)} = 0$

II. Dynamic error constants :-

$F(S) = \frac{1}{1+G(S)H(S)} = \frac{1}{1+\frac{100}{S(S+2)}} = \frac{S(S+2)}{S^2+2S+100}$ //

$K_0 = \lim_{S\to 0} F(S) = \lim_{S\to 0} \frac{S(S+2)}{S^2+2S+100} = 0$ //

(or)

$K_0 = \lim_{S\to 0} \frac{1}{1+\underbrace{\lim_{S\to 0} \frac{100}{S(S+2)}}_{G(S)H(S)}}$

$= \frac{1}{1+\infty}$

$= 0$ ...

$\boxed{\therefore K_0 = \frac{1}{1+K_p}}$ //... where $K_p = \lim_{S\to 0} G(S)H(S)$ ....

$K_1 = \lim_{S\to 0} \frac{d}{dS} F(S) = ?$

$\frac{d}{dS} F(S) = \frac{d}{dS}\left[\frac{S(S+2)}{S^2+2S+100}\right] = \frac{(S^2+2S+100)(2S+2)-(S^2+2S)(2S+2)}{(S^2+2S+100)^2}$

$\therefore K_1 = \lim_{S\to 0} \frac{d}{dS} F(S) = \frac{100(2)-0}{(100)^2} = \frac{1}{50}$ //...

$\boxed{\therefore K_1 = \frac{1}{50}}$ //...

$\Rightarrow \boxed{K_1 = \frac{1}{K_v}}$

$K_2 = \lim_{S\to 0} \frac{d^2 F(S)}{dS^2}$

Similarly $\boxed{K_2 = \frac{1}{K_a}}$ //....

## II. ERROR SERIES :—

$$e_{ss} = \lim_{t \to \infty} \left[ K_0 r(t) + K_1 \dot{r}(t) + \frac{K_2}{2!} \ddot{r}(t) + \cdots \right]$$

$r(t) = 5 + 2t \Rightarrow K_0 = 0$

$\dot{r}(t) = 0 + 2 = 2$ ; $K_1 = 1/50$ ....

$\ddot{r}(t) = 0$

$$\therefore e_{ss} = \lim_{t \to \infty} \left[ 0(5+2t) + \frac{1}{50} \times 2 + 0 \right]$$

$$e_{ss} = \frac{2}{50} = \frac{1}{25} \text{ units}$$

Note :— In IES if they specially asked to solve the problem in Error series then only go for this method, otherwise Never go for.

Shortcut Method :—

III. $e_{ss} = 0 + \frac{A}{K}$ $\left( \because 5 + 2t \right.$ ⇓ step i/p + Ramp i/p for type I $\left. = 0 + \frac{A}{K} \right)$ ...

$= 0 + \frac{2}{50}$

$= 2/50$ units

$$\left( K = \lim_{s \to 0} s \cdot \frac{100}{s(s+2)} = 50 \ldots \right)$$

## Transient State Analysis :—

It deals with the Nature of response of the system when subjected to an input and depends on order of the control system.

### I. Zero Order System :—

Generalized transfer function

$$\boxed{\frac{X_0(S)}{X_i(S)} = \frac{b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}}$$ ....

for zero order system

$$\frac{X_0(S)}{X_i(S)} = \frac{b_0}{a_0} \ldots$$

when all the 's' co-efficients are terms are taken as zeros, then we will get zero order system.

# EFFCTS OF FEEDBACK

OLTF → F·B not connected  CLTF → F·B connected
conditional system (No disturbance)

for any disturbance → OLTF does not exist
Internal/external → CLTF can stabilizes, because of F·B is connected.

This analysis is called "sensitivity".

Note :-
suppose if there is a internal/extn disturbances, if the system becomes unstable. we say the system is said to be highly sensitive systems. → Ex:- OLCS

suppose if there is a internal/external disturbance, if the system becomes stable, we say that system is "less sensitive systems". → Ex: CLCS

* for any disturbance → control does not exist ⇒ unstable.
so OLCS are highly sensitive systems.
* where as CLCS are less senn sysn.

SENSITIVITY ANALYSIS:-

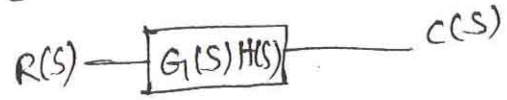
let $\alpha$ = A variable that changes its value.
$\beta$ = A parameter that changes the value of '$\alpha$'

$$S_\beta^\alpha = \frac{\%\text{ change in } \alpha}{\%\text{ change in } \beta} = \left(\frac{d\alpha}{\alpha}\right) \Big/ \left(\frac{d\beta}{\beta}\right)$$

$$\boxed{S_\beta^\alpha = \frac{d\alpha}{d\beta} \cdot \frac{\beta}{\alpha}}$$ //...

using this funn we are trying to find the sensitivity of OLCS.

OLCS:-



$M(S)$ = O.L.C.S

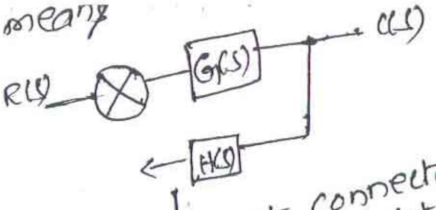
$= \frac{C(S)}{R(S)}$ → mathematically express or define a C.S

$$\overline{R(S)}$$

let

$\alpha = M(S)$

$\beta = G(S)$ → why we are taking only G(S) means




It is not connected to summing point. That means it doesn't provide any effect on OLCS.

(or)

In OLCS F.B is absent so not requir to take H(S).

$$\therefore S^{M(S)}_{G(S)} = \frac{G(S)}{M(S)} \times \frac{dM(S)}{dG(S)}$$

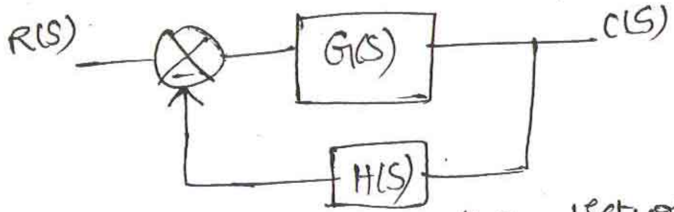
$$= \frac{\cancel{G(S)}}{\cancel{G(S)}H(S)} \times \frac{d\,[G(S)H(S)]}{dG(S)}$$

$$= \frac{1}{H(S)} \times [G(S)(0) + H(S)(1)]$$

$$= \frac{1}{\cancel{H(S)}} [\cancel{H(S)}]$$

$$\boxed{S^{M(S)}_{G(S)} = 1} \longrightarrow \text{For OLCS}$$

closed loop control system :-




Here we can take the disturbance in either G(S) (or) H(S).

(i) sensitivity of CLCS w.r.to G(S) :- we have to take the disturbance but to compare with OLCS, we have to take the disturbance in G(S) only. so

$$S^{M(S)}_{G(S)} = \frac{dM(S)}{dG(S)} \times \frac{G(S)}{M(S)}$$

where $M(S) = \dfrac{G(S)}{1 + G(S)H(S)}$

$$= \frac{d\left[\frac{G(S)}{1+G(S)H(S)}\right]}{dG(S)} \times \frac{\cancel{G(S)}}{\frac{\cancel{G(S)}}{1+G(S)H(S)}}$$

$$S_{G(S)}^{M(S)} = \frac{(1+G(S)H(S))(1) - [\cancel{G(S)}[H(S)]+0]\,G(S)}{(1+G(S)H(S))^2} \times (G(S)H(S))+1$$

$$= \left[\frac{1+\cancel{G(S)H(S)} - \cancel{G(S)H(S)}}{(1+G(S)H(S))^{\cancel{2}}}\right] \times [1+\cancel{G(S)H(S)}]$$

$$\boxed{S_{G(S)}^{M(S)} = \frac{1}{1+G(S)H(S)}} \rightarrow \text{for CLCS}$$ //....

where $\boxed{1+G(S)H(S) = \text{Noise Reduction factor (or) Return difference}}$

so the conclusion is that, OLCS are highly sensitive to any internal or external disturbances.

The sensitivity of a closed loop system is reduced by a factor of $(1+G(S)H(S))$ known as "Noise reduction factor" (or) "Return difference".

(ii) sensitivity of CLCS w.r.to H(S) :-

$\alpha = M(S)$
$\beta = H(S)$

$$S_{H(S)}^{M(S)} = \frac{dM(S)}{dH(S)} \times \frac{H(S)}{M(S)}$$

$$= \frac{d}{dH(S)}\left(\frac{G(S)}{1+G(S)H(S)}\right) \times \frac{H(S)}{\frac{G(S)}{(1+G(S)H(S))}}$$

$$= \left[\frac{(1+G(S)H(S))(0) - G(S)(G(S))}{(1+G(S)H(S))^{\cancel{2}}}\right] \times \frac{H(S)(1+\cancel{G(S)H(S)})}{G(S)}$$

$$= \frac{-(G(S)\times \cancel{G(S)})}{(1+G(S)H(S))} \times \frac{H(S)}{\cancel{G(S)}}$$

$$\boxed{S_{H(S)}^{M(S)} = -\frac{G(S)H(S)}{1+G(S)H(S)}}$$ //....

Note :-
Comparision is always absolute terms. of course Answer is write in Negative.

Page 60 In both the cases return difference = $(1+G(s)H(s))$

Q:2 Ans : (b)

Q:4

Sol: $G(s) = \frac{K}{s(s+a)}$

$$e_{ss} = \lim_{s \to 0} \frac{s.R(s)}{1+G(s)H(s)}$$

$$= \lim_{s \to 0} \frac{\cancel{s} \cdot (1/s^2) \cdot s}{1 + \frac{K}{s(s+a)}}$$

$$e_{ss} = a/K$$

i. $$S_K^{e_{ss}} = \frac{K}{e_{ss}} \cdot \frac{de_{ss}}{dK}$$

$$= \frac{K}{(a/K)} \cdot \left[ a(-1/K^2) \right]$$

$$= \frac{K^2}{a} \cdot \left[a(-1/K^2)\right]$$

$$= -1$$

ii. $$S_a^{e_{ss}} = \frac{a}{e_{ss}} \cdot \frac{de_{ss}}{a}$$

$$= \frac{a}{(a/K)} \cdot \left[\frac{1}{K}\right]$$

$$= \frac{Ka}{a} \times \frac{1}{K}$$

$$= 1$$

$\therefore$ Answer is : $\boxed{-1, 1}$ → (b) option.

If he given the options like (i) sensitivity $S_K^{ess} > S_a^{ess}$

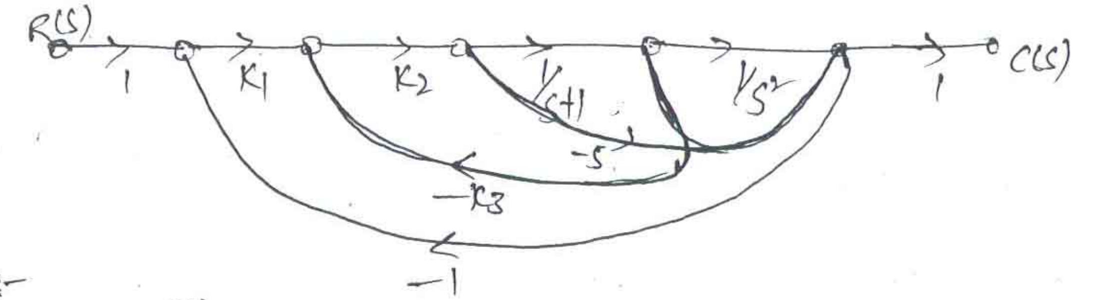
(ii) $S_K^{ess} < S_a^{ess}$

~~(iii)~~ $S_K^{ess} = S_a^{ess}$

(iv) both (a) & (b)

Note:-
Always comparision is absolute values are taken.
for Comparision There is no negative & +ve values ; if -ve values are comes take it as +ve.

Q:- Find the sensitivity of the system w.r.to $K_2$ at $\omega=0$!




**Sol:-**

$$\frac{C(S)}{R(S)} = \frac{\dfrac{K_1K_2}{(S+1)(S^2)}}{1 + \left(\frac{1}{S+1}\right)\left(\frac{1}{S^2}\right)(5) + \dfrac{K_2K_3}{(S+1)\cancel{S}} + \dfrac{K_1K_2}{(S+1)S^2}}$$

$$\frac{C(S)}{R(S)} = \frac{K_1K_2}{(S+1)S^2 + 5 + S^2K_2K_3 + K_1K_2}$$

At $\omega=0$

$$F(S)\Big|_{\omega=0} = \frac{C(S)}{R(S)}\Big|_{\omega=0} = \frac{K_1K_2}{5+K_1K_2}$$

$$S_{K_2}^{F(S)} = \frac{dF(S)}{dK_2} \times \frac{K_2}{F(S)} = \frac{(K_1K_2+5)K_1 - K_1K_2(K_1)}{(5+K_1K_2)^{\cancel{2}}} \times \frac{\cancel{K_2}}{\dfrac{K_1\cancel{K_2}}{\cancel{(5+K_1K_2)}}}$$

$$= \frac{5\cancel{K_1}}{(5+K_1K_2)\cancel{K_1}}$$

$$= \frac{5}{5+K_1K_2}$$

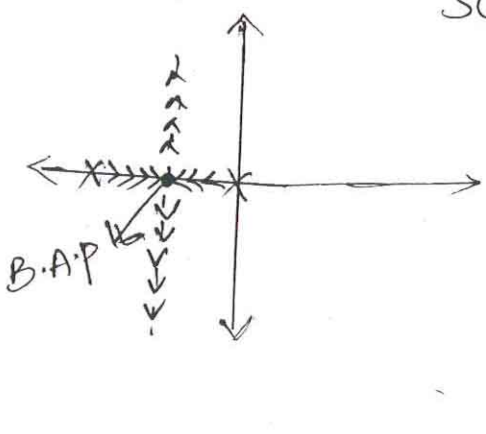
**Note:-**

Sensitivity concept can also be applied for Root locus.

Now we have drawn the root locus for the given T.F

$$G(S)H(S) = \frac{K}{S(S+2)}$$

Now I want the value of 'k' at B.A.P.



We are vary the values of 'K' from 0 to ∞.

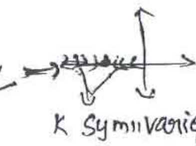
* As it B.A.P (or) B.I.P of a system will occur for higher values of K (or) lower values of K?

Ans:- lower values of 'K' ⇒ as shown in any RL in the Pblms.

Now we apply sensitivity concept to the root locus. This concept is called "Root sensitivity"

ROOT SENSITIVITY:-

As we vary K from 0 to ∞ symmetrycally



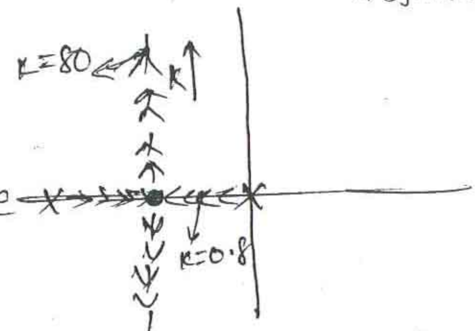

Suppose

K = 0.8

next value is K = 1.0 for a -

each change in K = 0.2.

Now for eg I am suddenly change the K value to 80. then the roots are disturbed (or) going in the same way we have to check.




Are the roots highly sensitive for higher value of K ?

" " ~~lower~~ higher " " lower " " K ?

we have to check.

ROOT SENSITIVITY:-

$\alpha$ = roots (S)

$\beta$ = System Gain (K)

Root sensitivity = S

C.E:

$\Rightarrow S(S+2)+K=0$

$K = -S(S+2)$

$\frac{K}{S} = -(S+2) \rightarrow ①$

$\frac{dK}{dS} = \frac{d}{dS}(-S^2-2S) = -2S-2 \rightarrow ②$

$\Rightarrow \frac{dS}{dK} = \frac{1}{-(2S+2)} \rightarrow ③$

from ① & ③

$\left(\because S_K^S = \frac{dS}{dK} \times \frac{K}{S}\right)$

$S_K^S = \frac{-(S+2)}{-(2S+2)} = \frac{S+2}{2S+2}$

The B.A.P is occurs at lower values of K.

$$S_K^S \Big|_{S=-1\,(B.A.P)} = \frac{-1+2}{2(-1)+2} = \infty$$

That means highly ~~stable~~ sensitivity. That means the poles will deviated from original path.

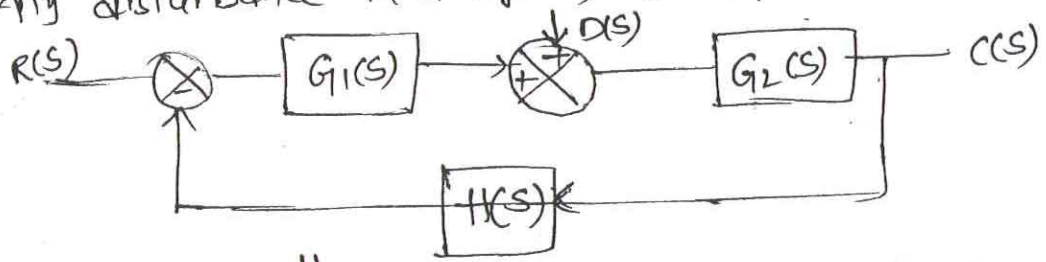
**Note :-**
This is also one of the reason, why critically damped systems are not design. For a critically damped system the roots are real & equal. So if any disturbance is occur they are deviated from its actual RL, since highly sensitive at lower values of K.

**NOTE :-**

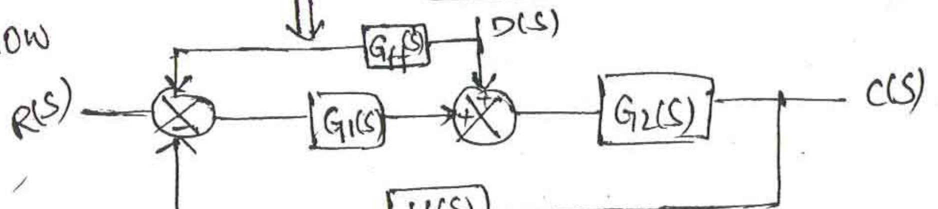
* Root sensitivity implies the sensitivity of roots on the locus path w.r.to sudden variations w.r. in system Gain 'K'.
* The Root Sensitivity is inversely proportional to the system Gain 'K' i.e the root sensitivity is higher for lower values of K & lower for higher values of 'K'.
* The Root sensitivity is max at Breakaway and Break in points.

## FEED FORWARD COMPENSATION :-

Any disturbance in a system is represent like this.




NOW




} ⇒ This will be getting by adding feed forward compensation.

$$\frac{C(S)}{R(S)}\Big|_{D(S)=0} = \frac{G_1(S)\,G_2(S)}{1+G_1(S)\,G_2(S)\,H(S)}$$

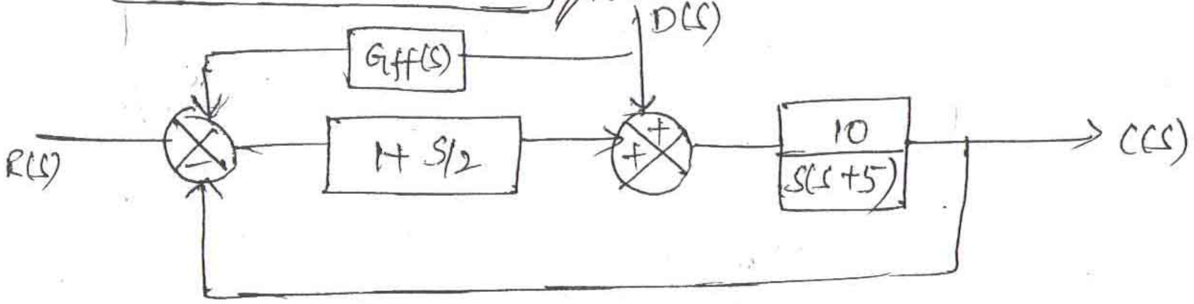
$$\frac{C(S)}{D(S)}\Big|_{R(S)=0} = \frac{G_2(S) + G_{ff}(S)\cdot G_1(S)\cdot G_2(S)}{1+G_1(S)\,G_{12}(S)\,H(S)}$$

$$\Rightarrow \quad C(S) = \frac{G_1(S)\cdot G_2(S) R(S) + D(S)\left[ G_2(S) + G_{ff}(S)\cdot G_1(S)\, G_2(S)\right]}{1 + G_1(S)\cdot G_2(S)\cdot H(S)}$$

To eliminate the effect of disturbances $(D(S))$, the condition for feed forward controller $G_{ff}(S)$ is

$$\boxed{G_{ff}(S) = \frac{-1}{G_1(S)}}$$

**Q:-**




The condition for feed forward compensator FF $G_{ff}(S)$ to eliminate the effect of disturbances in a system is

(a) $-\frac{(S+2)}{2}$ (b) $\frac{S+2}{2}$ (c) $\frac{-2}{S+2}$ (d) $\frac{2}{S+2}$

**Sol:-**

$$G_{ff}(S) = \frac{-1}{G_1(S)} = \frac{-1}{1+S/2} = \frac{-2}{S+2}$$

**Note:-** feed forward compensation is the effective way to eliminate the disturbance.

The way

## Introduction to "Industrial Controllers"

We already discuss that, an Introduction class

Controller O/P $(P) = f(e)$

where $e$ = Error

**1. Proportional controller :-**

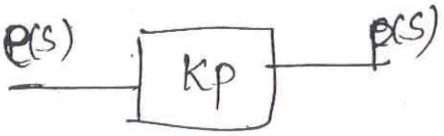
$$P \propto e$$

$$P = K_p e$$

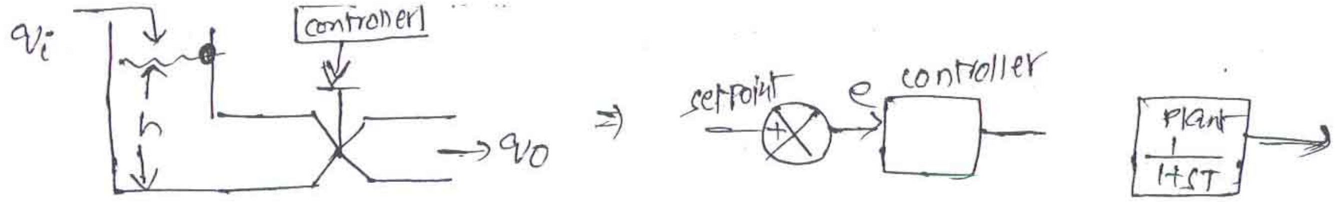
where

$K_p$ = Proportional gain

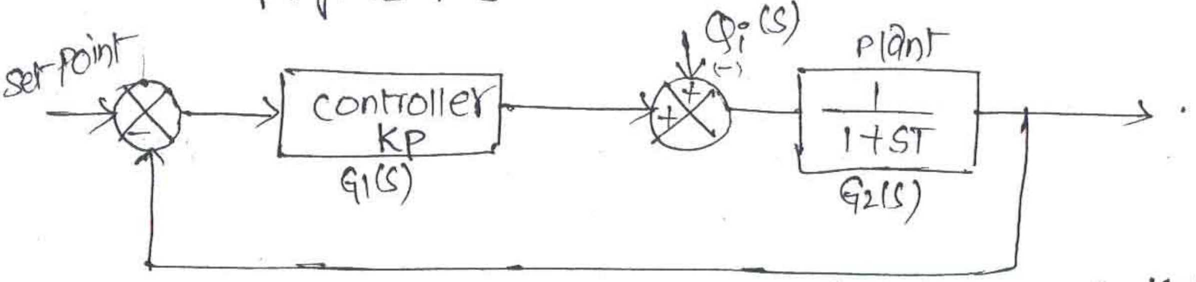
$$\therefore P(S) = K_p E(S)$$




$$\Rightarrow \frac{P(S)}{E(S)} = K_p$$




when ever set point change error will occur.
" " inlet " in plant " ". This will be shown in below. And level transeducer as shown in the figu as F.B.

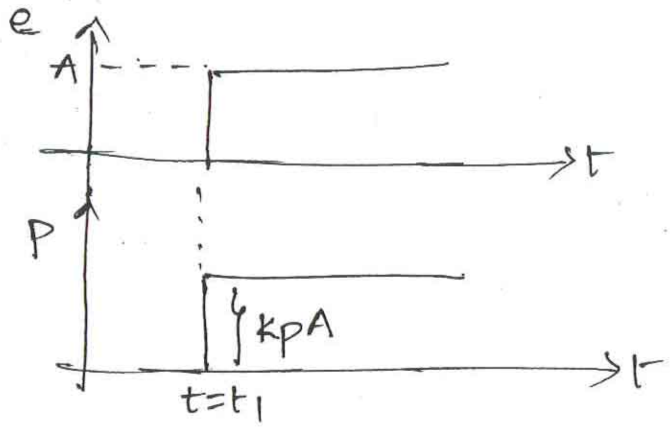



Now we have to analyze both disturbances individually.

$$e_{ss} = \lim_{s\to 0} - \frac{S \cdot Q_i(S) \cdot G_2(S)}{1+G_1(S) \cdot G_2(S)}$$

When suddenly water input inf increases ⇒ water level suddenly increases. ⇒ which will cause for sudden error this will be indicate as shown in below.

large qty of water of came & fill in the Tank.




$P = K_p e$

$\boxed{P = K_p A}$ $(\because e = A)$

o/p T/F i/p

Basically a controller will improves the steadystate & transcient behaviour.

Trancient response improves means the controller will respond quickly to the sudden change in i/p. that means suddenly opens the tap.

steadystate response improves means this error (as shown in fig) should be zero. That means water level returns back to its set point.

$$e_{ss} = \lim_{s \to 0} \frac{-s \cdot \frac{A}{s} \cdot \frac{1}{(1+sT)}}{1 + \frac{K_p}{1+sT}}$$

$$\boxed{e_{ss} = \frac{-A}{1+K_p}}$$

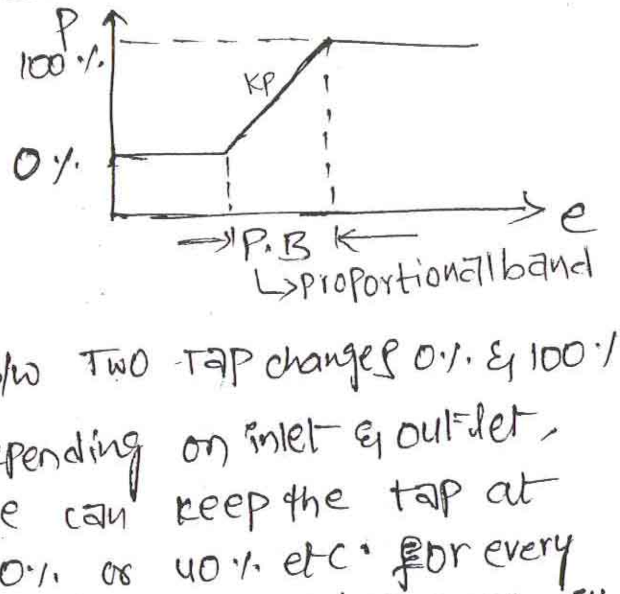
$$\boxed{|e_{ss}|_{OFFSET} = \frac{A}{1+K_p}} \longrightarrow$$ this residual error is called as "OFFSET".

That means the steady state error will not be zero. So that the proportional controller will improves only the transient response, but it does n't improves the steady state response since error $\neq 0$. This is the drawback of proportional controller.

For making $|e_{ss}|$ minimum $\Rightarrow K_p$ should be high. but we don't take high $K_p$. How it will be we will see.

Why $K_p$ can not be increased :-

Reason:- Now plot the chart b/w P & e.




Let us consider the liquid level control system.

we are the controller ($K_p$).

When the Tap (R) is completely closed, then any thing is below that? NO

O/P:- is saturated at 0%.

If Tap completely open then any this above that? NO, so it is saturated at 100%.

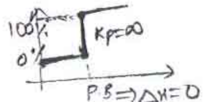
b/w Two Tap changes 0% & 100%.

Depending on inlet & outlet, we can keep the tap at 30% or 40% etc. for every value some band of error will present. This is shown in above fig.

for ex
3 rounds → for completely open the tank.
automatically 3 rounds → " " close "
} for both operations it is same. similarly

$$\text{Proportional Band (P.B)} = \frac{100}{K_p}$$ → Here for 0% & 100% same. that's why we are taking 100.

simply say → slope $= \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ $\Rightarrow K_p = \frac{100}{P.B} \Rightarrow P.B = \frac{100}{K_p}$ if slope↑ $\Rightarrow$



As $K_p \uparrow \Rightarrow$ P.B. $\downarrow \Rightarrow$ controller exhibits only "on & OFF operations"

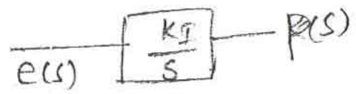
so that As $K_p \uparrow \Rightarrow$ Proportional band $\downarrow$ $\Rightarrow$ the controller having only on & OFF states only. Not having Proportional controller.

Note :-
large value of $K_p$, they reduce OFFSET ; but it also reduces the width of proportional band converting in to ON-OFF Controller.

2. Integral / Reset controller :-
Here the rate of change controller o/p is directly proportional to error.

$$\frac{dP}{dt} \propto e$$

$$\frac{dP}{dt} = K_I e$$



where $K_I$ = integral scaling

$$P = K_I \int e\,dt$$

$$P(s) = \frac{K_I}{s} e(s) \Rightarrow \boxed{\frac{P(s)}{e(s)} = \frac{K_I}{s}}$$

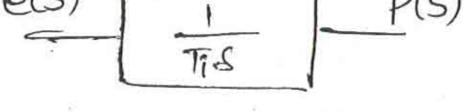
Now, we define "One important time i.e "Rise time" ", what is the significance of this we will discuss later.

$$\boxed{T_i = \frac{1}{K_I}}$$

$$P = \frac{1}{T_i} \int e\,dt$$

$$P(s) = \frac{1}{T_i s} \cdot e(s)$$

$$\Rightarrow \frac{P(s)}{e(s)} = \frac{1}{(T_i)(s)} = \frac{1}{T_i s} \;/\!/ \ldots$$



$$P = \frac{1}{T_i}\int A\,dt \quad [e = A]$$

$$P = \frac{A}{T_i}(t)$$

Analysis of this also same as Proportional controller [same Dgm].
when sudden error occurs ⇒ controller response = 0 & Response is very slow.

for checking that, response is slow (or) fast ?

Ans: for any system if i/p sinusoidal ⇒ o/p sinusoidal. If the o/p is phase lead char" then the response is fast & if the response is phase lag " " " slow.

let $e = \sin\omega t$

$$P = \frac{1}{T_i}\int \sin\omega t\,dt$$

$$P = \frac{-1}{\omega T_i}\cos\omega t = \frac{1}{\omega T_i}\sin(\omega t - \pi/2)$$

that means o/p is lag by $\pi/2$ degrees. so it is lag char"

Note:- * Disadv" of integral control is response is slow. Though it it is slow we have to check whether it bring back to set point or not. that means error = 0 (or) not check.

Now

$$e_{ss} = \lim_{s\to 0} \frac{-s\cdot\frac{A}{s}\cdot\frac{1}{(1+sT)}}{1+\frac{1}{T_i s(1+sT)}} \qquad \left(\because \lim_{s\to 0}\frac{-s\cdot Q_i(s)\cdot G_2(s)}{1+G_1G_2}\right)$$

$$e_{ss} = \frac{-A}{1+\infty} = 0$$

though this fellow (integral controller) is slow, it makes the error as zero.

* Dis adv" of this is Response is slow.

* but The steady state error = 0. → This is the Adv".

By changing the value of '$T_i$' we can reset at any value. That's means why this is also called as "Reset controller" & time is known as "Reset time".

NOTE:-

1. The Disadv. of Integral controller is its response to errors is slow.
2. At any instant of time 't' the rate of change of controller o/p can be reset by changing the value of $T_i$. Hence it is also known as "Reset controller".

Derivative controller:-

It is also called as "rate controller" because controller o/p is proportional to rate of change of error.

$$P \propto \frac{de}{dt}$$

$$P = K_D \cdot \frac{de}{dt}$$

$K_D$ = Derivative scaling

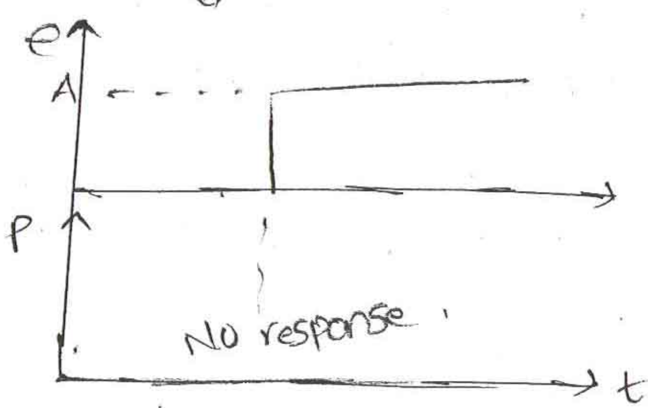
$$P(S) = K_D \cdot S \cdot E(S)$$

$$\Rightarrow \boxed{\frac{P(S)}{E(S)} = K_D \cdot S}$$

Now → let us take, a sudden change in input of Tank [step error].

Error

⇓




$$P = T_d \cdot \frac{d(A)}{dt} \qquad [\because e = A]$$

$$\underline{P = 0}$$

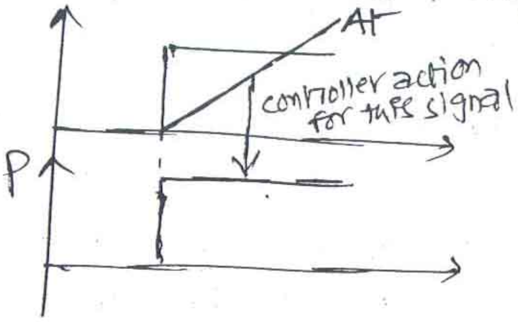
Analysis is same. [water level Analysis]. Suppose water level reaches to higher

It can not respond for certain type of errors.

Now if $q_i$ is continuously rise $\Rightarrow$ error is also rises continuously.

$$P = T_d \frac{d}{dt}(At)$$

$$= AT_d$$




water level just above the set point, Even it is not even to start to rise.

$\Rightarrow$ eventually it will make the Tap completely open. so Tank inlet < Tank outlet. at perticular point Tank Empty.

* Anticipation will not work at all ~~typ~~ times. It will work when the error is dynamic.

NOTE:- Anticipation means before the error signal occur, it will send the controller signal.

let $e = \sin\omega t$

$$P = T_d \frac{d}{dt}(\sin\omega t) = \omega T_d \cos\omega t$$

$$= \omega T_d \sin(\omega t + \pi/2)$$

Because of-

* Its Anticippation Nature will results in large error & instability in system:

NOTE:-

1. The Dis advn of this controller is it can not Respond to certain types of errors.
2. It is also known as "Anticipatory controller". Because it sends a control signal in anticipation of error.

NOTE: Anticipation = which is going to be started.

Composite Controller Modes :- first we learn (P + I) mode.

P + I Mode :-

$$P = k_p e + \frac{1}{T_i}\int e\,dt$$ $\rightarrow$ This is also correct:

$$P = k_p e + \frac{k_p}{T_i}\int e\,dt$$ $\rightarrow$ This is the correct way to represent any Analytic fun$^n$.

$$P(s) = \left\{K_p\left[1 + \frac{1}{T_i s}\right]\right\}E(s)$$

$$E(S) \longrightarrow \boxed{k_p\left(1+\frac{1}{T_i S}\right)} \longrightarrow P(S)$$

Effect on Transient state:— when controller is subjected to sinusoi i/p.

let $e = \sin\omega t$

$$p = k_p \sin\omega t + \frac{k_p}{T_i}\int \sin\omega t \, dt$$

$$P = k_p \sin\omega t + \frac{k_p}{T_i}\cdot\left[\frac{-\cos\omega t}{\omega}\right]$$

overall controller o/p now become

$$P = \sqrt{(k_p)^2 + \left(\frac{k_p}{\omega T_i}\right)^2}\ \sin\left[\omega t - \tan^{-1}\left(\frac{1}{\omega T_i}\right)\right]$$

where $\tan^{-1}\left(\frac{1}{\omega T_i}\right)$ is lag angle. so sluggish behaviour (slow).

therefore (P+I) controller will improves only transient steadystate error.

now question is:- In integral controller also steady state error improves. Then we do you are go for (P+I) controller, you can't change it.

Ans:- integral controller phase lag by 90°, you can't change it.
but here " " $\tan^{-1}\left(\frac{1}{\omega T_i}\right)$. By changing the $T_i$ value we can change the lagging.

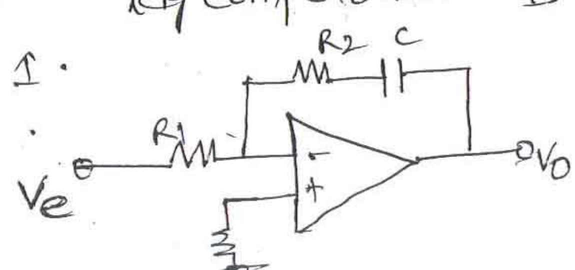
this is called "Tunning of controller". By chang $k_p$, $k_I$, $T_i$ we can change the phase lag. so this is called "Tunning"

but in individual controller if you change $k_I$, then sluggishne will not change.

But here by changing $k_I$ some better improvement will appear. That's why we are intrested to go for (P+I) mode for steady state response improvement [error=0; trub] → both are possible

this compe. (P+I) mode is similar to "lag compensator" since both will improve steady state response.

How (P+I) mode is similar to lag compensator means lag compensator is the combination of P & I.




$R_1$ → indicates proportional controller
$R_2 C$ → indicates integrator.

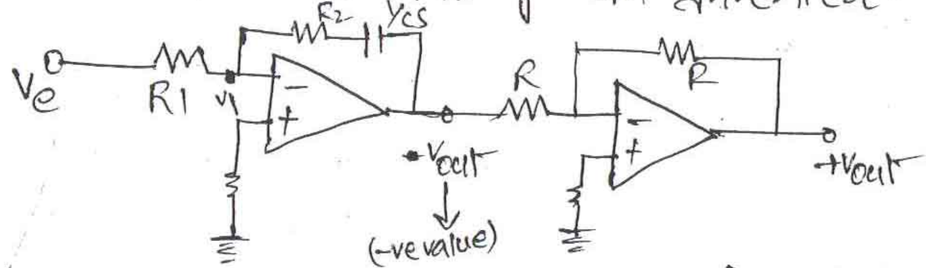
$$\frac{V_e - V_1}{R_1} = \frac{V_1 - V_{out}}{\frac{R_2CS+1}{CS}}$$

since $V_1 = 0$

$$\frac{V_e}{R_1} = \frac{-V_{out}\, CS}{R_2CS+1}$$

$$\Rightarrow -V_{out} = \frac{V_e(R_2CS+1)}{R_1CS} = V_e \cdot \frac{R_2CS}{R_1CS} + \frac{V_e}{R_1CS}$$

now compare this with $P(s) = K_p\left(1+\frac{1}{T_i s}\right)E(s)$. but here $-V_{out}$ is present. so for converting $-V_{out}$ into $+V_{out}$ we are taking an inverter.




$$\therefore +V_{out} = \frac{R_2}{R_1}\cdot V_e + \frac{R_2}{R_1}\cdot\frac{1}{(R_2C)}\int V_e\, dt \Rightarrow$$

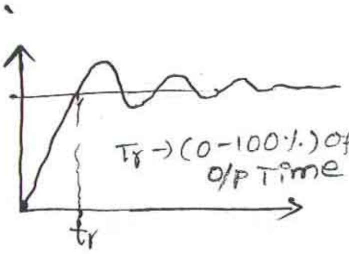
By comparing with standard eq$^n$ $\boxed{K_p = \frac{R_2}{R_1}}$ ; $\boxed{T_i = R_2C}$

→ This controller is capable of improving only steady state Response.

"<u>what is the effect of this controller on Rise time</u>"

<u>Rise time of the system will increases.</u>

An underdamped response As $t_r \uparrow$ what will happend to $M_p \downarrow\downarrow$.




Now <u>Adding lag compensator means adding a Pole</u> (at origin) to <u>the T.F</u> ⇒ stability ↓.

stability ⇒ not only BIBO ⇒ ( what's your name → ___ what's your name → ( ) After 5 mins ___ , so BIBO but speed of response)

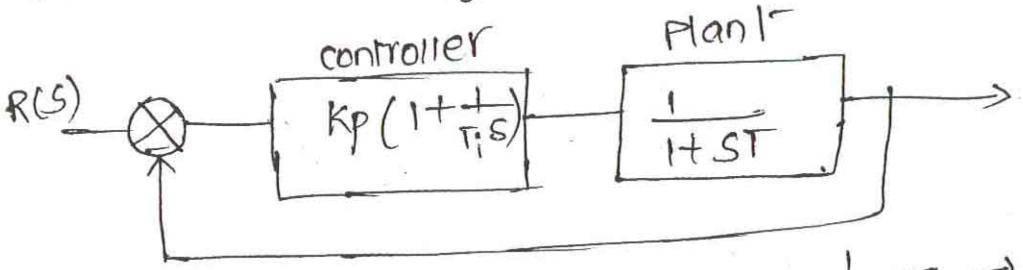
stability ⇒ <u>speed of response also</u>.

Therefore the Effect of lag compensator (or) PI controller stability will reduces & BW $\downarrow\downarrow$, $t_r \uparrow\uparrow$.

Note:-

1) It is capable of improving the steady state char" of the system only.
2) The integral controller eliminates the offset of proportional controller.
3) For sinusoidal input the phase of the controller o/p lags by $\tan^{-1}\left(\frac{1}{\omega T_i}\right)$. Hence it is similar to lag compensator
4) In terms of filtering property it is similar to lowpass filter.
5) It increases the rise time of the system.
6) It reduces the peak overshoot.
7) It reduces the Bandwidth of the system.
8) It reduces the stability of the system, because it adds the dominant pole.

Effect on order / Type of the system:-




without PI controller $G(S) = \frac{1}{(1+ST)} \rightarrow$ Type=0, order=1

with PI controller $G(S) = \frac{(K_p T_i S + K_p)}{(T_i S)(1+ST)}$

$\hookrightarrow$ Type=1 ; order=2

Effect on $\xi$ :-

$$1 + \frac{K_p T_i S + K_p}{T_i S(1+ST)} = 0$$

$$T_i T S^2 + S[T_i + K_p T_i] + K_p = 0$$

$$S^2 + \frac{S[\cancel{T_i} + K_p \cancel{T_i}]}{\cancel{T_i} T} + \frac{K_p}{T_i T} = 0$$

$$\boxed{\omega_n = \sqrt{\frac{K_p}{T_i T}} \text{ rad/sec}}$$

//...

$$2\xi\omega_n = \frac{1+K_P}{T}$$

$$2\xi\sqrt{\frac{K_P}{T_i T}} = \frac{1+K_P}{T} \Rightarrow \frac{4\xi^2 K_P}{T_i \cancel{T}} = \frac{(1+K_P)^2}{T^{\cancel{2}}}$$

$$\Rightarrow \boxed{\frac{4\xi^2 \cdot K_P T}{(1+K_P)^2} = T_i} \text{//...}$$

Note ***

$\boxed{\text{Rise time} \propto \xi^2}$

$$* \boxed{\therefore T_i \propto \xi^2} \text{//...}$$

(P+D) Mode :- $\rightarrow K_P + T_d \cdot \frac{de}{dt} = P$

The correct way to represent Analytical eqn (or) funn is

$$P = K_P e + K_P T_d \frac{de}{dt}$$

$$P(S) = \{ K_P(1+T_d S) \} E(S)$$

$$E(S) \longrightarrow \boxed{K_P(1+T_d S)} \longrightarrow P(S)$$

(P+D) ⇒ minm offset error definately meet.
maxm error depends on Derivative controller.

∴ This controller is not capable of improving steadystate response.

→ This controller is having the capability of improving Transient response.

Suppose we are using this controller to Boiler ⇒ speed of response ↑↑.

Note :-

(P+I) →
(P+I+D) → } most of controllers we are using.

(P+D) → rarely used.
⇓
This controller is used where Dead time (or) Delatime is large.

Effect on Transient state:-

$$P = K_P e + K_P T_d \frac{de}{dt}$$

let $e = \sin\omega t$

$\Rightarrow P = K_p \sin\omega t + K_p T_d \frac{d}{dt} \sin\omega t$

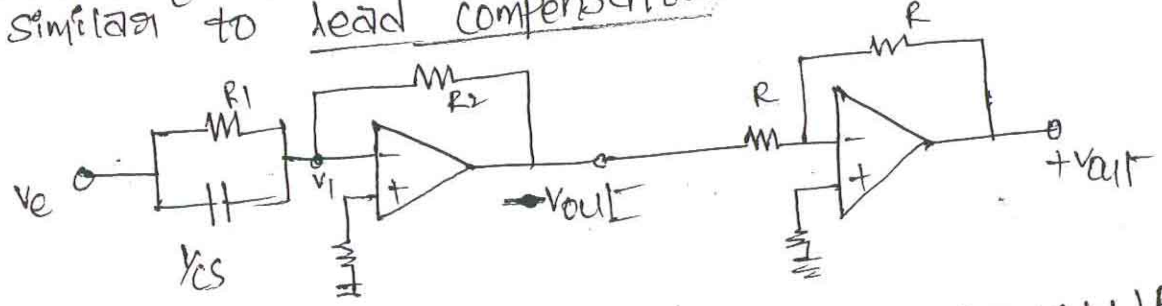
$P = K_p \sin\omega t + \omega K_p T_d \cos\omega t$

$$\boxed{P = \sqrt{K_p^2 + (\omega K_p T_d)^2} \cdot \sin(\omega t + \tan^{-1}(\omega T_d))}$$

∴ The phase of controller is lead by $\tan^{-1}(\omega T_d) \Rightarrow$ fast response.

$T_d \rightarrow$ by changing $T_d$ value we can change the anticipation response speed.

* since phase of this controller is lead by $\tan^{-1}(\omega T_d)$. This is similar to lead compensator.




$$\frac{V_e - V_1}{\frac{R_1}{R_1CS+1}} = \frac{V_1 - V_{out}}{R_2} \Rightarrow -V_{out} = \frac{V_e (R_1CS+1) R_2}{R_1}$$

$$-V_{out} = V_e \frac{R_2}{R_1} + \frac{R_2}{R_1} \cdot R_1CS \cdot V_e$$

$$+V_{out} = \frac{R_2}{R_1} \cdot V_e + \frac{R_2}{R_1} \cdot R_1C \cdot \frac{dV_e}{dt}$$

$$\boxed{\therefore K_p = \frac{R_2}{R_1}} ; \quad \boxed{T_d = R_1C}$$

chapter-3 Q.② conventional :- EFFECT ON TYPE & ORDER OF SYSTEM :-

without controller $G(s) = \frac{K}{s(sT+1)}$ ↳ Type = 1; order = 2

with PD controller $G(s) = \frac{K(K_p + sK_D)}{s(sT+1)}$ ↳ Type = 1; order = 2

NOW C.E ⇒ without controller:-

$TS^2 + S + K = 0$

$s^2 + \frac{1}{T}s + \frac{K}{T} = 0$

$\omega_n = \sqrt{\frac{K}{T}}$ ; $\xi = \frac{1}{2\sqrt{KT}}$

with (P+D) controller:

$C.E \Rightarrow 1+G(s)=0$

$Ts^2+s+KK_p+KK_D$

$$s^2 + \frac{s(1+KK_D)}{T} + \frac{KK_p}{T} = 0$$

$$\Rightarrow \boxed{\omega_n = \sqrt{\frac{KK_p}{T}} \text{ rad/sec}}$$

$$2\xi\sqrt{\frac{KK_p}{T}} = \frac{1+KK_D}{T}$$

$$\Rightarrow 4\xi^2\left(\frac{KK_p}{\cancel{T}}\right) = \frac{(1+KK_D)^2}{T^{\cancel{2}}}$$

$$\Rightarrow \boxed{\xi = \frac{1+KK_D}{2\sqrt{KK_pT}}}$$

Note:- It is a High pass filter.

note:-

$$\boxed{\therefore \xi \propto K_D \text{ (or) } \xi \propto T_d}$$

The speed of response is good. speed of response will be always interms of rise time only. so $t_r \downarrow\downarrow$.

$t_r\downarrow\downarrow \Rightarrow B.W\uparrow\uparrow$ ;

Here we are adding a dominant zero to the T.F

$\Rightarrow$ stability $\uparrow\uparrow$.

with only proportional:

$$\xi = \frac{1}{2\sqrt{K_pKT}}$$

with P+D :-

$$\xi = \frac{1+KK_D}{2\sqrt{K_pKT}}$$

$\Rightarrow \xi\uparrow$ with $(1+KK_D)$ Times.

As $\xi\uparrow \Rightarrow M_p\downarrow\downarrow$

Actually in underdamped case $t_r\downarrow \Rightarrow M_p\uparrow$. } so this is how can possible?

This will be understand in another way, we will discuss later.

chapter-7 Ans : (a)
Q : (11)
page (78)

page no : (62)

Q:(1) sol:- Ans (b) since does not improve steady state accuracy. means error = 0.

Q (7) Ans : (d)

Q:(9) sol:- $\omega_n$ not depends $T_D$ & $K_D$.
Ans : (C)

Q:(15) :-
Ans(d) } both will correct.
Ans (C) → since it is a HPF. so Attenuation/Amplification will do.

Among (c) & (d) c is more preference ✓✓

'C' will ~~domin~~ definately happen. but Mp is high or low depends on $\xi$. ($\xi \propto K_D$) or ($\xi \propto T_D$)

NOTE:-

Assum level control system.

Assum Tap open at 25%. ⇒ water continuously ~~flowing~~ falling in Tank. by controlling action we make the water level at set point.

Assum we are using (P+D) controller.

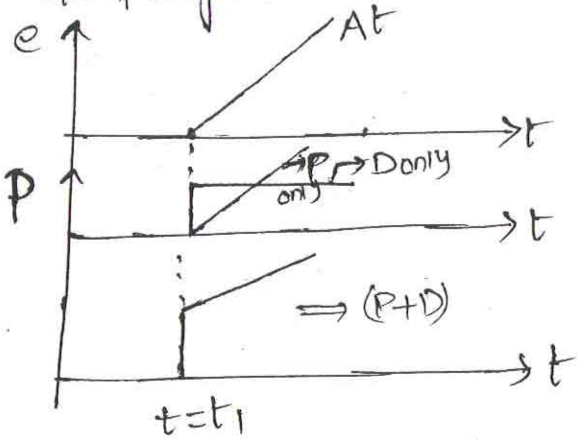
suddenly the inlet flow rate drops. but outlet is more. so water level decreases.

Actually inlet 25% ⇒ 10 m³/hr ; outlet = 20 m³/hr

Now ~~but~~ suddenly ⇒ 2 m³/hr

but outlet ⇒ 20 m³/hr so water level reduces.

water level is continuously reduces. so it indicates ramp signal.




At t=t₁ :-

Error was not occur. but the valve is closed. so the water level ↑↑. so this will indicates Peak over shoot.

As water level 2 m³/hr - ready to start fall, the 'D' controller action first will occur. so this will be cause for water level.

& And then 'P' controller comes into action.

This will mathematically can be express as Peak over shoot.

$$P = K_P At + K_P T_d \frac{d}{dt}(At)$$

$$P = K_P At + (AK_P) T_d$$

note:-

* If water level started to goes down, then The Derivative control acts and then closes the Tap ⇒ water level ↑ ⇒ Mp↑↑.

note: when 'D' control acts ⇒ there is a bound out of action from steady value.

Note:-

1. It is capable of improving the Transient state respn of the system only.
2. For sinusoidal input the phase of the controller O/P leads by $Tan^{-1}(\omega T_d)$. Hence it is similar to "lead compensator".
3. In terms of Filtering property it is a high pass filter.
4. It reduces rise time.
5. It increases peak over shoot.

As seen from the characteristics (fig ①) Derivative controller sends anticipatory control signal followed by Proportional Control signal. Hence increases Peak overshoot implies presence of maximum value due to anticipatory action which is proportional to properly tuned value of $T_D$. $[\xi \propto T_D]$.

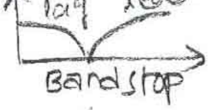
note: (This is applicable only for underdamped not for overdamp" since no overshoot there.)

6. It increases band width.
7. It increases stability of the system. because it adds a dominant zero.

Though Error ≠ 0. but still we are saying stability ↑ on what basis means "speed of response".

(P+I+D) mode:-

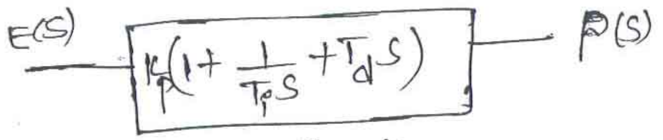
1) It is used for improving both Transient & ~~Transient~~ steady state response char" of the system.

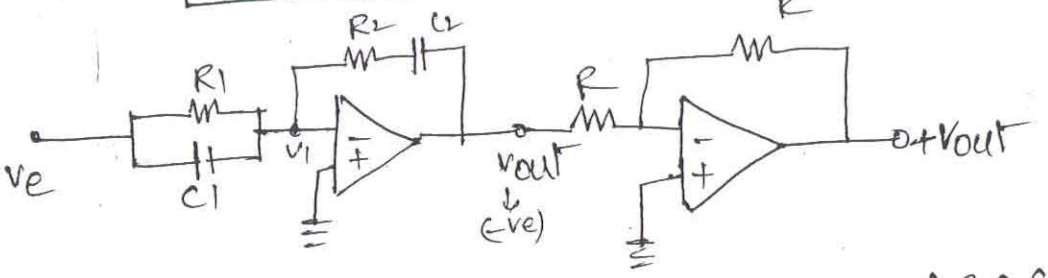



2) It is similar to "lead-lag compensator".
3) In terms of filtering property it exhibits "BANDSTOP" filter char"
4) reduces ~~Bandwidt~~ Risetime.
5) Increases Peak overshoot [The max" value depends on properly Tuned values of $T_D$ & $T_i$].
6) Increases Bandwidth.
7) Increases stability of the system.
8) Increases the Type & order of the system by 1.
9) It eliminates the steady state Error b/w I/P & O/P. Hence improves steady state response char" of the system.

$$P = K_p e + \frac{K_p}{T_i}\int e\,dt + K_p T_d \frac{de}{dt}$$

$$P(s) = K_p\left[1 + \frac{1}{T_i s} + T_d s\right] E(s)$$




$$\frac{V_e - V_1}{\frac{R_1}{R_1C_1S+1}} = \frac{V_1 - V_{out}}{\frac{R_2C_2S+1}{C_2S}} \Rightarrow -V_{out} = \frac{V_e[R_1C_1S+1][R_2C_2S+1]}{R_1C_2S}$$

$$\Rightarrow V_{out} = \frac{V_e[\cancel{R_1}C_1R_2\cancel{C_2}S^{\cancel{2}}]}{\cancel{R_1}\cancel{C_2}\cancel{S}} + \frac{V_e S[R_1C_1 + R_2C_2]}{R_1C_2S} + \frac{V_e}{R_1C_2S}$$

$$\Rightarrow -V_{out} = V_e\left[\frac{\cancel{R_1}C_1}{\cancel{R_1}C_2} + \frac{R_2\cancel{C_2}}{R_1\cancel{C_2}}\right] + \frac{V_e}{R_1C_2S} + R_2C_1SV_e$$

compared to $\frac{R_2}{R_1}$, $\frac{C_1}{C_2}$ ratio will be Negligible since capacitance will be in $\mu F$. so

$$-V_{out} = \frac{R_2}{R_1}V_e + \frac{R_2}{R_1}\cdot\frac{1}{R_2C_2}\int V_e\,dt + \frac{R_2}{R_1}R_1C_1\frac{dV_e}{dt}$$

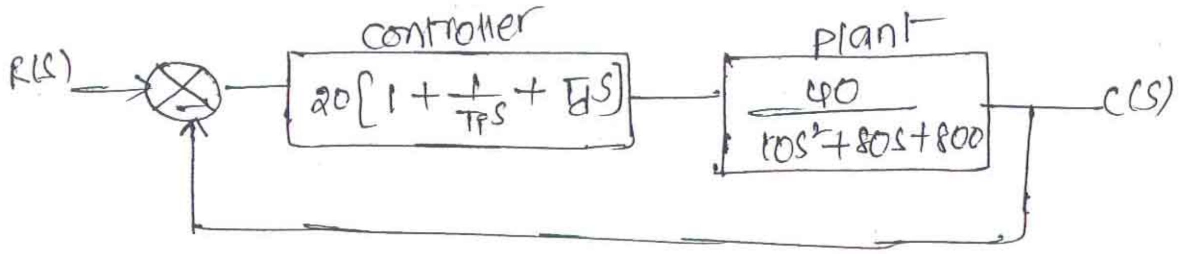
By comparing

$$\boxed{K_p = \frac{R_2}{R_1}\ ; \quad T_i = R_2C_2\ ; \quad T_d = R_1C_1}$$

Page no (7)
Conven Q: (1)
sol:-

$$\theta_c(t) = 20\left[e(t) + \frac{1}{T_r}\int_0^t e(t)\,dt + T_d\cdot\frac{de(t)}{dt}\right]$$

$$\Rightarrow \theta_c(s) = 20\left[E(s) + \frac{E(s)}{T_r S} + [T_d S]E(s)\right]$$




a) $G(S) = \dfrac{20(1+T_d S)\,4}{S^2+8S+80}$

$1+G(S)H(S)=0 \Rightarrow S^2+8S+80+20(1+T_d S)\,4=0$

$\Rightarrow S^2+S[8+80\,T_d]+160=0$

$\omega_n=\sqrt{160}=12.64$ rad/sec

$2\zeta \times 12.64 = 8+(80 * T_d)$

$\Rightarrow 2\times1\times12.64 = 8+(80T_d)$

$\Rightarrow T_d = 0.2$ sec ...

b) $G(s) = \dfrac{(20T_i s+20+20(0.2)\,T_i s^2)\,4}{T_i s\,(s^2+8s+80)}$

$= \dfrac{80\,T_i s+80+16T_i s^2}{T_i s\,[s^2+8s+80]}$

$1+G(S)H(S)=0$

$\Rightarrow (S^2+8S+80)\,T_i S + 80T_i S+80+16T_i S^2=0$

$T_i S^3+24T_i S^2+160T_i S+80=0$

$S^3+24S^2+160S+\frac{80}{T_i}=0$

| | | |
|---|---|---|
| $S^3$ | 1 | 160 |
| $S^2$ | 24 | $\frac{80}{T_i}$ |
| $S^1$ | $\dfrac{(24\times160)-\frac{80}{T_i}}{24}$ | 0 |
| $S^0$ | $\frac{80}{T_i}$ | 0 |

$\dfrac{(24\times160)-\frac{80}{T_i}}{24}=0$

$\Rightarrow \frac{80}{T_i}=24\times160$

$\Rightarrow \boxed{T_i = 0.02\ \text{sec}}$

(Here we are solving like Kmar. If K > Kmar ⇒ system stable like that.) ...

If $T_i > 0.02$ → system will unstable

$T_i < 0.02$ → stable

$T_i = 0.02$ → marginally stable

Q:8 sol:- which is a proportional controller since P.B exist
7M page only in 'P' controller.

$P = K_p e$

$100 = K_p \times 1$

$\Rightarrow K_p = 100$ ...

$P.B = \frac{100}{K_p} = \frac{100}{100} = 1$

$\therefore \% P.B = 1 \times 100 = 100\%$ ...

for 100% P.B → 100 V o/p for 1V error.

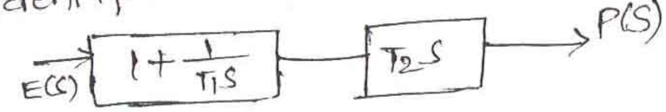
$20\% P.B \rightarrow [(20)\%] \times 100 = 0.2 \times 100 = 20V$ ...

Ans:- $\boxed{1V, 20V}$ //...

Q:10:- lead → High pass filter
lag → Low pass
compensator

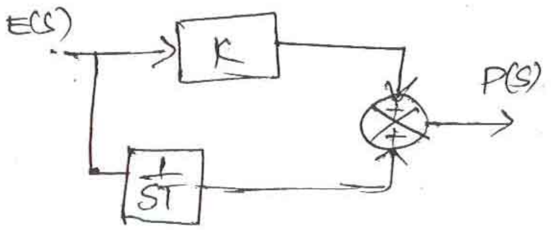
Q:- Identify the controller & obtain controller constants.

(a)



(b)

(c)



sol:-

(a) $\left(T_2 S + \frac{T_2 S}{T_1 S}\right) E(S) = P(S)$

$\left(\frac{T_2}{T_1} + T_2 S\right) E(S) = P(S)$

$[\because (K_p + K_p T_D S) E(S) = P(S)]$

↳ It is a (P+D) controller.

$\left\{\frac{T_2}{T_1}\left[1 + \frac{T_1}{T_2} \times T_2 S\right]\right\} E(S) = P(S)$

now compare this with $P(S) = [K_p (1 + T_d S)] E(S)$

$$\boxed{K_p = \frac{T_2}{T_1} ; \; T_d = T_1}$$ //

(b) $P(S) = \left[K + \frac{1}{ST}\right] \cdot E(S)$

It is a PI controller.

$P(S) = \left[K\left[1 + \frac{1}{KST}\right]\right] E(S)$

Compare it with

$$\boxed{P(S) = K_p\left(1 + \frac{1}{T_i S}\right) E(S)}$$

$\boxed{\therefore K_p = K}$ ; $\boxed{T_i = KT}$ ....

(c) $P(S) = (1+ST)^2 \cdot E(S)$

$P(S) = [T^2S^2 + 2TS + 1]\, E(S)$

$$P = T^2 \frac{d^2e}{dt^2} + 2T\frac{de}{dt} + e$$

let $\frac{de}{dt} = e_p \Rightarrow e = \int e_p\, dt$

$$p = T^2 \frac{de_p}{dt} + 2Te_p + \int e_p\, dt$$

$$P(S) = \left(T^2 S + 2T + \frac{1}{S}\right) E_p(S)$$

$$= \left\{2T\left(1 + \frac{T^{\cancel{2}}}{2\cancel{T}} S + \frac{1}{2TS}\right)\right\} E_p(S)$$

now compare it with standard form

$\Rightarrow \boxed{K_p = 2T}$ ; $\boxed{T_d = T/2}$ ; $\boxed{T_i = 2T}$ .....

= x =

# SERVO MECHANISM

They are electro mechanical systems whose input is electrical voltage and output is mechanical position (or) its time-derivative.

these are also known as "Inverse Transducers".

Note:- most commonly used servo mechanism in control applications are "servo motors".

SERVO MOTORS

- Ac servo motor
- DC servo motor
  - Armature controlled
  - Field controlled

In this Analysis, we discussing about T.F of diff" motors (AC & DC)

note:- why word servo we are using in control system.

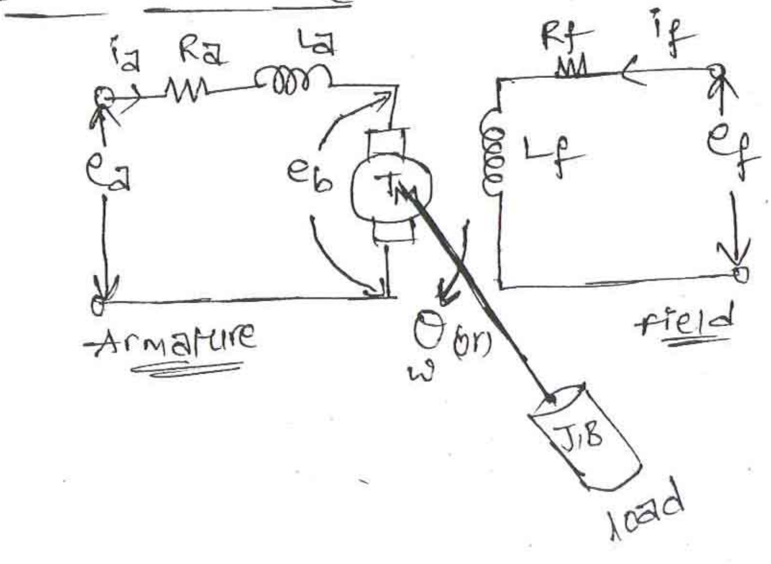
explanation:- Basically

1) servo → low power applications this word is used. so we use that word control systems are low power ckts.

2) These all are control system components. we make all of them combine & then make a complete control system. overall C.S should exhibit linear char ⇒ for that individual char" of each component should be linear.

so servo means obtaining approximate linear char".

DC servomotors :-



servo applications implies low power applications (or)
the charu b/w the input & output should be approximately linear.

In control systems AC (or) DC motors having low power Ratings are known as "servo Motors".

→ Now to write T.F ⇒ first of all identify what is the i/p & what is the o/p.

Armature controlled DC servomotor :-

Input = $e_a$ ; output = $\theta$ (or) $\omega$

→ Airgap flux ($\phi$) $\propto$ field current ($I_f$)

$$\phi = K_f i_f \rightarrow (1)$$

→ $T_m \propto \phi I_a$

$$T_m \propto K_f I_f I_a$$

$$T_m = \underbrace{K_1 K_f I_f}\, I_a$$

$K_T = K_1 K_f I_f$ = Motor Torque constant

$$\therefore T_m = K_T I_a$$

→ Motor Backemf $\propto$ speed

$$e_b \propto \frac{d\theta}{dt} \Rightarrow e_b = K_b \frac{d\theta}{dt} \rightarrow (3)$$

→ Analysis of Armature circuit :

$$e_a = i_a R_a + L_a \frac{di_a}{dt} + e_b \Rightarrow e_a - e_b = i_a R_a + L_a \frac{di_a}{dt} \rightarrow (4)$$

→ At load:- $T_M = B \frac{d\theta}{dt} + J \frac{d^2\theta}{dt} \rightarrow (5)$

→ Transfer fun:-

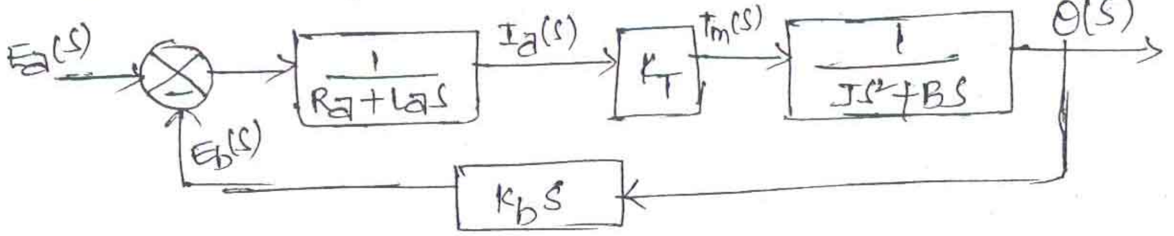
$$T_M(S) = K_T I_a(S) \rightarrow (6)$$

$$E_b(S) = K_b S\, \theta(S) \rightarrow (7)$$

$$E_a(S) - E_b(S) = [R_a + L_a S] \cdot I_a(S) \rightarrow (8)$$

$$T_M(S) = \theta(S) [JS^2 + BS] \rightarrow (9)$$

you have to see, in which eqn overall i/p will present. so take eqn ⑧ first. next take which eqn having $I_a$ ⇒ eqn ⑥ having draw it like that blockdgm is drawn.




$$T.F \quad \frac{\theta(S)}{E_a(S)} = \frac{\dfrac{K_T}{(R_a+L_aS)(JS^2+BS)}}{1+\dfrac{K_T K_b S}{(R_a+L_aS)(JS^2+BS)}}$$

$$= \frac{K_T}{(R_a+L_aS)(JS^2+BS)+K_T K_b S}$$

since for Armature $L_a \simeq 0$

$$T.F = \frac{K_T}{R_a(JS^2+BS)+K_T K_b S}$$

$$\boxed{T.F = \frac{K_T}{R_a S\left[\left(JS+B+\frac{K_T K_b}{R_a}\right)\right]}}$$

This T.F of Arm. Contrl servo motor having only one time constant. since $L_a=0$.

Note:- single time constant - Armature controlled servo motor is feed back control system.

Field controlled servo Motor:-

Input $= e_f$ ; output $= \theta$ (or) $\omega$

Airgap flux ($\phi$) $\propto$ field current ($I_f$)

$\phi \propto i_f \Rightarrow \phi = k_f i_f \rightarrow ①$

$T_M \propto \phi i_a$

$T_m \propto k_f i_f i_a$

$T_m = \underbrace{k_1 k_f i_f}\, i_a = \underbrace{k_1 k_f}_{K_T}\, i_a \cdot i_f$

$k_1 k_f i_a =$ motor torque constant

$T_M = K_T I_f \rightarrow (2)$ where $K_T = K \cdot K_f I_a$

Back emf $(e_b) \propto \frac{d\theta}{dt}$

$\Rightarrow e_b = K_b \frac{d\theta}{dt} \rightarrow (3)$

Analysis of field current

$e_f = i_f R_f + L_f \frac{di_f}{dt} \rightarrow (4)$

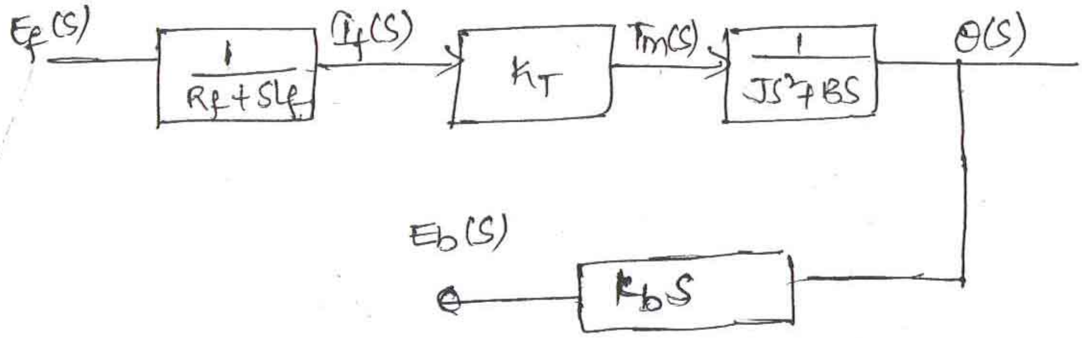
At load $T_M = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} \rightarrow (5)$

Transfer function: $T_M(s) = K_T I_f(s) \rightarrow (6)$

$E_b(s) = K_b s\theta(s) \rightarrow (7)$

$E_f(s) = I_f(s)[R_f + L_f s] \rightarrow (8)$

$T_m(s) = \theta(s)[Js^2 + Bs] \rightarrow (9)$



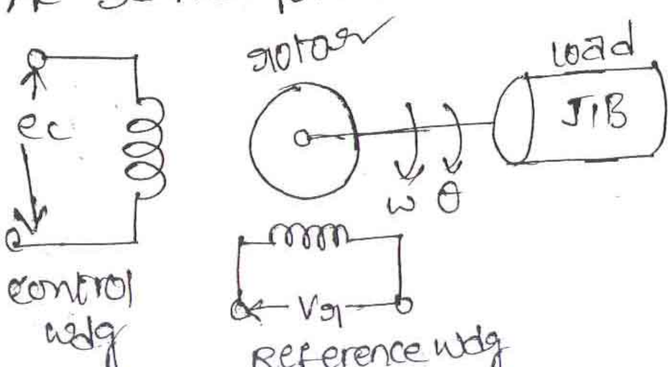

NO f.B. so it is OLCS.

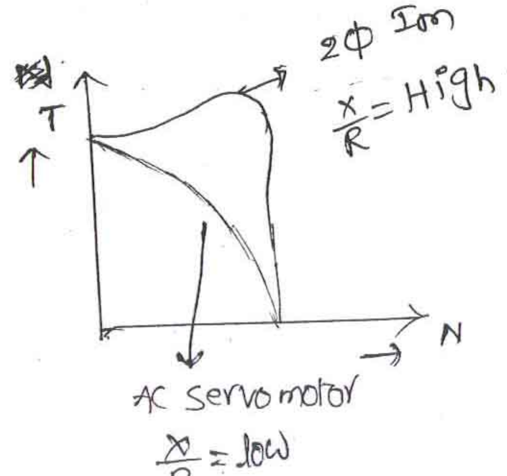
mostly used is CLCS i.e Armature controlled.

The diff b/w OLCS & CLCS is insignificance of f.B. This insignificance of feedback in OLCS is called as "removal (or) elimination" of f.B." so this exn (field controlled method) is the perfect exn for OLCS. In which conclude that we can't remove $e_b$ (back EMF). Are you remove back emf in a motor? no we can't remove. so f.B is exist in OLCS & CLCS but insignificant in OLCS.

· we can't remove the f.B.

## AC Servomotor :-



1) In Induction motor for change Torque → we have to change both the wdgs polarities. In servo motor only one wdg will be changes.

2) when $\frac{X}{R}$ = low ⇒ char" are nearly linear. There are the two diff" b/w Induction motor & servomotor.

<u>limitations of AC servo motor</u> <u>compared to DC servomotor</u>:-

→ when AC i/p involved direct T.F evaluation is difficult. We will write approximate model.

→ " " " Freq" involved. So Freq" Elements are involved i.e coils & capacitors involved.

<u>Note</u>:- 1) It is constructionally similar to a "2-φ induction motor".

2) In AC servo motor the rotor is either squrrel cage (or) Drag cup Type. And Torque developed by it is proportional to control winding voltage $V_c$. The other wdg in quadrature is excited by a constant voltage and is known as reference winding.

3) The rotor of an AC servo motor is built with high resistance so that its $\frac{X}{R}$ ratio is small and N-T char" (Speed-Torque) are approximately linear.

$$T_m \propto e_c$$

$$T_m = K_m e_c \rightarrow (1)$$

where $K_m = \frac{T_o}{e_c}$ where $T_o$ = stall Torque

At load :-

$$T_M = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt}$$

At Equilibrium

$$K_m e_c = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt}$$

$$K_m E_c(S) = [JS^2 + BS]\,\theta(S)$$

$$\boxed{\frac{\theta(S)}{E_c(S)} = \frac{K_m}{S[JS+B]}}$$

//....

This is the approximate T.F only.

$$\boxed{\frac{\theta(s)}{E_c(s)} = \frac{K_m}{s(Js+B \pm m)}}$$

$\pm M$ = correction factor
$\simeq$ slope of N-T chara

Here $\pm m$ take → because in some books it is +ve, & in some books it is -ve.
If the slope is a +ve slope you will take $+M$, if the slope is -ve slope, you have to take $-M$.

If +M is given that is correct. ✓
-M " " ✓
±M " " " , but we have give preference to this ✓✓

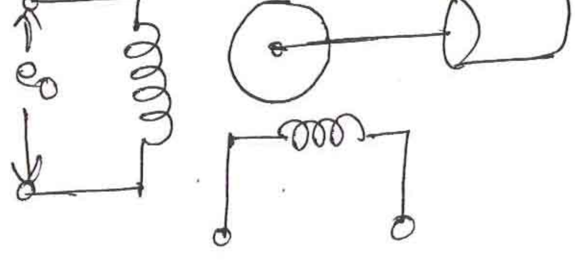
## TACHOMETER :-

They are speed Transeducers and are used as feedback element in control system applications. They are of TWO types.
1) DC Tachometer
2) AC Tachometer

DC Tachometer :-
A DC Tachometer is a small DC Generator, whose input is shaft speed & output is electrical voltage directly proportional to the speed.

AC Tachometer :-
An AC Tachometer is also known as "Drag cup Generator", because its rotor is drag cup type and it is constructionally similar to AC servo motor.

(As load constant ⇒ rotor constant ; when load is moving ⇒ rotor rotates ⇒ $e_0$ produces)




out of the TWO wdgs only one of the wdg is excited by a constant voltage '$V_r$' as shown in figure.

when the rotor is stationary the pheripheral flux will be linking reference winding only. As the rotor rotor rotates rate of change of flux induces an emf '$e_0$' directly proportional to speed of the rotor.

Input = $\omega$

o/p = $e_0$

$e_0 \propto \omega$

$e_0 = K_T \omega$

$E_0(s) = K_T\, \omega(s)$

$$\omega(S) \longrightarrow \boxed{K_T} \longrightarrow E_0(S)$$

suppose I/P = $\theta$

o/p = $e_0$

$e_0 \propto$ speed

$e_0 \propto \frac{d\theta}{dt}$

$e_0 = K_T \frac{d\theta}{dt}$

$E_0(S) = K_T S \cdot \theta(S)$

$$\theta(S) \longrightarrow \boxed{K_T S} \longrightarrow E_0(S)$$

where $K_T$ = Tachometer constant.

$E_0(S) = S K_T \cdot \theta(S) \implies E_0(S) = K_T[S\,\theta(S)]$

As a T.F you can write $K_T S$. for operation analysation you can take only $K_T$ i.e zero order system.

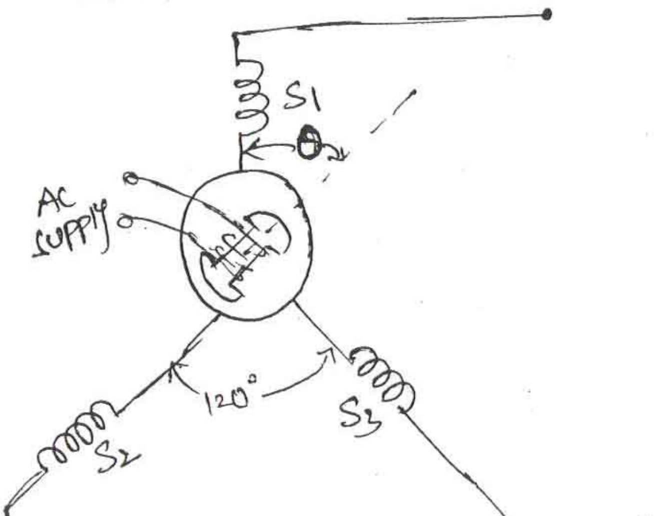
note:-

So T.F of an AC Tachometer is

$K_T \rightarrow$ for i/p is $\omega(s)$

$K_T S \rightarrow$ for i/p is $\theta(S)$.

SYNCHRO:-




The commercial Name for synchro is "SELSYN" (or) "AUTOSYN".

The main application of synchro is "error detector".

synchro transmitter — Receiver

↓ SERMO mechanism

↓ Transeducer

It is constructionally similar to Alternator not operationally not same, rotor is Dumble shape.

→ when we rotate the rotor in a Perticular position then EMF will be max, & at remaining positions it is min.

operationally similar to T/F.

Suppose we align rotor at $S_1$, max voltage will be induced at $S_1$ & min voltage at $S_2, S_3 \cong 0$.

for a perticular position it will produce 3 voltages. That means 1 i/p ⇒ 3 o/ps. Can you write the T.F?

Ans: No: & you don't take remaing o/ps as 0 since all are exist or occur at one time. → ($S_1S_2, S_2S_3, S_3S_1$)

Transmitter voltages these voltages will transmitted & given to 2nd synchro. So it will produces Displacement since Transeducer. therefore the rotor in control T/F is cylinder shape.

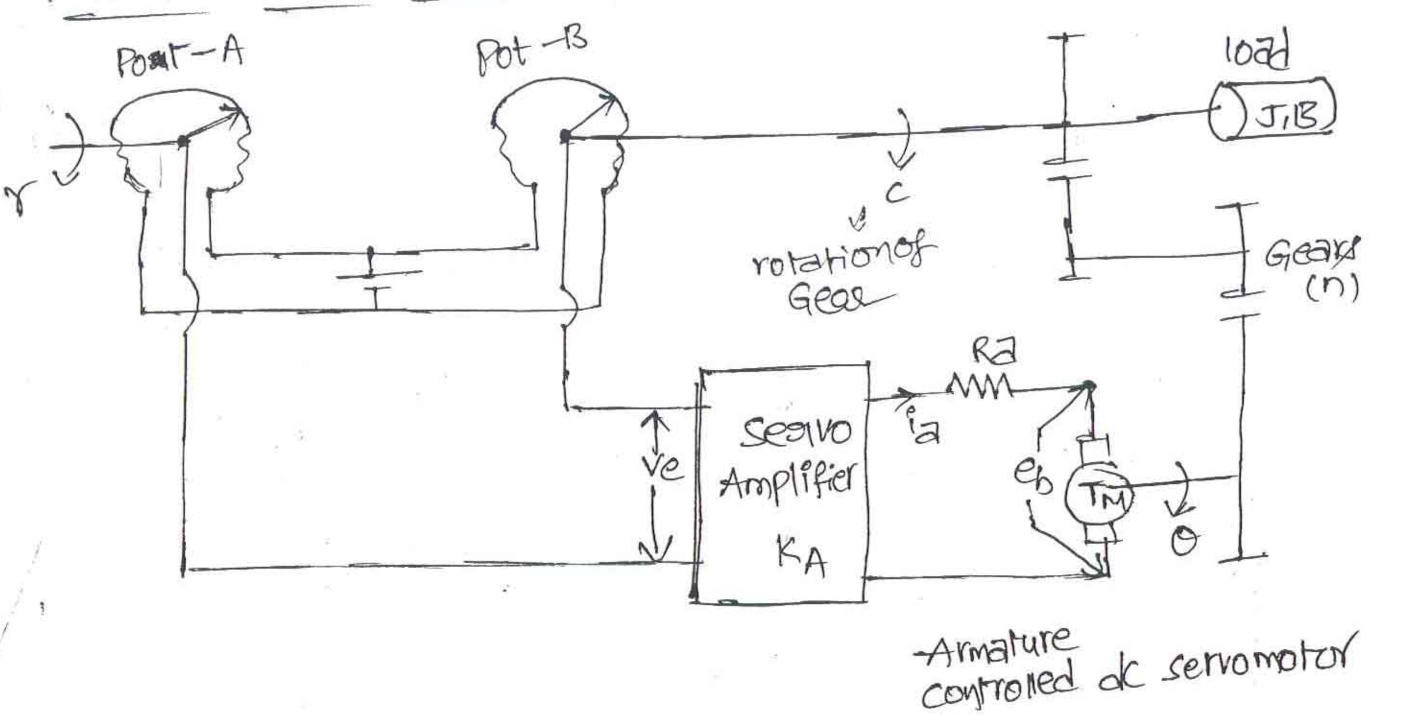
NOTE:-

1) It is commeartially called as "SELSYN (or) AUTOSYN"
2) It is constructionally similar to 3-φ Alternator and operationally it is based on T/F (Transformer) action.
3) The basic synchro units known as "Synchro Transmitter" is a Transeducer which converts angular position of the rotor converts into proportional voltages.
4) When the Dumbel shape rotor is excited by an AC voltage it induces sinusoidally time varying fluxes b/w the stator - rotor Pheriphery. Depending on the Angular position of the rotor voltages are induced across $S_1$ & $S_2$ & $S_3$ windings with unequal magnitudes under the phase difference of 120°.
5) When the rotor is align with respect to any one of the stator wdg then max voltage will be induced across that winding and minimum (almost zero) voltages will induces across other Two windings. This position of rotor is known as "Electrical zero position".
6) The main use of synchro in control system applications is as Error Detector.

The pair of Synchros known as "Synchro Transmitter" and "Synchro controlled Transformer" (Receiver), acts as "Error Detector".

The synchro control T/F is a servo mechanism which converts voltages from synchro Transmitter into angular displacement of rotor. The rotor of control T/F is "circular in shape".

POSITION CONTROL SYSTEM :- this is the application what we are learn




* we have two potentiometers. The pair of potentiometers are acts as "error detector". (Already we well know)

The Aim of the Position c.s is to transfer the i/p shaft $\gamma$ to the output (or) load.

If $\gamma = c \Rightarrow$ Then there is no Error signal $V_e$. we have only Here Gear will not having any mass & friction. we have only J,B of load. 'c' is the rotation of Gear. when motor rotates (θ) then gear mechansim rotation is 'c'....

Input $= r = [R(S)]$ ; output $= c = [C(S)]$

→ At Potentiometer

$V_e \propto \gamma - c$

$V_e = K_p(\gamma - c)$

$V_e(s) = K_p(R(S) - C(S)) \rightarrow ①$

→ At Amplifier

$K_a = \dfrac{V_a}{V_e} = \dfrac{V_a(s)}{V_e(s)} \rightarrow ②$

→ Analysis of Armature controlled DC Servomotor

$$T_M(S) = K_T\, I_a(S) \rightarrow ③$$

$$E_b(S) = K_b\, S\, \theta(S) \rightarrow ④$$

$$[V_a(S)] - [E_b(S)] = I_a(S)\, R_a \rightarrow ⑤$$

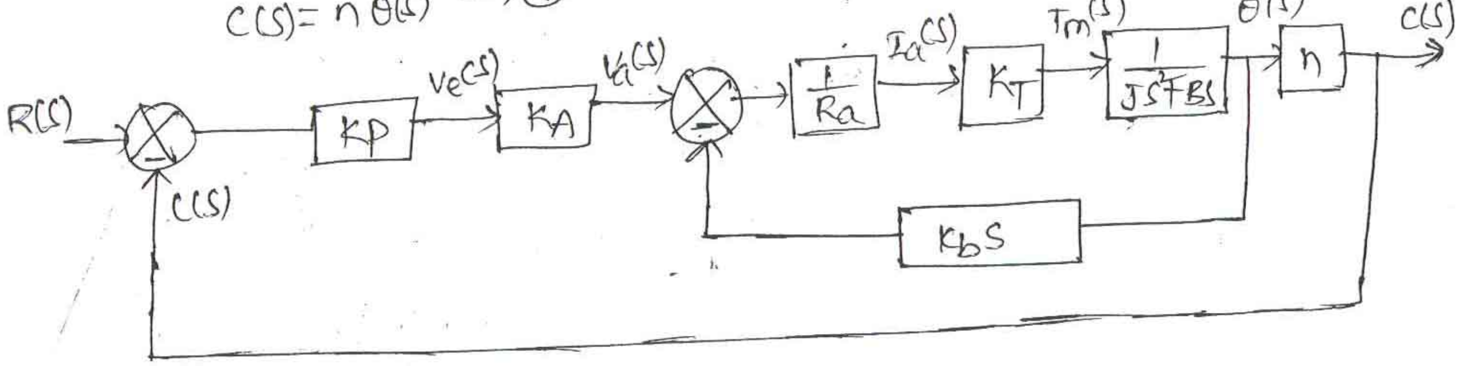
$$T_M(S) = \theta(S)[JS^2 + BS] \rightarrow ⑥$$

→ At Gears:-

$$C \propto \theta$$

$$C = n\theta$$

$$C(S) = n\,\theta(S) \rightarrow ⑦$$



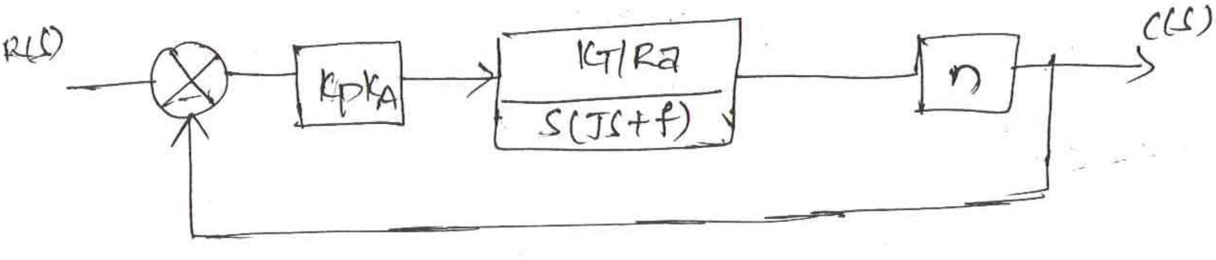

$$T \cdot F = \frac{C(S)}{R(S)} = ?$$

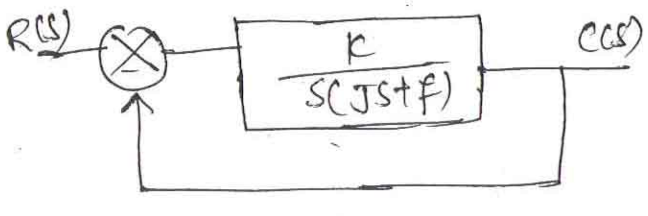
F.B Path :-

$$\frac{\dfrac{K_T}{R_a(BS+JS^2)}}{1 + \dfrac{K_T K_b S}{R_a(JS^2+BS)}} = \frac{K_T}{R_a(JS^2+BS) + (K_T K_b S)}$$

$$= \frac{K_T/R_a}{JS^2 + BS + \underbrace{\dfrac{K_T K_b S}{R_a}}_{fS}}$$

$$\Rightarrow \frac{K_T/R_a}{S(JS+f)} \;||\ldots$$




Where $\boxed{K = \dfrac{K_p K_A K_T n}{R_a}}$

where

$K_p$ → you can't change

$K_T$ → motor const" you can't change.

$n$ → we can't change → Are you change no: of teeths ?? NO

$K_A$ / $R_a$ → can be change

- As we maintain $R_a$ = const.

⇒ So we are varying only '$K_A$', we are Analyse the system Performance. what what values of $K_A$ system will stable (or) critically stable (Root locus).

So By using the above (T.F) eq" mathematical model we are Analyse the sys performance of Practically systems theoritically. (Root locus for what values of Amplifier gain system could be stable. In that range only we are taking).

# PART - IV

## FREQUENCY DOMINE ANALYSIS

* When any system is subjected to sinusoidal input - $r(t) = A \sin \omega t$. The output is also sinusoidal having different magnitude and phase angle but same input frequ '$\omega$' rad/sec.

i.e $c(t) = B \sin(\omega t \pm \phi)$.

* Freq" response Analysis amplies vary '$\omega$' from '0' to '∞' and observing the corresponding variations in the magnitude and phase angle of the response.

Let $F(S) = \dfrac{C(S)}{R(S)}$ = T.F

Put $S = j\omega$. (when we replace 's' by '$j\omega$', - It means that we are subjected to sin" i/p to the system.

$f(j\omega)$ = sinusoidal T.F

≅ sinusoidal response

$$\boxed{F(j\omega) = |F(j\omega)| \angle F(j\omega)}$$

$|F(j\omega)| \rightarrow \sqrt{(I.P)^2 + (R.P)^2}$

$\angle F(j\omega) \rightarrow \tan^{-1}\left(\dfrac{I.P}{R.P}\right)$

(Pb) $x(t) \leftarrow \boxed{\frac{1}{s+1}} - y(t)$ Given $x(t) = \sin t$

Sol:

$$F(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+1}$$

$$F(j\omega) = \frac{1}{j\omega+1} = \frac{1+j0}{1+j\omega}$$

$$|F(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$$

$$\angle F(j\omega) = -\tan^{-1}\left(\frac{\omega}{1}\right)$$

$$\therefore F(j\omega) = \frac{1}{\sqrt{1+\omega^2}} \cdot \angle -\tan^{-1}(\omega)$$

Given $x(t) = \sin t$
$\cong \sin \omega t$
$\Rightarrow \omega = 1$ rad/sec

$$F(j\omega) = \frac{1}{\sqrt{2}} \angle -45^\circ$$

$$\boxed{\therefore y(t) = \frac{1}{\sqrt{2}} \sin(t - 45^\circ)}$$ //...

(Pb) GATE 2012

$\frac{C(s)}{R(s)} = \frac{(s^2+9)(s+2)}{(s+3)(s+4)}$ ; for $R(s) = \frac{\omega}{s^2+\omega^2}$, the freq$^n$ at which the system becomes zero is ——?

Sol:-

$$F(j\omega) = \frac{(-\omega^2+9)(\omega+2)}{(\omega+3)(\omega+4)} = 0 \Rightarrow (-\omega^2+9)(\omega+2) = 0$$

$$-\omega^2+9 = 0$$

$$\omega = \pm(3)$$

$$\boxed{\omega = 3 \text{ rad/sec}}$$ //...

note:-

The main use of freq$^n$ res$^n$ is to find the stability.
Already some methods we are learn for stability.
By using this also we can find stability, but difference is sinusoidal input.

freq$^n$ Res $\rightarrow$ variation of $|M|$ & $\phi$ w.r.to freq$^n$.

1) <u>Polar Plots</u>:—

These are the first & foremost plots.

Absolute values of $|F(j\omega)|$ Vs $\omega$ & $\angle F(j\omega)$ (degrees) Vs $\omega$

2) <u>Bode plot</u>:—

decible (db) values of $|F(j\omega)|$.

$20 \log |F(j\omega)|$ Vs $\log \omega$..

& $\angle F(j\omega)$ (degrees)

The word decible is ~~nothing~~ but power related to Power.

Power = $VI$ ✗ → Two var^ll are present. but —

$= I^2R$ — In freq^ll Analysis we are varying only one

$= V^2/R$ Term i.e freq^ll.

decible value of power is

$10 \log (I^2 R)$

$= 20 \log I + 10 \log R$

$= mx + c$

So we can directly plot the magnitude in the ~~magnitude~~ form of straight line. we don't need need all the points i.e that is the Adv^ll of representing the T.F in decibles.

Frequency response of second order system:—

The T.F of a second order system is

$$\boxed{F(S) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}}$$

Before subjected to sinusoidal i/p, Always it is converted into Time constant form.

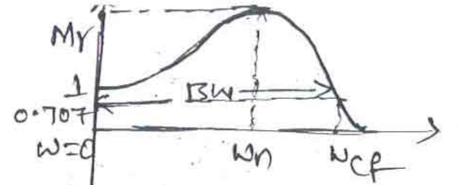
$$F(S) = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1}$$

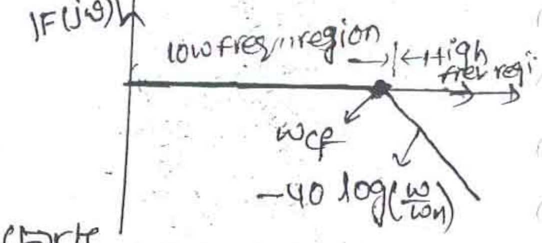
$$F(j\omega) = \frac{1}{\frac{(j\omega)^2}{\omega_n^2} + \frac{2\zeta(j\omega)}{\omega_n} + 1}$$

$$= \frac{1}{\left(1 - \frac{\omega^2}{\omega_n^2}\right) + \frac{2\zeta(j\omega)}{\omega_n}}$$

$$|F(j\omega)| = 1 \Big/ \sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}$$

The frequency at which magnitude $=0.707$, is called "cutoff freq". & range of freq " " " bandwidth".




The above (nonlinear) chart can be converted into straight line by taking logarithmic scale.

log (0) we can't calculate, so we are starts from some freq i.e Ex: 1g=1.

---

The adv of log scale is we can calculate the mag at $\omega=0.1, 0.2\ldots$ (low freq) & $\omega=10^3, 10^4\ldots$ (High freq)

$$|F(j\omega)| = 1/\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(\frac{2\zeta\omega}{\omega_n}\right)^2}$$

Its decible value is

$$-20\log\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(\frac{2\zeta\omega}{\omega_n}\right)^2} \rightarrow (1)$$

Now we can compare this with $y=\underline{m}x+c \Rightarrow$ $m=$slope, $x=\log\omega$ (since x-axis $\log\omega$)

its not represent a straight line. that's why we going for -

- Asymptotic Approximation.

* whereeve $\zeta$ is present that effect should neglected.

~~Here only~~ $\Rightarrow -20\log\sqrt{\left(1-\frac{\omega}{\omega_n}\right)^2)^2}$ .

Hence '$\omega$' only variable.

case (i):- Low frequency region:-

$1 >> (\omega/\omega_n)$

$\Rightarrow -20\log\sqrt{1} = 0$ db

case (ii):- High frequency region:

$$-20\log\left(\sqrt{\left(\left(\frac{\omega}{\omega_n}\right)^2\right)^2}\right)$$

$$= -20\log\left(\frac{\omega}{\omega_n}\right)^2$$

$= -40 \log (\omega/\omega_n)$

$= -40 \log \omega + 40 \log \omega_n$

$[mx + c]$

$\boxed{\therefore \text{slope} = -40 \text{ db/dec}}$

Conclusion:-

So the entire magnitude can be converted into two straight lines one is at low freq". & another one at High freq".

Corner frequency :-

$0 = -40 \log (\omega/\omega_n)$

$\log (\omega/\omega_n) = 0 \Rightarrow \frac{\omega}{\omega_n} = \log^{-1}(0) = 1$

$\Rightarrow \therefore \boxed{\omega = \omega_{cf} = \omega_n \text{ rad/sec}}$

upto $\omega_n$ this magnitude will be 0, after corner freq" its magnitude will be drawn as a straight line having a slope of -40 dB/dec.

So the Nonlinear char" are converted into straight line by taking decible the (T.F) in decible (or) magnitude

As seen from the above at corner freq" $|M| = 0$; whether it is correct or not let us check.

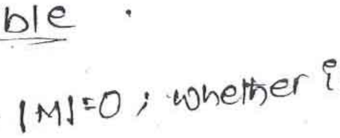
At $\omega = \omega_{cf} = \omega_n$:-

$-20 \log \sqrt{(1-1)^2 + (2\zeta)^2} \Rightarrow \boxed{-20 \log (2\zeta)}$

This much of error will be present at corner freq".

This is called "ERROR at corner freq".

for making the error as minimum (or) zero we eliminating the 'ζ'. (in eq①) so that error will be reduces.




Note :-

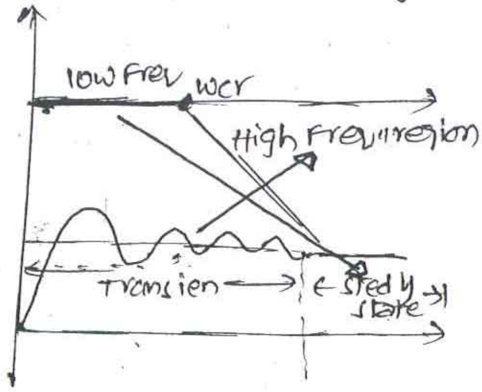
Now the question I am asking is?

In which region the O/P is quantitatively available?

Ans:- In Transient state we can apply for analyse for any I/P, while the steady state response is unity (1) for any input.

Similarly for any input at low frequency region the magnitude is zero. we can apply any i/p for high freq" region.

** Therefore in Transient response Transient state is similar to High freq" region in the Frequency response and steady state response is similar to low freq" region.




Frequency Domine specifications :-

1) Resonant frequency ($\omega_r$) :-

* It is the frequency at which the magnitude has maximum value.

$$|F(j\omega)| = \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$$

let $\frac{\omega}{\omega_n} = u$

$$= \frac{1}{\sqrt{(1-(u)^2)^2 + (2\zeta u)^2}}$$

At $\omega = \omega_r$ = resonant freq"

$$u = u_r = \frac{\omega_r}{\omega_n}$$

$$\frac{d}{du_r}\left(\frac{1}{\sqrt{(1-u_r^2)^2 + (2\zeta u_r)^2}}\right) = 0 \Rightarrow \frac{d}{du_r}\left[(1-u_r^2)^2 + (2\zeta u_r)^2\right]^{-1/2} = 0$$

$$\Rightarrow \left(-\frac{1}{2}\right)\left[(1-u_r^2)^2 + (2\zeta u_r)^2\right]^{-3/2} \cdot \frac{d}{du_r}(1-u_r^2)^2 + (2\zeta u_r)^2 = 0$$

$$\Rightarrow 2(1-u_r^2)(-2u_r) + 4\zeta^2 \cdot 2u_r = 0$$

$$\Rightarrow (2-2u_r^2)(-2u_r) + 8\zeta^2 u_r = 0$$

$$\Rightarrow -4u_r + 4u_r^3 + 8\zeta^2 u_r = 0$$

$$\Rightarrow -1 + u_r^2 + 2\zeta^2 = 0$$

$$\Rightarrow u_r^2 = 1 - 2\zeta^2$$

$$\Rightarrow u_r = \sqrt{1-2\zeta^2}$$

$\therefore$ $\boxed{\omega_r = \omega_n\sqrt{1-2\zeta^2}}$ rad/sec

It is the correlated with "Damped natural frequency".

$\omega_d = \omega_n\sqrt{1-\zeta^2}$ rad/sec

** note:-

$\therefore$ for '$\omega_r$' to be real & +ve

$2\zeta^2 < 1$

$\Rightarrow \boxed{\zeta < \frac{1}{\sqrt{2}}}$ ** //...

for '$\omega_d$' to be real & +ve

$1-\zeta^2 > 0$

$\Rightarrow \zeta^2 < 1$

$\Rightarrow \boxed{\zeta < 1}$ **

2) Resonant peak (or) Peak magnitude:-

It is the ~~mag~~ max$^{m}$ value of magnitude occuring at resonant freq$^{n}$ $\omega_r$.

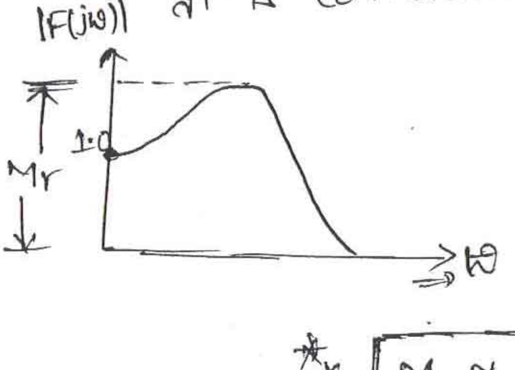
At $\omega = \omega_r = \omega_n\sqrt{1-2\zeta^2}$ rad/sec

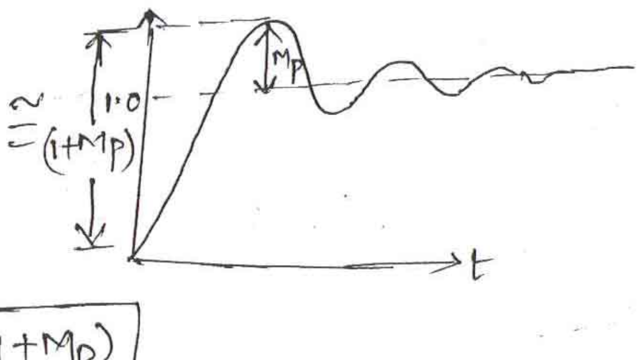
$\boxed{|F(j\omega)| = M_r}$.

$$M_r = \frac{1}{\sqrt{\left[1-\left(\frac{\cancel{\omega_n}\sqrt{1-2\zeta^2}}{\cancel{\omega_n}}\right)^2\right]^2 + \left[\frac{2\zeta\cancel{\omega_n}\sqrt{1-2\zeta^2}}{\cancel{\omega_n}}\right]^2}}$$

$$\boxed{M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}}$$ ** //...

It is correlated with max$^{m}$ peak overshoot




** $\boxed{M_r \cong (1+M_p)}$ //...

when $\zeta < 1/\sqrt{2}$ ; $M_r > 1$

$\zeta = 1/\sqrt{2}$ ; $M_r = 1$

$\zeta > 1/\sqrt{2}$ ; $M_r = 0$ (No $M_r$)

3) Bandwidth :-

It is the range of frequencies over which the magnitude has the value of $1/\sqrt{2}$. It indicates the speed of response of the system.

wider B.W $\Rightarrow$ Faster response.

note:-

$f = \frac{1}{T}$ As $f\uparrow \Rightarrow T\downarrow$

for low freq" (or) radio freq" $\Rightarrow$ B.W = cutoff freq" only.

$$\boxed{B.W \propto \frac{1}{t_r}}$$

4) Cutoff freq":-

It is defined as the frequency at which the magnitude has a value of $1/\sqrt{2}$. It indicates the ability of the system to distinguish signal from Noise.

In low frequency range Band width = cutoff frequency.

$$|F(j\omega)| = \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}} = \frac{1}{\sqrt{(1-u^2)^2 + (2\xi u)^2}}$$

where $u = \frac{\omega}{\omega_n}$

At $\omega = \omega_c$ = Cutoff freq"

$u = u_c = \frac{\omega_c}{\omega_n}$

$$\frac{1}{\sqrt{(1-u_c^2)^2 + (2\xi u_c)^2}} = 1/\sqrt{2}$$

$$\Rightarrow (1-u_c^2)^2 + (2\xi u_c)^2 = 2$$

$$\Rightarrow u_c^4 + u_c^2(4\xi^2 - 2) - 1 = 0$$

$$\text{roots} = \frac{-(4\xi^2-2) \pm \sqrt{(4\xi^2-2)^2+4}}{2(1)}$$

$$= \frac{1-2\zeta^2 \pm \sqrt{16\zeta^4 - 16\zeta^2 + 8}}{2}$$

$$= 1-2\zeta^2 \pm \sqrt{4\zeta^4 - 4\zeta^2 + 2}$$

$$u_c^2 = 1-2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2}$$ → (∵ freq never be −ve)

since actually we are take $u_c^2 = x$ & find root for x then substitute here directly taken

$$\Rightarrow u_c = \sqrt{(1-2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

$$\boxed{\therefore \text{B.W (or) } \omega_c = \omega_n\sqrt{(1-2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}}$$ rad/sec //....

## Stability From Frequency Response

Chareq'' $1 + G(s)H(s) = 0$.

$G(s)H(s) = -1$

Put $s = j\omega$ since sinusoidal i/p.

$G(j\omega)H(j\omega) = -1 + j0$

↓ critical point

Note:-

If you want to draw Bodeplot, Nyquist, Polar plots, you don't need to find the CLTF. whatever the stability we are finding for OLTF similar to CLTF. In the same way even to draw polar plot, Nyquist plot, Bode plot, we can draw directly for OLTF. what stability criteria is calculated that valid for CLTF also.

STABILITY CRITERIA:-

1) Gain cross over frequency ($\omega_{gc}$):-

$$|G(j\omega)H(j\omega)|\Big|_{\omega=\omega_{gc}} = 1 \text{ (or) } 0 \text{ db}$$

2) Phase cross over frequency ($\omega_{pc}$):-

$$\angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{pc}} = -180^\circ$$

for any minimum phase system $P > Z$. so the resultant phase angle is −ve. so you can never get +ve.

Gain Margin :- It is the allowable Gain.

Procedure:-

i) $|G(j\omega)H(j\omega)|\Big|_{\omega=\omega_{pc}} = X$ → If we can add this much of gain to the system then the system becomes unstable

ii) $G.M = \frac{1}{X}$

iii) $G.M = 20\log\left(\frac{1}{X}\right)$ db

similarly for P.M.

Phase Margin :- It is the "allowable phase lag"

i) $\angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{gc}} = \cancel{48}\ \phi$

ii) $P.M = 180 + \phi$

NOTE:-

STABLE ⇒ G.M & P.M = +ve ⇒ $\boxed{\omega_{gc} < \omega_{pc}}$ ***

Marginally stable ⇒ G.M & P.M = 0 ⇒ $\omega_{gc} = \omega_{pc}$

unstable ⇒ G.M & P.M = −ve ⇒ $\omega_{gc} > \omega_{pc}$

G.M & P.M for second order system :-

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2}$$

for calculating the G.M & P.M we require OLTF.

$$\frac{G(S)}{1+G(S)H(S)} = \frac{\omega_n^2}{S^2+2\zeta\omega_n S+\omega_n^2}$$

$$G(S)\left[S^2+2\zeta\omega_n S+\omega_n^2\right] = \omega_n^2\left[1+G(S)H(S)\right]$$

$$\Rightarrow G(S) = \frac{\omega_n^2}{S(S+2\zeta\omega_n)}$$

$$G(j\omega) = \frac{\omega_n^2 + j0}{-\omega^2 + j2\zeta\omega\omega_n} \rightarrow ①$$

$$G(j\omega) = \frac{\omega_n^2+j0}{j\omega(j\omega+2\zeta\omega_n)} = \frac{\omega_n^2+j0}{(0+j\omega)(2\zeta\omega_n+j\omega)} \rightarrow ②$$

$$|G(j\omega)| = \frac{\omega_n^2}{\omega\sqrt{4\xi^2\omega_n^2+\omega^2}}$$

$$\angle G(j\omega) = \frac{0^\circ}{180 - \tan^{-1}\left(\frac{2\xi\omega\omega_n}{\omega^2}\right)}$$ for eq ①

$$= -180 + \tan^{-1}\left(\frac{2\xi\omega_n}{\omega}\right) \ldots$$

Now At $\omega=\infty \Rightarrow \angle G(j\omega) = -180$

$$\angle G(j\omega) = \frac{0^\circ}{90^\circ\left(\tan^{-1}\left(\frac{\omega}{2\xi\omega_n}\right)\right)}$$ for eq ②

(we can write like eq ① or eq ② you can use eq ① or eq ①)

$$= -90^\circ - \tan^{-1}\left(\frac{\omega}{2\xi\omega_n}\right) \ldots$$

At $\omega=\infty \Rightarrow \angle G(j\omega) = -90-90 = -180$

At $\omega = \omega_{pc} = \infty$ rad/sec ;

$$\angle G(j\omega) = -180^\circ$$

$$|G(j\omega)|\Big|_{\omega=\omega_{pc}=\infty} = X$$

$\Rightarrow X = 0$ ; $\therefore G.M = \frac{1}{X} = \infty$

$$G.M = 20\log\left(\frac{1}{X}\right) = \infty \text{ db} \ldots$$

$\therefore$ For a second order system

$$\boxed{\begin{aligned} \omega = \omega_{pc} &= \infty \\ G.M &= +\infty \text{ db} \end{aligned}}$$

(or)

$$\angle G(j\omega)\Big|_{\omega=\omega_{pc}} = -180$$

$$\cancel{-180} + \tan^{-1}\left(\frac{2\xi\omega_n}{\omega_{pc}}\right) = \cancel{-180}$$

$$\frac{2\xi\omega_n}{\omega_{pc}} = 0$$

$$\Rightarrow \boxed{\omega = \omega_{pc} = \infty}$$

$$\therefore \quad |G(j\omega)| = \frac{\omega_n^2}{\omega\sqrt{\omega^2 + 4\xi^2\omega_n^2}}$$

$$\angle G(j\omega) = -180 + \tan^{-1}\left(\frac{2\xi\omega_n}{\omega}\right)$$

$$= -90 - \tan^{-1}\left(\frac{\omega}{2\xi\omega_n}\right)$$

NOW P.M ?

At $\omega = \omega_{gc}$ :
$$\frac{\omega_n^2}{\omega\sqrt{\omega^2 + 4\xi^2\omega_n^2}} = 1$$

$$\Rightarrow \omega^4 + \omega^2 4\xi^2\omega_n^2 - \omega_n^4 = 0$$

roots :

$$\omega^2 = \frac{-4\xi^2\omega_n^2 \pm \sqrt{16\xi^4\omega_n^4 + 4\omega_n^4}}{2}$$

$x = \omega^2$

similarly at last $x = \omega^2$ replace

$$= -2\xi^2\omega_n^2 \pm \omega_n^2\sqrt{4\xi^4 + 1}$$

$$\Rightarrow \omega^2 = -2\xi^2\omega_n^2 + \omega_n^2\sqrt{4\xi^4 + 1}$$

$$\boxed{\therefore \omega = \omega_{gc} = \omega_n\sqrt{-2\xi^2 + \sqrt{4\xi^4 + 1}}} \text{ rad/sec}$$

$$\angle G(j\omega)\Big|_{\omega = \omega_{gc}} = \phi^\circ$$

$$\phi^\circ = -180^\circ + \tan^{-1}\left[\frac{2\xi\omega_n}{\omega_n\sqrt{-2\xi^2 + \sqrt{4\xi^4 + 1}}}\right]$$

$$\therefore P.M = 180 + \phi$$

$$\boxed{P.M = \tan^{-1}\left(\frac{2\xi}{\sqrt{(-2\xi^2 + \sqrt{4\xi^2 + 1}}}\right)}$$

NOW

$$\angle G(j\omega)\Big|_{\omega = \omega_{gc}} = \phi = -90 - \tan^{-1}\left(\frac{\omega_n\sqrt{-2\xi^2 + \sqrt{4\xi^4 + 1}}}{2\xi\omega_n}\right)$$

$P.M = 180 + \phi$

$$\boxed{P.M = 90 - \tan^{-1}\left[\frac{\sqrt{-2\zeta^2 + \sqrt{4\zeta^4+1}}}{2\zeta}\right]}$$ //...

we can use either relation from the above ② eqn's.

Now the short cut method for finding the P.M of second order system is

$$\boxed{P.M \simeq 100\zeta}$$ //... → This is valid only for $\zeta < 1$.
**

Ex:- If $\zeta = 0.5$ then P.M = ? $P.M = 100 \times 0.5 = \underline{50^\circ}$

**(we can use this short cut method where the P.M is +ve).

Page (72) Q (7) :- Ans: (a)

Q (13):- $G(S)H(S) = \dfrac{2\sqrt{3}}{S(S+1)}$

Sol:- Char eqn $1 + G(S)H(S) = 0 \Rightarrow S^2 + S + 2\sqrt{3} = 0$

$\therefore \omega_n = \sqrt{3.4} = 1.8$ ; $\zeta = \frac{1}{3.6} = 0.27$

$\therefore \zeta = \underline{0.27}$

$P.M = 100\zeta = \underline{27^\circ} \simeq \underline{+30^\circ}$ ....

Q:- The value of 'a' to give P.M = 45° will be

$G(S) = \dfrac{aS+1}{S^2}$ (a) $\sqrt{2}$ 1.414 (b) $1/\sqrt{2}$ 0.707 (c) $\sqrt{2}$ 1.18 (d) $1/\sqrt{2}$ 0.84

Sol:- char eqn
$S^2 + aS + 1 = 0$
$\omega_n = 1$ rad/sec ; $2\zeta \times 1 = a$

$\therefore P.M = 45^\circ$

$100\zeta = 45 \Rightarrow \zeta = \underline{0.45}$

$\Rightarrow \underline{a = 0.9}$

$\therefore$ Ans : (b)

Q: (14)

G.M?

$G(S)H(S) = \dfrac{1}{S(S^2+S+1)}$

Sol:- $G(j\omega) = \dfrac{1}{j\omega(-\omega^2+1+j\omega)}$

$|G(j\omega)| = \dfrac{1}{\omega\sqrt{(1-\omega^2)^2+\omega^2}}$

$\angle G(j\omega) = -90 - \tan^{-1}\left(\dfrac{\omega}{1-\omega^2}\right)$

At $\omega = \omega_{gc} = 1$ rad/sec

$\angle G(j\omega)H(j\omega) = -90-90 = -180^\circ$

Now

$$X = \frac{1}{1\sqrt{(1-1)^2 + (1)^2}} = 1$$

$$G.M = \frac{1}{X} = 1$$

$$GM = 20\log(1) = 0 \text{ db}$$

(33) What is the G.M of the system having OLTF $G(s) = \frac{2(1+s)}{s^2}$

(a) 0 (b) 1 (c) $\infty$ (d) $-\infty$

Sol:

$$G(j\omega) = \frac{2(1+j\omega)}{(j\omega)^2}$$

$$|G(j\omega)| = \frac{2\sqrt{1+\omega^2}}{\omega^2}$$

$$\angle G(j\omega) = -180 - \tan^{-1}\left(\frac{\omega}{2}\right)$$

At $\omega = \omega_{pc} = 0$ rad/sec

$\angle G(j\omega) = -180 + 0^\circ = -180^\circ$

Now

$X = |G(j\omega)|$ at $\omega = \omega_{pc} = 0$ rad/sec

$= \infty$ ...

$\therefore G.M = \frac{1}{\infty} = 0$

$G.M = 20\log 0$ $= 20\log[1/\infty]$

undefined. And also we don't have this option. That's why this can be written as $20\log(1/\infty)$

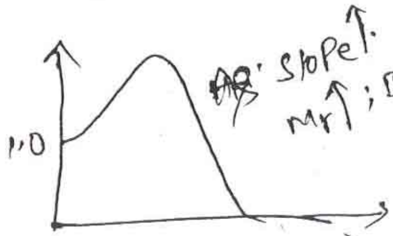
$= 20\log(1) - 20\log(\infty)$

$= 0 - \infty$

$= -\infty$ ...

Q:(11)

Sol:




slope↑ 
$M_r \uparrow$; B.W↓

Ans:- option (c) ...

Q8
sol:-
**

$$e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.5$$

$$\Rightarrow \zeta = 0.215 \ldots$$

$$\cancel{T = \frac{1}{\zeta\omega_n} = 0.2 \Rightarrow \omega_n = \frac{1}{0.215\times 0.2}} =$$

$$T_d = \frac{1}{f_d} \Rightarrow f_d = \frac{1}{0.2} = 2\,Hz$$

$$\omega_d = 2\pi f_d = 2\pi\times 5 = 31.41\,rad/sec$$

$$\therefore \omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\Rightarrow \omega_n = \frac{31.41}{\sqrt{1-(0.215)^2}} = 32.16\,rad/sec$$

$$\omega_r = \omega_n\sqrt{1-2\zeta^2} = 32.16\sqrt{1-(2(0.215)^2)} = 30.63\,rad/sec \ldots$$

Q:17
sol:-

Poles at = 0.01 Hz, 1 Hz, 80 Hz
zeros at = 5 Hz, 100 Hz, 200 Hz

$$\frac{(1+\cancel{T_a}s)(1+\cancel{T_b}s)(1+\cancel{T_c}s)}{(1+T_1 s)(\cancel{1+T_2 s})(\cancel{1+T_3 s})}$$ → we are Neglecting 3 zeros & 2 poles since insignifant.

$$\text{phase angle} = -\tan^{-1}(\omega T_1)$$

$$f = 20\,Hz \Rightarrow \omega = 2\pi\times 20 = 124\,rad/sec$$

The maxm angle for 'ω' is ∞.

$$-\tan^{-1}(\infty) = -90^\circ$$ i.e for any value of 'ω' we can't get the angle more than 90°. so ~~(a)~~ is wrong.

Q:(19).

$$G(s) = \frac{3e^{-2s}}{s(s+2)}$$

$$G(j\omega) = \frac{3e^{-2j\omega}}{j\omega(j\omega+2)}$$

$$|G(j\omega)| = \frac{3}{\omega\sqrt{\omega^2+4}} \quad ; \quad \angle G(j\omega) = \frac{0^\circ[-57.3\times 2\omega]}{[+90][\tan^{-1}(\omega/2)]}$$

$$\angle G(j\omega) = 0^\circ - 90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - [2\omega\times 57.3]$$

$$= -90 - 114.6\omega - \tan^{-1}\left(\frac{\omega}{2}\right)$$

At $w = w_{gc}$

$$\frac{3}{w\sqrt{w^2+4}} = 1$$

$$9 = w^2(w^2+4)$$

$$w^4 + 4w^2 - 9 = 0$$

$$\frac{-4 \pm \sqrt{16+36}}{2}$$

$$-2 \pm 3.6$$

$$-5.6, +1.6$$

$$w^2 = 1.6$$

$$w = w_{gc} = \sqrt{1.6} = 1.26 \text{ rad/sec}$$

$$\angle G(j\omega)\Big|_{w=w_{gc}=1.26} = \phi^\circ = -90 - 114.6 \times 1.26 - \tan^{-1}(0.5 \times 1.26)$$

$$= -267.5^\circ$$

$$\therefore P.M = 180 + (-267.5)$$

$$= -87.5^\circ$$

we can't have ~~P~~ P.M = +ve, G.M = −ve & viceversa.

so G.M = −ve.

so Ans is (d).

for finding the G.M we have to use Trial and Error method.

for what values of 'w' the angle is −180°. which is $w_{pc}$. Then you will get G.M = −ve.

(Q:21)

Mp (Peak magnitude) not take $e^{-\zeta\pi/\sqrt{1-\zeta^2}}$.

↓

i.e ~~Peak~~ resonant Peak.

$$M(j\omega) = \frac{100}{100 - w^2 + \sqrt{2}(10)j\omega}$$

$$w_n = 10; \quad \zeta = \frac{10\sqrt{2}}{2w_n(\cancel{10\sqrt{2}})} \Rightarrow \zeta = \frac{\cancel{\sqrt{2}}}{\cancel{20\sqrt{2}}} \frac{1}{\sqrt{2}}$$

$\boxed{\therefore M_r = 1}$ for $\zeta = 1/\sqrt{2}$.

(24) sol:

$$\angle G(j\omega) = \tan^{-1}\left(\frac{\omega}{5}\right) - \tan^{-1}\left(\frac{4\omega}{9-\omega^2}\right)$$

for $\omega < 3 \Rightarrow \phi = \tan^{-1}\left(\frac{\omega}{5}\right) - \tan^{-1}\left(\frac{4\omega}{9-\omega^2}\right)$

At $\omega = 0 \Rightarrow \phi = 0°$

for $\omega > 3 \Rightarrow \phi = \tan^{-1}\left(\frac{\omega}{5}\right) + \left[\tan^{-1}\left(\frac{4\omega}{9-\omega^2}\right) - 180°\right]$

($\because$ II$^{nd}$ quadrant)

At $\omega = \infty$ $\phi = 90° + \left(-180 + \tan^{-1}\left(\frac{4\omega}{\omega^2(-1+\frac{9}{\omega^2})}\right)\right)$

$$= 90 + \left(-180 + \tan^{-1}\left(\frac{4}{\infty}\right)\right).$$

$$= -180 + 90 + 0°$$

$$= -90°\ldots$$

$\therefore$ phase Angle varies b/w $0°$ & $-90°$.

(28) sol:

$$G(S)\,H(S) = \frac{e^{-TS}}{S(S+1)}$$

$$G(j\omega) = \frac{e^{-j\omega T}}{j\omega(j\omega+1)}$$

$$\angle G(j\omega) = (-\omega T \times 57.3) - 90 - \tan^{-1}\left(\frac{\omega}{1}\right)$$

(or)

$$= \underset{\text{radians}}{(\omega T)} - 90° - \tan^{-1}(\omega)$$

At $\omega = \omega_1$

$$\angle G(j\omega) = 0$$

$$-\frac{\pi}{2} - \omega_1 T - \tan^{-1}(\omega_1) = 0$$

$$-\omega_1 = \tan\left(\frac{\pi}{2} + \omega_1 T\right)$$

$$+\omega_1 = +\cot(\omega_1 T)$$

$$\boxed{\omega_1 = \cot(\omega_1 T)}$$ //...

Frequency domine Approximation :-

$F(s) = \frac{Y(s)}{X(s)} = e^{-Ts}$ → you Never Take $e^{-Ts}$ in any plot. $|e^{-Ts}| = 1$ & $\angle e^{-Ts} = -\omega T$ radians

[Polar, Nyquist, Bode plots] //

$F(j\omega) = e^{-T(j\omega)} = \cos(\omega T) - j\sin(\omega T)$

$e^{-Ts} = (1 - sT)$ only for Time response Analysis i.e R-H, root locus etc.

$|F(j\omega)| = \sqrt{(\cos(\omega T))^2 + (\sin(\omega T))^2} = 1$ //...

$\angle F(j\omega) = \tan^{-1}\left(\frac{-\sin\omega T}{\cos\omega T}\right)$

$= \tan^{-1}(-\tan\omega T)$

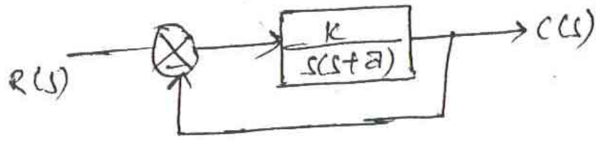
$\angle F(j\omega) = -\omega T$ radians

$= \left(-\omega T \times \frac{180}{\pi}\right)^\circ$

$= (-\omega T \times 57.3)$ degrees ....

Page (76) conventional Q:(2)

find K & a ; for satisfy $M_r = 1.04$, $\omega_r = 11.55$ rad/sec



Sol:-

$1 + G(s)H(s) = 0$

$s^2 + as + K = 0$

$\omega_n^2 = K$

$2\xi\omega_n = a$

Given:- $M_r = 1.04$

$\frac{1}{2\xi\sqrt{1-\xi^2}} = 1.04$

$\Rightarrow \xi = 0.6, 0.8$

$\xi = 0.6$ ✓ since $\xi < 0.707 (1/\sqrt{2})$

$= 0.8$ ✗

$\omega_r = 11.55$ rad/sec

$11.55 = \omega_n\sqrt{1 - 2\xi^2}$

$\Rightarrow \omega_n = \frac{11.55}{\sqrt{1 - 2(0.6)^2}} = 22.2$ rad/sec

$\therefore K = (22.2)^2 = 492.84$

$a = 2 \times 0.6 \times 22.2$

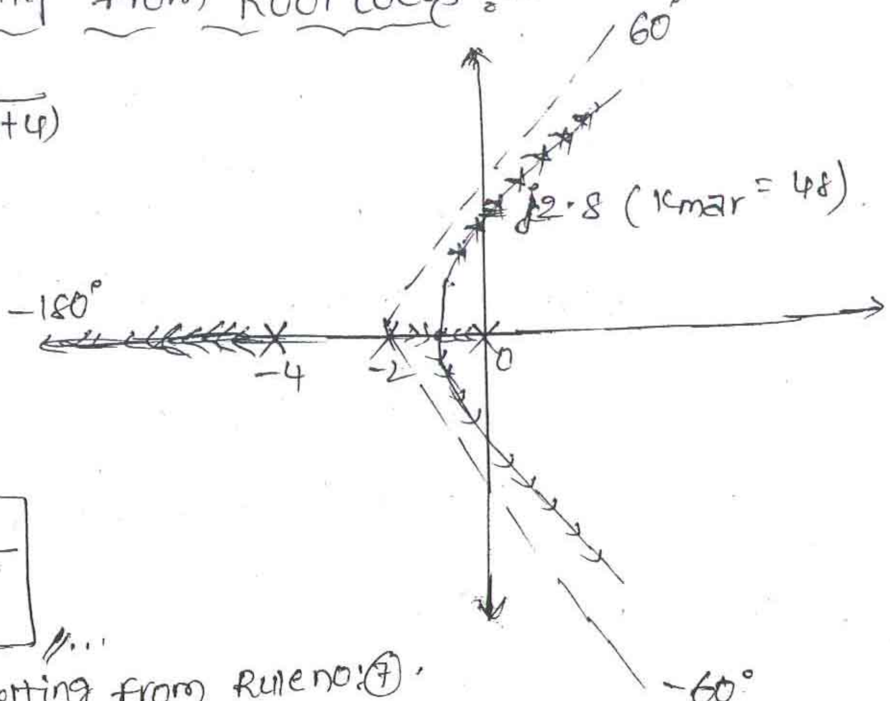
$= 26.4$ ....

find G.M & P.M from ROOT LOCUS :-

(Pb) let $G(s) = \frac{K}{s(s+2)(s+4)}$

sol:-

TO Find G.M :-




find G.M for $K_{(desired)} = 20$

$$\boxed{X = \frac{K_{marginal}}{K_{desired}}}$$

Kmarginal is getting from Rule no:(7).

TO Find P.M :-

Analytic method :-
By Trial & error method for a perticular value of jω axis
The 'K' value $= K_{desired} = 20$ ; where $|G(j\omega)H(j\omega)| = 1$
which will taken as $\omega_{gc}$. Next remaining procedure
is same.

Graphical method :- find 'K' value for a perticular point j3.
If K value is greater than $K_{des} = 20$ then take j2.

$$K = \frac{l_{P_1} \times l_{P_2} \times \cdots}{l_{z_1} \times l_{z_2} \cdots}$$

P.M:-

1) By Trail & Error locate a point on jω axis which gives $K_{desired}$ value using Magnitude condition.
2) compare jω axis point with jω to obtain ω value.
This frequency is known as Gain cross over frequ.
find P.M using usual Method.

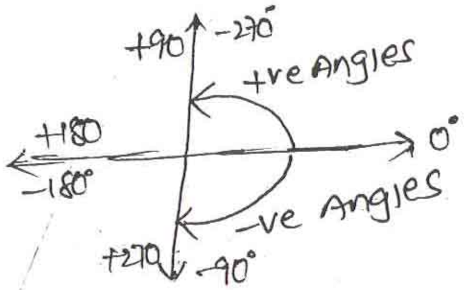
# POLAR PLOTS

It is the plot of the absolute values of magnitude and phase angle in degrees of openloop T.F $G(j\omega)\,H(j\omega)$ versus '$\omega$' drawn on polar co-ordinates.

w.r.t s-plane co-ordinate plane

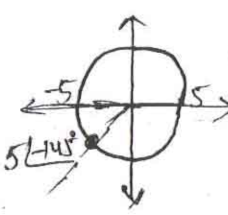
x → axis real axis
y → " Img axis

For marking polar co-ord" in s-plane, represent all +ve angles in CCW & all -ve angles represent in CW wise. Magnitudes will be calibrated according to x-axis.





Ex:- $5\angle -145^\circ$

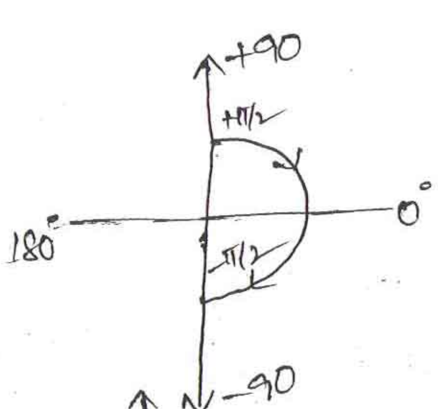




There is a time waste for converting into co-ordinate planes & then draw in s-plane.

Now the procedure for draw the polar plot directly is draw a unit circle with radious 5 units & represents the angle with a line from origin.

Accurate plot of polar plot we never drawn. How it will be discuss later.



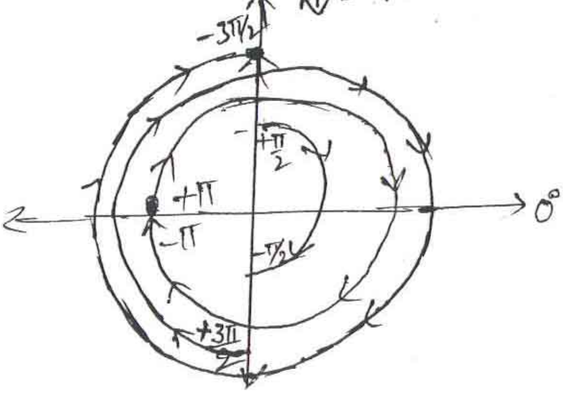


If you want to travel from +90 to -90.

First go from where you want to start.

+90 to 0° & 0° to -90°.

& then for going -90° to +90° same reverse direction.





which means A CW circle is from +π to -π.

CCW circle means -π to +π.

For Ex: If you want draw a CW circle from $+\frac{3\pi}{2}$ to $-3\pi/2$. starts from $+\frac{3\pi}{2}$ to $+\pi$ to $+\frac{\pi}{2}$ to 0° to $+\frac{3\pi}{2}$ (or $-\frac{3\pi}{2}$) to $-\frac{3\pi}{2}$. //...

• Draw polar plot $G(s)=\frac{1}{s+1}$

Sol:- NOTE :-

1) To draw any plot (polar plot, Bodeplot, Nyquist plot) convert T.F into Time constant form.

2) Because we are defining the stability for OLTF, we have to take all the plots for $G(s)H(s)$.

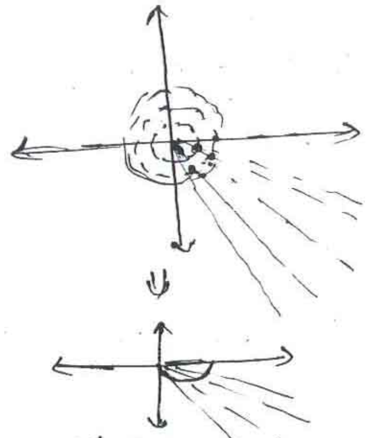
$$G(s)=\frac{1}{1+s}$$

$$G(j\omega)=\frac{1}{1+j\omega}$$

$$|G(j\omega)|=\frac{1}{\sqrt{\omega^2+1}}$$

$$\angle G(j\omega)=-\tan^{-1}(\omega)$$

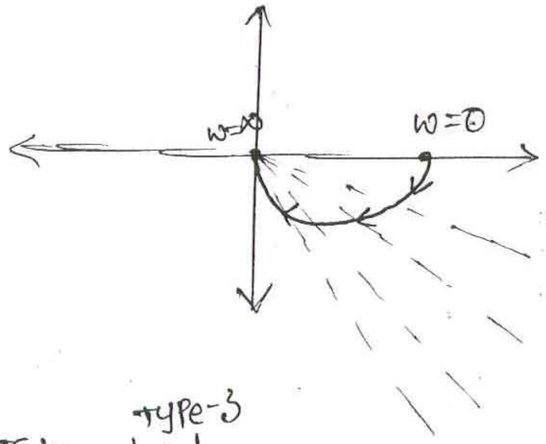
| $\omega$ | 0 | ... | $\infty$ |
|---|---|---|---|
| $\lvert G(j\omega)\rvert$ | 1 | | 0 |
| $\angle G(j\omega)$ | $0^\circ$ | | $-90^\circ$ |



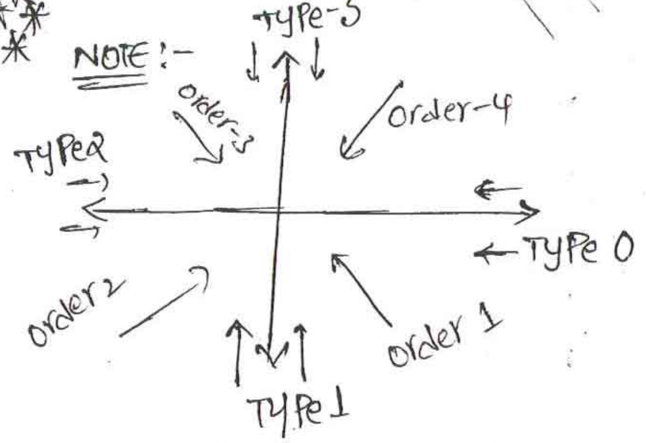
this is the polar plot for this. we can't able to see. so Don't go for importa accuracy, shape is important.

Note:- To draw polar plot accurately x-axis should be accurately taken. since magnitudes are taken w.r.to x-axis.

Accuracy is not important for us, shape of the plot is important.




** NOTE :-




The given T.F is a first order type 0 system. that means it is starts from 0°. If one pole is added at the origin then starting angle is −90°. so starting from −90° for Type 2. Similarly for order-01, it ends at 4th quadrant origin. for Type-2 system ends at 3rd quadrant origin. similarly for higher order systems.

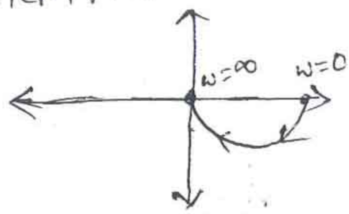
## General Shapes Of Polar Plots :—

We will see General shapes of polarplots for diff[n] Type & order T.F's.

TYPE / Order | Polar Plot

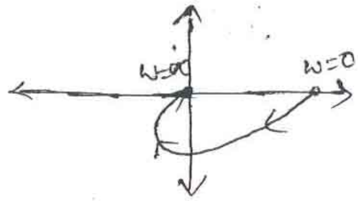
1) Type-0 / order-1

$G(s) = T.F = \frac{1}{1+sT}$



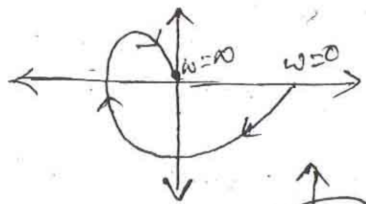

2. Type-0 / order-2

$G(s) = T.F = \frac{1}{(1+sT_1)(1+sT_2)}$



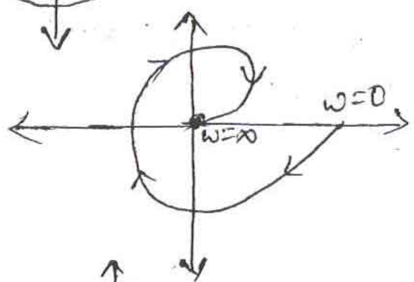

3. Type-0 / order-3

$G(s) = \frac{1}{(1+sT_1)(1+sT_2)(1+sT_3)}$



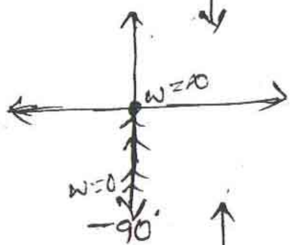

4. Type-0 / order-4

$G(s) = \frac{1}{(1+sT_1)(1+sT_2)(1+sT_3)(1+sT_4)}$



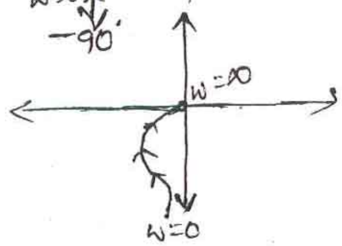

5. Type-1 / order-1

$G(s) = \frac{1}{s} = \frac{1}{j\omega} = \frac{1}{\omega} \angle -90^\circ$



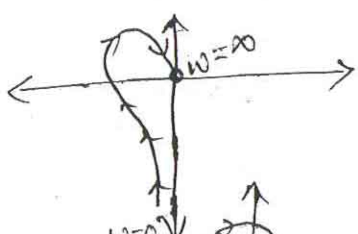

6. Type-1 / order-2

$G(s) = \frac{1}{s(1+sT)}$



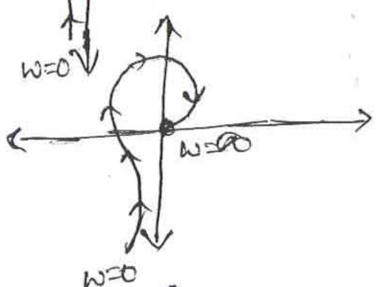

7. Type-1 / order-3

$G(s) = \frac{1}{s(1+T_1 s)(1+T_2 s)}$



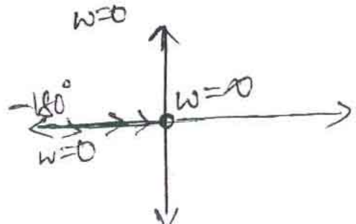

8. Type-1 / order-4

$G(s) = \frac{1}{s(1+sT_1)(1+sT_2)(1+sT_3)}$



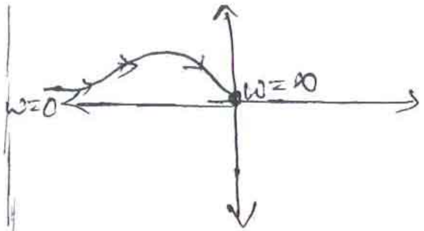

9. Type-2 / order-2

$G(s) = \frac{1}{s^2} = \frac{1}{-\omega^2} \angle 180$



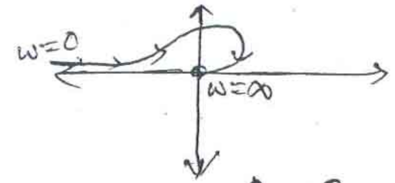
10. Type-2 / order-3

$$\frac{1}{s^2(1+sT)}$$



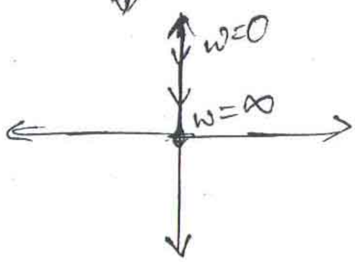

11. Type-2 / order-4

$$\frac{1}{s^2(1+sT_1)(1+sT_2)}$$



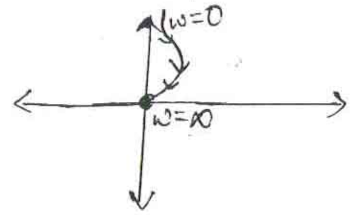

12. Type3 / order-3

$$\frac{1}{s^3 \cancel{(1+s)}} = \frac{1}{(j\omega)^3} = \frac{1}{\omega^3}\angle -270$$




13. Type-3 / order-4

$$\frac{1}{s^3(1+sT)}$$




**Note:-** Ex: $G(s) = \frac{(1+T_a s)}{s(1+T_1 s)(1+T_2 s)}$

Type = 1
order = 3 ...

**Note:-** Generalization for Type-2 / Type-3 minimum phase system :-

for Type-2 & Type-3 which is having one pole in L.H.S of s-plane, The polar plot cuts the -ve real axis as many times which equal to the no. of zeros present in G(s).
(not applicable for $1/s^2$, $1/s^3$ ...)

EFFECT OF ZEROS ON THE SHAPE OF POLAR PLOT :-

for Type-2 and Type-3 Minimum Phase functions the polar plot will intersect Negative real axis as many times as there are zeros b/w $\omega = 0$ and $\omega = \infty$ points.

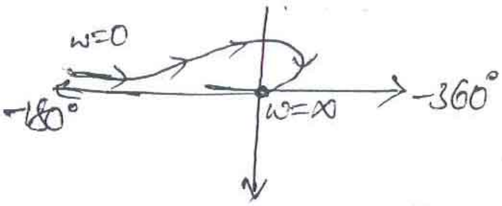
Ex:- $G(s) = \frac{1}{s^2(1+s)(1+2s)}$

$$G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega)(1+2j\omega)}$$

$$\angle G(j\omega) = -180 - \tan^{-1}(\omega) - \tan^{-1}(2\omega)$$

$\omega = 0$ ; $\angle G(j\omega) = -180^\circ$ ; $|G(j\omega)| = \infty$

$\omega = \infty$ ; $\angle G(j\omega) = -360^\circ$ ; $|G(j\omega)| = 0$



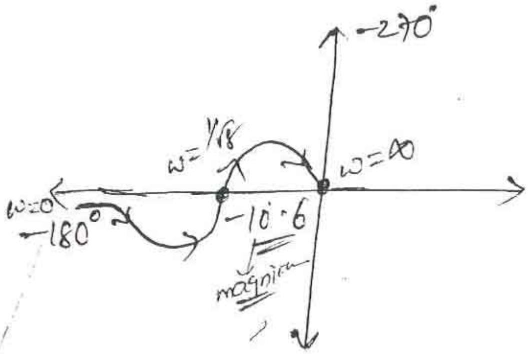

Now let us take a zero in T.F.

$$G(s) = \frac{(1+4s)}{s^2(1+s)(1+2s)}$$

$$\angle G(j\omega) = -180° - \tan^{-1}(\omega) - \tan^{-1}(2\omega) + \tan^{-1}(4\omega)$$

$\omega = 0 \Rightarrow \angle G(j\omega) = -180°$ ; $|G(j\omega)| = \infty$

$\omega = \infty \Rightarrow \angle G(j\omega) = -270°$ ; $|G(j\omega)| = 0$




checking whether it cuts or not :-

$$-180 - \tan^{-1}(\omega) - \tan^{-1}(2\omega) + \tan^{-1}(4\omega) = -180°$$

$$\tan^{-1}(4\omega) = \tan^{-1}(\omega) + \tan^{-1}(2\omega)$$

$$4\omega = \frac{\omega + 2\omega}{1 - 2\omega^2}$$

$$\Rightarrow -8\omega^2 + 4 = 3$$

$$\Rightarrow 8\omega^2 = 1 \Rightarrow \omega = \pm 1/\sqrt{8}$$

$$\Rightarrow \boxed{\omega = +\frac{1}{\sqrt{8}} \text{ rad/sec}}$$

Now the magnitude at intersection point is

$$\frac{\sqrt{1+(4\omega)^2}}{\omega^2\sqrt{1+\omega^2}\sqrt{1+(2\omega)^2}}$$

$$\Rightarrow \frac{\sqrt{1+(4/\sqrt{8})^2}}{(1/\sqrt{8})^2\left(\sqrt{1+(1/\sqrt{8})^2}\sqrt{1+(2(\frac{1}{\sqrt{8}}))^2}\right)}$$

$$= \underline{10.6} \ldots$$

If $\omega = 0 \Rightarrow$ then the polar plot doesn't cuts. since already we seen at $\omega = 0$; Angle $= -180°$ $|M| = \infty$.

$\omega = 1/\sqrt{8} \neq 0$ so it will cuts the $-180°$ axis.

another case is if freq. is only $-$ve, then the plot will not cuts the $-$ve real axis.

Note :- Any system whether it is a Minimum phase (or) Non Minimum phase system whenever a zero is present in the T.F then we have to check whether the polar plot intersects or not at $-$ve real axis. It may cuts or may not cuts.

Note :-- In the above T.F poles = −1 , −0.5
zero = −0.25

The zeros should be added before the pole locations. then only Generalized method is valid. for zero location is away (or) beyond from pole location from img axis ( ) then the generalization is may be valid (or) not valid. The Generalized method is only for Type-2, & Type-3 systems.

Note :- the Effect of adding zeros on the shape of the polar plot for any type Minimum phase (or) Non Minimum phase system is that the polar plot may (or) may not intersect the −ve real axes b/w $\omega = 0$ & $\omega = \infty$ points which needs to be check due to the presence of zeros in the T.F.

Ex:-2 : $G(s) = \dfrac{(s+2)}{(s+1)(s-4)}$

Sol :- It is a Non minimum phase system.
First step is convert the T.F into Time constant form.

$$G(s) = \frac{2\left(1+\frac{s}{2}\right)}{(-4)(1+s)(1-0.25s)}$$

$$\Rightarrow G(j\omega) = \frac{-0.5\,[1+0.5j\omega]}{(1+j\omega)(1-0.25j\omega)}$$

( Actually −ve gain = ±180° contributes for all $\omega$ values )

$$|G(j\omega)| = \frac{0.5\sqrt{1+(0.5\omega)^2}}{\sqrt{(1+\omega^2)}\,\sqrt{1+(0.25\omega)^2}}$$

But
( Note :- In freq" resp" Analysis (−ve(signal)) −ve gain have to be taken as −180°. how ?
P.M = 180 + φ
= ~~180~~
P.M = φ − [−180]
this is the actual Expression of P.M)
= 180 + φ we are writing

$\angle G(j\omega) = ?$

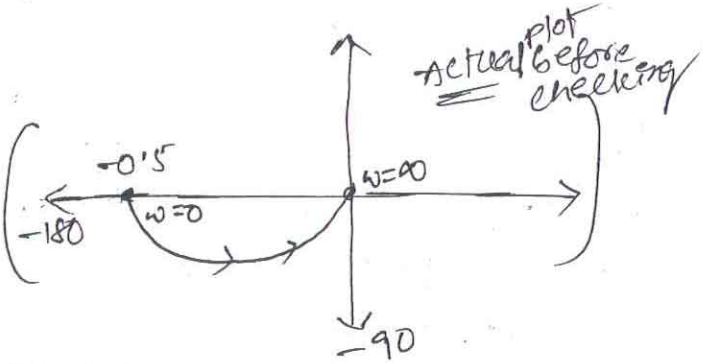
The −ve gain configuration is taken as −180°.
So
$$\angle G(j\omega) = \frac{[-180^\circ]\,[\tan^{-1}(0.5\omega)]}{[\tan^{-1}\omega]\,[\tan^{-1}(0.25\omega)]}$$

$$= -180^\circ + \tan^{-1}(0.5\omega) - \tan^{-1}(\omega) + \tan^{-1}(0.25\omega)$$

> NOTE :- −ve Gain [−K] will contribute −180° for all $\omega$ in freq" response " plots.

//..

| $\omega$ | 0 | — | $\infty$ |
|---|---|---|---|
| $\lvert G(j\omega)\rvert$ | 0.5 | | 0 |
| $\angle G(j\omega)$ | $-180^\circ$ | | $-90^\circ$ |




**checking whether it intersects (or) not :-**

$$-180 + \tan^{-1}(0.5\omega) - \tan^{-1}(\omega) + \tan^{-1}(0.25\omega) = -180$$

$$\tan^{-1}(0.5\omega) + \tan^{-1}(0.25\omega) = \tan^{-1}\omega$$

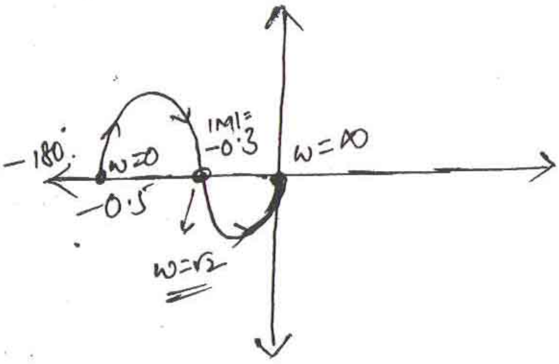
$$\Rightarrow \cancel{\omega} = \frac{0.5\cancel{\omega} + 0.25\cancel{\omega}}{1-(0.125\omega^2)}$$

$$\Rightarrow 1 - 0.125\omega^2 = 0.75$$

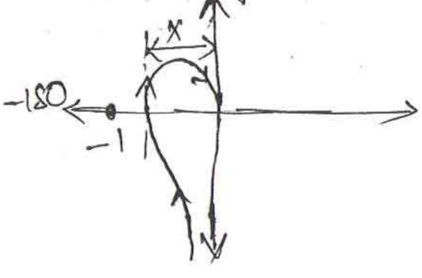
$$\Rightarrow \omega = \omega_{pc} = \sqrt{\frac{0.25}{0.125}} = \sqrt{2} \text{ rad/sec}$$

So it intersects the -ve real axis.

$$\text{Now } \lvert G(j\omega)\rvert_{\omega=\sqrt{2}} = \frac{0.5\sqrt{(1+0.5\sqrt{2})^2}}{\sqrt{1+(\sqrt{2})^2}\sqrt{1+(0.25\sqrt{2})^2}} = 0.3$$



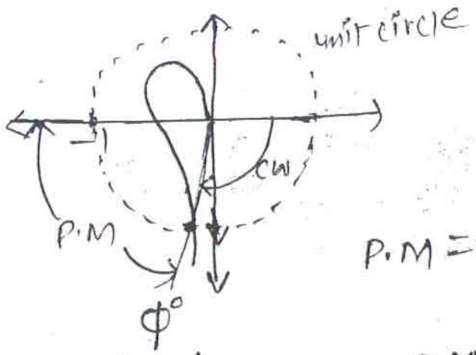

**Stability from polar plot :-**




$$G.M = \frac{1}{x}$$ where 'x' is always absolute value i.e +ve value.

$$G.M(db) = 20\log\left(\frac{1}{x}\right) \text{ db}$$

Now for finding the P.M draw a unit circle with radious 1 units. And take the angle at which the polar plot cuts the unit circle in CW only. The distance b/w -180° & the cutting line is called P.M.

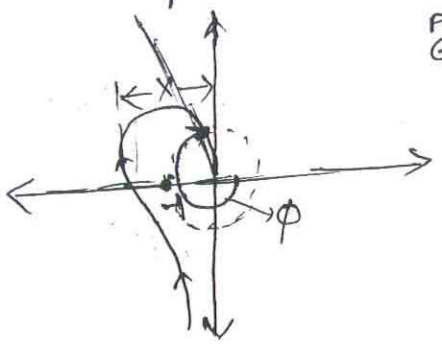



$\boxed{\text{The Angle '}\phi\text{' must be measured in CW only.}}$

$P.M = \phi - (-180^\circ) = 180 + \phi = \underline{\underline{+ve}} \ldots$

(since angle b/w these two is $-180$ in cw)

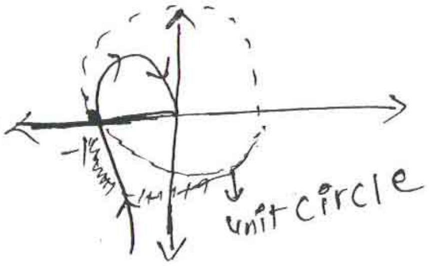
P.M = +ve, G.M = +ve } so system becomes stable.




$G.M = \frac{1}{x} \Rightarrow G.M(db) = 20\log\left(\frac{1}{x}\right) = \underline{-ve}$

$P.M = 180 + \phi = \underline{\underline{-ve}} \ldots$

so unstable.




$G.M = \frac{1}{1} = 1 \Rightarrow G.M(db) = 20\log(1) = 0\,db$

$P.M = 180 + \phi = 0^\circ$

$\therefore$ so GM = 0db, PM = $0^\circ$ } system becomes marginally stable.

Note:- If the G.M = +ve; and PM is can't be determined, (or) P.M = ∞, & GM " " in that situation for finding the stability we are going to one special method. i.e Concept of Enclosement & Encirclements.

(Contour → means some shape)

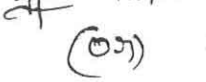



fig ①


fig ②

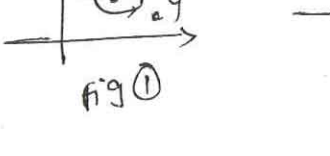
The concept of Enclosement is nothing but we are standing in the direction contour. we are standing in the right side part. Now we have to see the right side part. If a point is in right side then it is said to enclosed otherwise not enclosed

in fig① x → Enclosed ; y = not enclosed

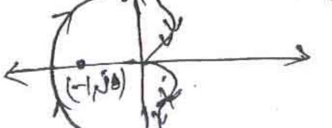
fig② x → not " , y = Enclosed.

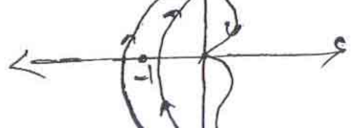
In fig② 'y' is said be not encircled & x is said to be encircled in ACW direction.

* The Polar plot is not fully encircled.




Is it the point is enclosed or not. In this case for saying the point enclosed or not is difficult. That's why we are extending this. which will be called as "Nyquist plot".



NOTE:-

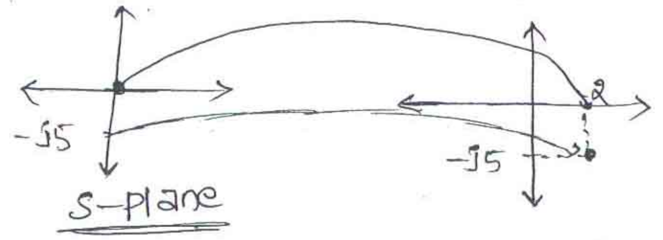
* A point is said to be enclosed by a contour if it lies to the right side of the direction of the contour.
* A point is said to be encircled if the contour is a closed path.

  In fig ② point 'y' is said to be enclosed; whereas point 'x' is said to be encircled in anticlockwise direction.
* In polar plots if the critical point ~~is~~ $(-1+j0)$ is not enclosed then the system is said to be stable.

THEORY OF ~~N~~ Nyquist plot :-

⟹ x ====

## Principle of Mapping :-




S-plane

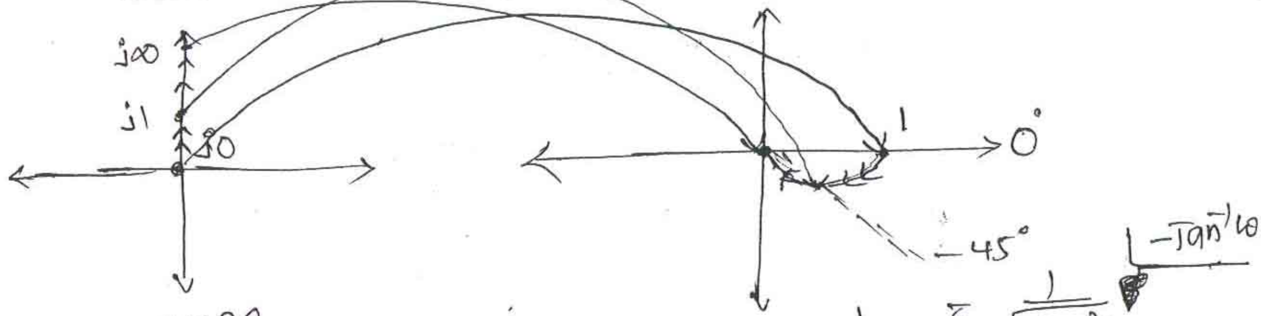
P(S) plane

$P(S) = S+2$

$P(0) = 0+2 = 2$

$P(-j5) = -j5+2 \ldots$

Ex:- Polar plot :-




S-plane

$G(s)H(s) = P(S) = \frac{1}{S+1} = \frac{1}{\sqrt{1+\omega^2}}$

Just substitute $j0, j1, \ldots j\infty$ points, then $P(S) = 1/S+1$, map those points.

* simplifying polar co-ordinates is easy compared to " rect " " .

since in polar we can multiply & add devide the magnitudes and angles are added (or) subtracted.

* The mapping theorem states that every point in s-plane is map on to a corresponding point in P(S) plane, where P(S) is any function of 'S'.

The examples are shown in above.

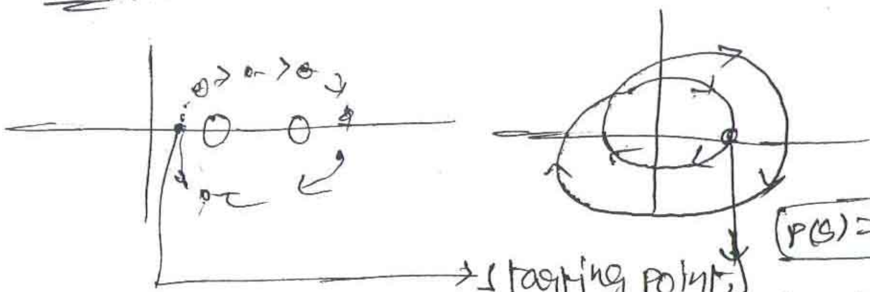
## Principle of Argument :-

Principle of Argument is defined using this principle of Mapping..

case(i) :- ~~This~~ let us take s-plane, we are choosing any where any no. of zeros in right-hand side, which must be RHS. & also zeros can be choose, which must be we have to choose in CW direction. we are starting from one point and move in CW direction as shown figure.

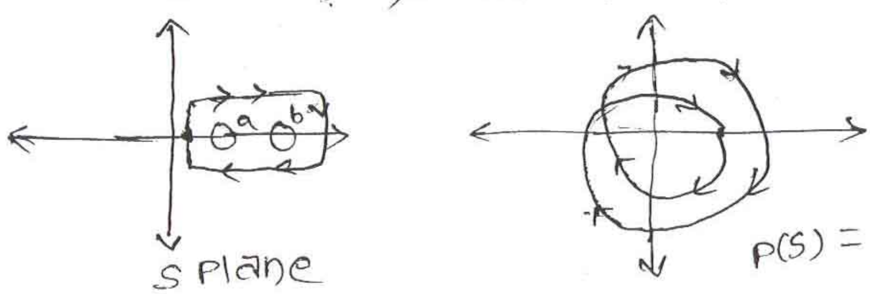

when we are mapping these into P(s) plane, then they are not occurs at same point, they will be at diff. points.

we are putting all these points then we are getting diff points, then we can get a shape like polar plot:



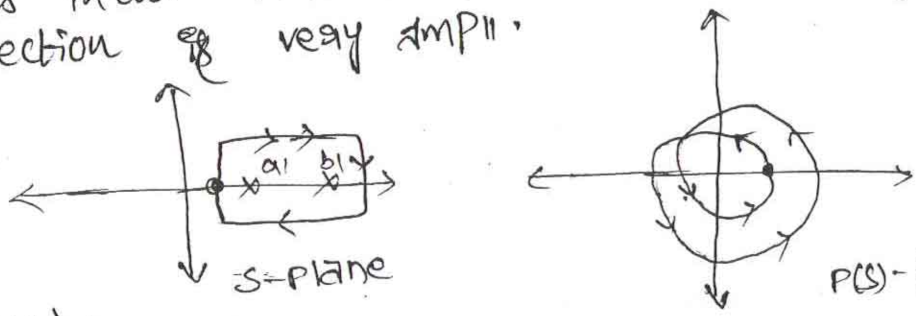

$P(s) = (s-a)(s-b)$

then all points are mapped as like in this shape:



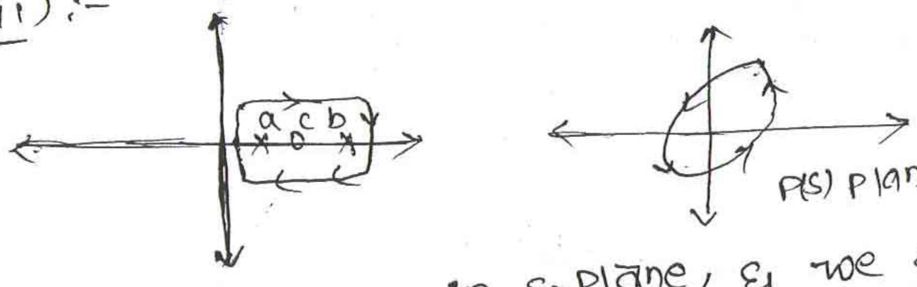

$P(S) = (S-a)(S-b)$

case(ii):- we are choosing some poles in RHS, choose some points in CW. All these points are putting in CW direction, direction is very ampl.




P(S)-Plane $= \dfrac{1}{(s-a)(s-b)}$

case(iii):-




P(S) plane $= \dfrac{(s-c)}{(s-a)(s-b)}$

we are choosen P>Z in s-plane, & we are choosing all points which encloses the poles & zeros in CW. when we mapping these points into P(s) then it will form a shape in ACW direction. since P>Z.

so from this Analysis we can make a conclusion,

$\boxed{N = P - Z}$ where N = no. of encirclements +ve 'N' ⇒ ACW
−ve 'N' ⇒ CW

$\boxed{N = P - Z}$ N = No: of encirclements

N = +ve → Anticlockwise direction
N = −ve → clockwise direction

P = No: of poles in RHS of s-plane
Z = No: of zeros in RHS of s-plane.

* The principle of arguments may be stated as if the s-plane closed contour encloses 'P'-poles & 'Z'-zeros [ P > Z ] in the RHS of s-plane then the origin of P(s) plane is encircled (P−Z) times in Anticlockwise direction.

This is called "principle of argument".

## Nyquist stability criteria :-

$$G(s)H(s) = \frac{K(s \pm z_1)}{s(s \pm P_1)} \rightarrow (1)$$

conventional method of finding stability is

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s \pm z_1)}{s(s \pm P_1)} = 0 \Rightarrow \frac{s(s \pm P_1) + K(s \pm z_1)}{s(s \pm P_1)} = 0 \rightarrow (2)$$

$$\Rightarrow s(s \pm P_1) + K(s \pm z_1) = 0 \rightarrow \text{closed loop poles...}$$

for time being, we are not multiplying $s(s \pm P_1)$ to zero, and we are are taking that as eq(1).

Now from (1) & (2)

Numerator of eq(2) → closed loop poles
dinominator of eq(2) ⇒ openloop poles.

in control system in general Numerator is called as zeros and dinominator part is called as poles.

for eq(2) ⇒ Poles = O.L. poles
zeros = C.L. poles

Applying $\boxed{N = P - Z}$ to eq(1);

P = No: of openloop poles in RHS of s-plane
Z = No: of closed loop poles in RHS of s-plane

for a system to be stable, the CL poles should not be in RHS of s-plane ∴ so 'z' must be zero.

$\Rightarrow \boxed{Z=0}$
$\Rightarrow \therefore \boxed{N=P}$ } → condition for stability of a system.

→ suppose he is given T.F as → $\dfrac{s^2+4s+6}{s^3+8s^2+10s+15}$.

Now we want to find stability for this T.F by this $\boxed{N=P-Z}$ phinominal.

*** To apply $N=P-Z$ concept to this T.F, I have to choose "some finite points" around poles & zeros and map these points into P(s) plane.

By looking Transfer funn, the no: of poles & zeros in RHS of s-plane is → ?

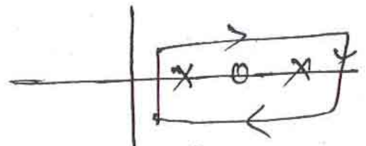
| | | |
|---|---|---|
| $+\ s^3$ | 1 | 10 |
| $+\ s^2$ | 8 | 15 |
| $+\ s^1$ | $\frac{80-15}{8}$ | 0 |
| $+\ s^0$ | 15 | |

∴ so no poles in R.H.S plane.

we can't take finite points, like in case(iii) a, b, c are poles & zeros.

we have to take infinite No. of points in s-plane.

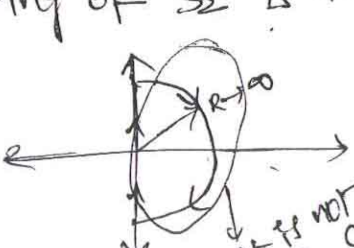
since we don't know, the where the poles & zeros are there, in s-plane. That is we are taking entire RHS plane with radious infinity.



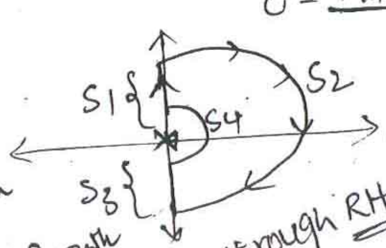
finite countour because we don't know where the poles & zeros are there in s-plane.

→ mapping of $S_1$ is nothing but drawing polar plot

mapping of $S_2$ is nothing but put $s = Re^{j\theta}$ with $R \to \infty$ & $\theta = +\pi/2$ to $-\pi/2$.



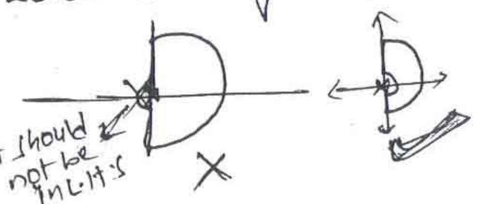

at is not a closed path so that we are go through two axis to make closed path




don't go through RHS.

* If any pole (or) zero in $j\omega$ axis, the mapping of that pole (or) zero is → magnitude is almost 0 & angle is +90 to −90.

NOTE:- If any point lying at origin, you can't go through that point, you should choose points around the poles and zeros. but the points poles (or) zeros at origin it can't possible since it lies in L.H.S.



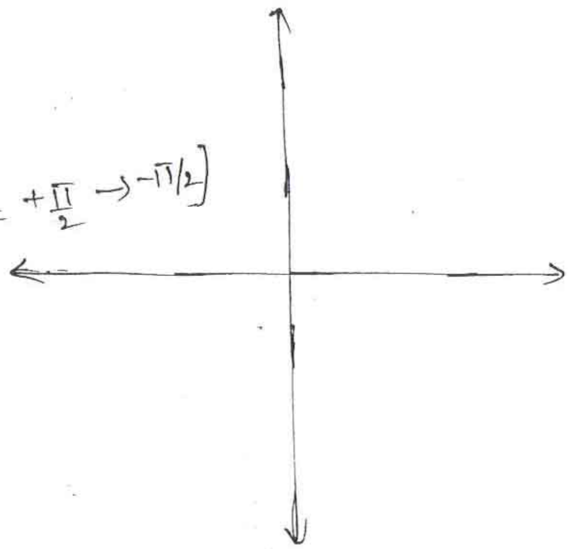

So, bypass the poles (or) zeros lying at origin.

* similarly for any T.F some poles may present on $j\omega$ axis (or) repeated poles or zeros may present, in that cases also bypass that.

1) To map $S_1$:- polar plot

2) To map $S_2$:- put $s = \lim_{R\to\infty} Re^{j\theta}$ $\left[\theta = +\frac{\pi}{2} \to -\pi/2\right]$

3) To map $S_3$:-



NOTE:-

* for poles (or) zeros lying on $j\omega$ axis only Nyquist path can be shown by putting $\lim_{R\to 0} R\cdot e^{j\theta} \pm j\varepsilon$.

$j\varepsilon$ → is location of poles (or) zeros...

(Pb) Draw Nyquist plot for the given T.F.

$$G(s) = \frac{10}{(s+2)(s+4)}$$

Sol:- No poles (or) zeros at origin... so $S_1, S_2, S_3$ have to be map & $S_4$ no need to map.

Note: There is no procedure for poles on $j\omega$ axis just bypass that, but there is a procedure for $S_1, S_2, S_3$...

Procedure :-

To map $S_1$ :- Draw polar plot.

To map $S_2$ :- $G(S) \cong \frac{10}{s^2} = \frac{10}{\lim_{R\to\infty} (Re^{j\theta})^2}$

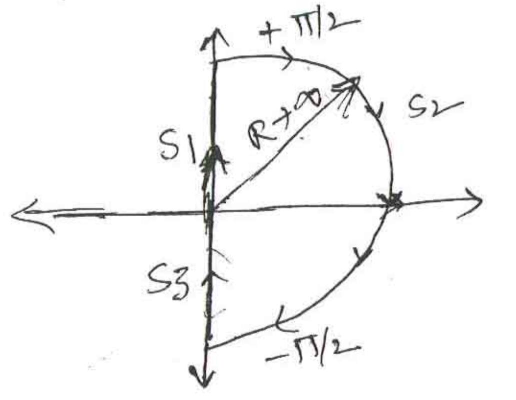
$\frac{10}{(s+2)(s+4)}$ infinity is higher than 2, 4. so this cause treated as $\frac{10}{s^2}$.

$$= \frac{10}{\lim_{R\to\infty} R^2 . e^{j2\theta}}$$

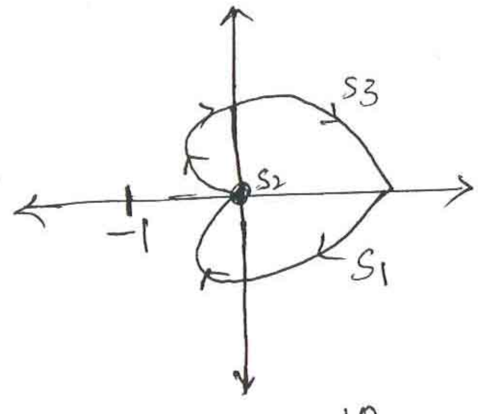
$$= 0 . e^{-j2\theta}$$

that means all points in '$S_2$' will have zero magnitude. so these $S_2$ plot is maped to origin.

To map $S_3$ :- Draw the mirror image of polar plot.




S-plane




$G(S) = \frac{10}{(s+2)(s+4)}$

$\therefore N = P - Z$

$0 = 0 - Z \Rightarrow \boxed{Z = 0}$ so stable.

no: of encirclements of $(-1, j0)$

NOTE:- If you draw the Nyquist plot for $1 + G(S)H(S)$ you should observe the encirclements of origin.

If you draw the Nyquist plot for $G(s)H(s)$ you should observe the encirclements of $(-1, j0)$ point.

pb). $G(s) = \frac{100}{(s+2)(s+4)(s+8)}$

Sol:- For this T.F also we have to map $S_1, S_2, S_3$ :.

procedure:- 1) mapping of $S_1 \rightarrow$ polar plot.

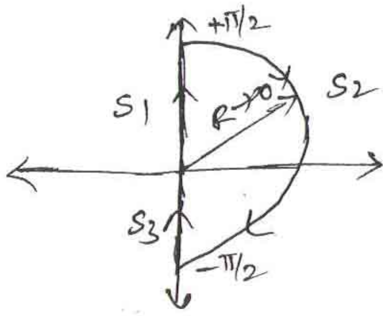
2) $G(s) = \frac{100}{(s+2)(s+4)(s+8)}$

if $s = \infty \Rightarrow \infty$ is dominant that is 's' points are dominant. so this can be approximated as $\frac{100}{s^3}$

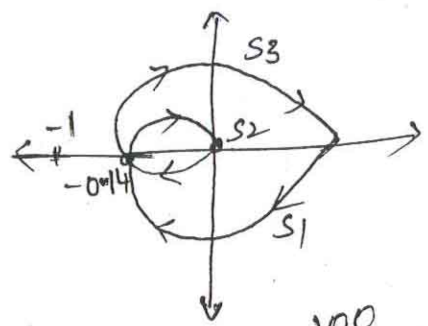
$$\therefore G(s) = \frac{100}{s^3} = \frac{100}{\lim_{R\to\infty} (Re^{j\theta})^3} = \frac{100}{\lim_{R\to\infty} R^3 e^{3j\theta}}$$

$$= 0 \cdot e^{-3j\theta}$$

3) To map $S_3$, Draw mirror image of polar plot.




s-plane




$G(s) = \frac{100}{(s+2)(s+4)(s+8)}$

Short-cut for finding $\omega_{pc}$ & 'x' value:-

Now for finding $\omega_{pc}$:-

replace 's' by '$(j\omega)$'.

Note:- This shortcut procedure is valid only for minimum phase system & standard T.Fs which we can easy to draw polar plot..

$$G(j\omega) = \frac{100}{(-\omega^2 + 6j\omega + 8)(j\omega + 8)}$$

$$= \frac{100}{-j\omega^3 + (-8)j\omega^2 - 6\omega^2 + 48j\omega + 8j\omega + 64}$$

$$= \frac{100}{64 - 14\omega^2 + j(56\omega - \omega^3)}$$

$56\omega - \omega^3 = 0$

$\Rightarrow \omega^2 = 56 \Rightarrow \omega = \omega_{pc} = \sqrt{56} = 7.4$ rad/sec

$$\Rightarrow \frac{100}{64 - 14(7.4)^2} = -0.14$$

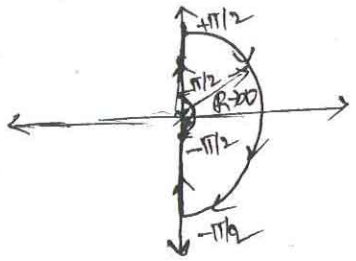
$\therefore$ So, $-0.14$ is not encloses the $(-1, j0)$ point.

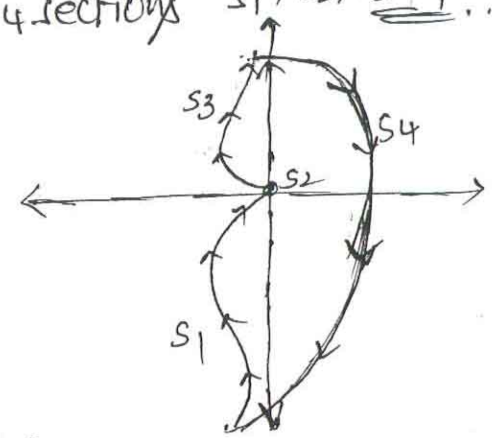
So $\Rightarrow N=0$

$\therefore 0 = 0 - Z \Rightarrow \boxed{Z=0}$ stable...

Pb) $G(S) = \dfrac{5}{S(S+2)}$

Sol:- Here we have to map 4 sections $S_1, S_2, S_3, S_4$...




To map $S_4$, we are placing the points around the small semi circle. which are nearly zero so 's' is not dominant. '2' dominant.

This $G(s)$ can be approximated as $G(s) = \dfrac{5}{S(2)} = \dfrac{2.5}{S}$

$$= \frac{2.5}{\underset{R\to 0}{Lt}\ Re^{j\theta}} = \infty \cdot e^{-j\theta}$$

magnitude

$\theta = -\frac{\pi}{2}$ to $+\pi/2$

$$= \infty e^{j\pi/2} \xrightarrow{to} \infty e^{-j\pi/2}$$

$\cdot N = P - Z$

$0 = 0 - Z \Rightarrow \boxed{Z=0}$ stable....

Pb) $G(s) = \dfrac{K}{s(s+1)(s+2)}$ find the range of 'K' for stability?

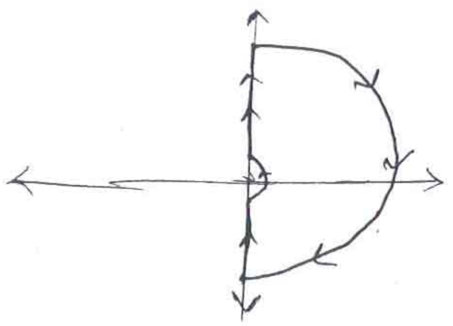
Sol:- Again we have '4' sections.

Now $S_1 \rightarrow$ polar plot

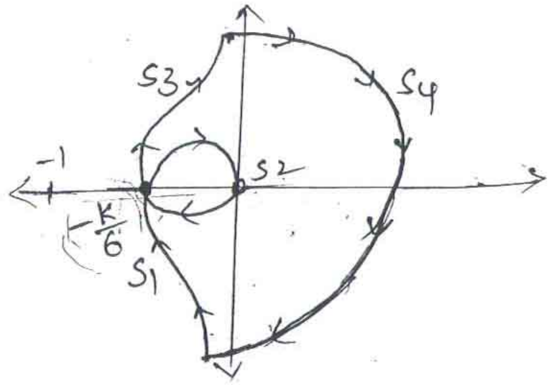
$S_2 \rightarrow$ map to origin

$S_3 \rightarrow$ Mirror image

$S_4 \rightarrow$ map to semicircle with $R\to\infty$.



s-plane




$$G(s) = \frac{K}{s(s+1)(s+2)}$$

To map $S_4$:-

$$G(s) = \frac{K}{s(1)(2)} = \frac{0.5K}{s} = \frac{0.5K}{\lim_{R \to 0} Re^{j\theta}} = \infty e^{-j\theta}$$

$$[\theta = -\tfrac{\pi}{2} \to +\tfrac{\pi}{2}]$$

$$= \infty e^{j\pi/2} \to \infty \cdot e^{-j\pi/2}$$

Range of 'K' for stability :-

Now replace 's' by 'jω'.

$$G(j\omega) = \frac{K}{j\omega(-\omega^2 + 3j\omega + 2)}$$

$$= \frac{K}{-3\omega^2 + j(2\omega - \omega^3)}$$

⇒ Imaginary part should be zero, since $\theta = -180^\circ$ axis.

$$2\omega - \omega^3 = 0 \Rightarrow \omega^2 = 2 \Rightarrow \omega = \omega_{pc} = \sqrt{2} \text{ rad/sec}$$

$$\frac{K}{-3(\sqrt{2})^2} = -K/6$$

$$\therefore \boxed{X = \frac{K}{6}}$$

$$G \cdot M = \frac{1}{X} = \frac{6}{K}$$

$$\boxed{G \cdot M \propto 1/K} \to$$ Ex:- If 'K' double then G·M half ***

To find range of 'K' for stability

$$-\frac{K}{6} = -1 \Rightarrow K_{mar} = 6$$

case(i):- $K > 6 \Rightarrow -2 = 0 - Z$

$\boxed{Z = 2}$ unstable

case(ii): $K < 6$

$0 = 0 - Z$

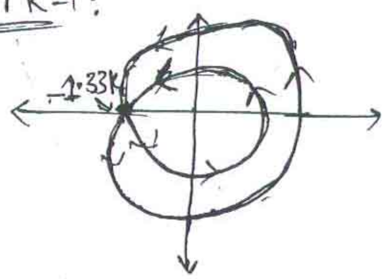
$\boxed{Z = 0}$

stable

workbook
Q:27
chapter 6

Q. $G(s) = \frac{K(s+3)(s+5)}{(s-2)(s-4)}$

for K=1:




$-1.33K = -1$

$\Rightarrow K_{mar} = \frac{1}{1.33} = 0.75$ ...

case(i) :-

$K < 1/1.33$

$0 = 2 - Z$

$\boxed{Z = 2} \Rightarrow$ unstable.

case(ii) :-

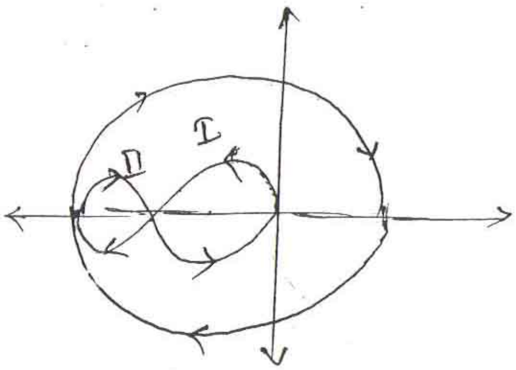
$K > 1/1.33$

$2 = 2 - Z$

$\Rightarrow \boxed{Z=0}$ stable ...

Ans :- (d)

Q : 26




$\boxed{N = P - Z}$

given $\boxed{P=0}$ $\Rightarrow$ (-1, j0) in 1st region ...

$\boxed{N=0}$ for region ①

$\therefore 0 = 0 - Z \Rightarrow \boxed{Z=0}$

stable

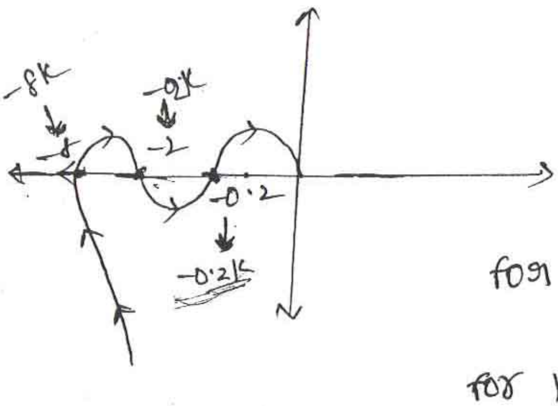
If (-1, j0) is in 2nd region

$\boxed{N=-2}$ $\Rightarrow -2 = 0 - Z \Rightarrow \boxed{Z=2}$

unstable ...

Ans :- stable to unstable ...

18
pb)




for $K=1 \Rightarrow$ crossover points are $-0.2, -2, -8$.

for $K=K \Rightarrow$ " " $-0.2K, -2K, -8K$ ...

case ① :

$-0.2K = -1 \Rightarrow K_{mar} = 5$

$K < 5 \Rightarrow$ stable

$K > 5 \Rightarrow$ unstable

case(ii) :- $-2K = -1 \Rightarrow K_{mar} = 1/2$

$K > 1/2 \Rightarrow$ stable

~~1/2 < K<5 & K<1/8 ⇒~~ :

case(iii): $-8K = -1 \Rightarrow K_{max} = 1/8$

$K < 1/8 \Rightarrow$ stable

obviously for $K > 1/8 \Rightarrow$ unstable.

$\therefore \boxed{\frac{1}{2} < K < 5 \ \& \ K < 1/8} \rightarrow$ Ans....

Q:3:- $G(s) = \dfrac{1}{s(1+sT_1)(1+sT_2)}$

Sol:- 

$$G(j\omega) = \frac{1}{j\omega(1+j\omega T_1)(1+j\omega T_2)}$$

$$= \frac{1}{j\omega\left(1 + j\omega(T_1+T_2) - \omega^2 T_1T_2\right)}$$

$$= \frac{1}{-\omega^2(T_1+T_2) + j[\omega - \omega^3 T_1T_2]}$$

$$\omega - \omega^3 T_1T_2 = 0 \Rightarrow 1 - \omega^2 T_1T_2 = 0$$

$$\therefore \boxed{\omega = \omega_{pc} = \frac{1}{\sqrt{T_1T_2}} \text{ rad/sec}}$$

$$-x = \frac{1}{-\left[\frac{1}{\sqrt{T_1T_2}}\right]^2 (T_1+T_2)} = \frac{-T_1T_2}{T_1+T_2}$$

$$\therefore x = \frac{T_1T_2}{T_1+T_2}$$

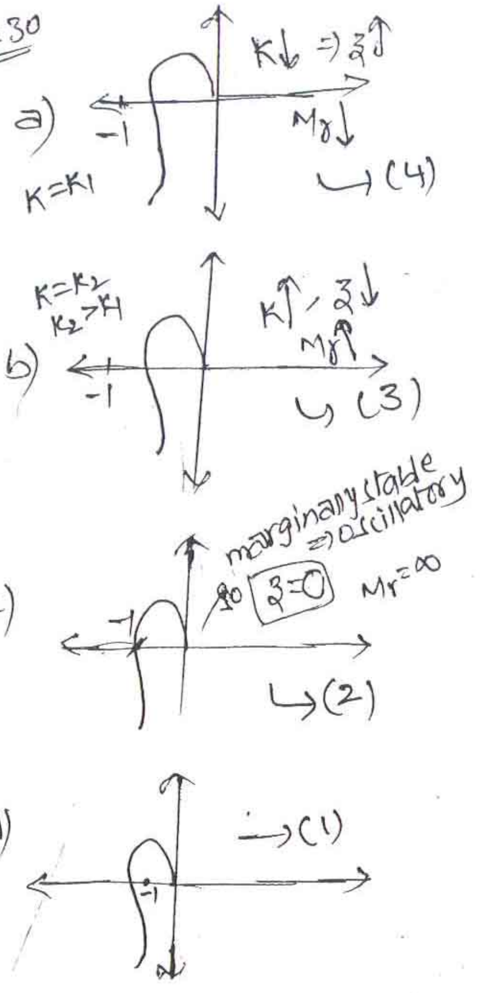
$$\boxed{G\cdot M = \frac{1}{x} = \frac{T_1+T_2}{T_1T_2}}$$ //...

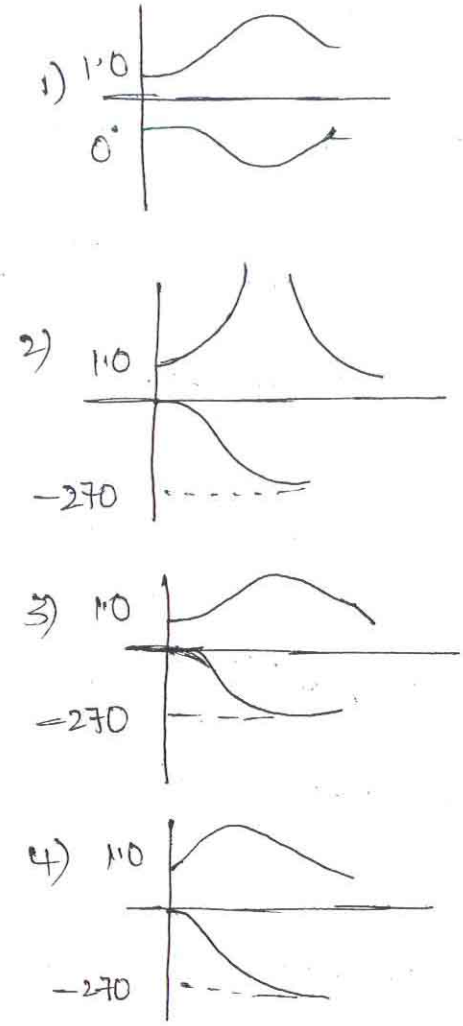
Q:2: $x = 0.25$

Sol: $G\cdot M = \frac{1}{0.25} = 4$

$\phi = -30^\circ \Rightarrow P.M = 180 + \phi = 180 - 30 = 150^\circ$

Q:30




NOTE:-

$$\zeta \propto \frac{1}{\sqrt{K}}$$

As $K\uparrow \Rightarrow \zeta\downarrow \Rightarrow M_r\uparrow$

↓

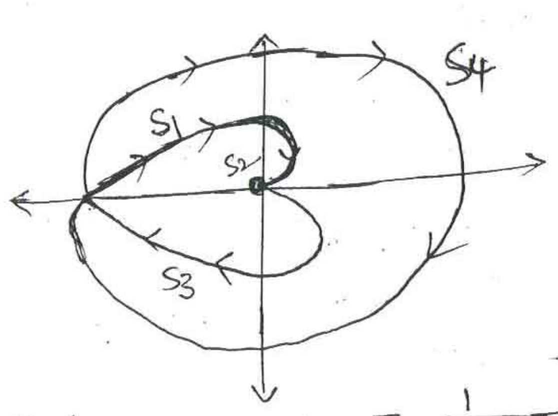
This concept is used to solve this problem.

Ans:- a → 4
b → 3
c → 2
d → 1

conventional

Q 4 $\quad G(s)H(s) = \dfrac{1}{s^2(1+s)(1+2s)}$

Sol:-




To map S4:- $G(s) = \frac{1}{s^2} = \lim_{R\to 0} \dfrac{1}{(Re^{j\theta})^2} = \lim_{R\to 0} \dfrac{1}{R^2 e^{2j\theta}}$

$= \infty \cdot e^{-j2\theta} \quad \theta = (+\pi/2 \to -\pi/2)$

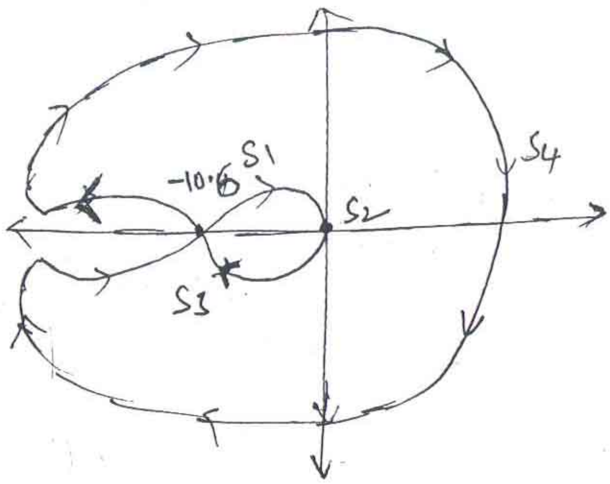
$= \infty \cdot e^{j\pi} \longrightarrow \infty\, e^{-j\pi}$

$\therefore N = P - Z$

$-2 = 0 - Z \Rightarrow \boxed{Z=2}$ unstable.

$$G(s) = \frac{1+4s}{s^2(1+s)(1+2s)}$$

Sol:




$N = P - Z$

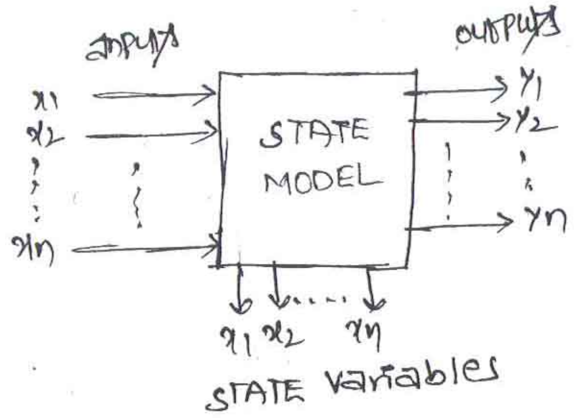
$-2 = P - Z$

$\boxed{Z=2} \rightarrow$ unstable

X

# Part - V

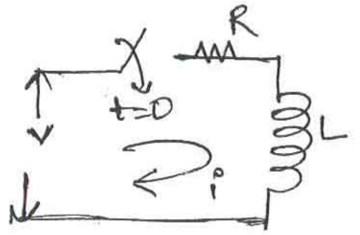
## STATE SPACE Analysis....




→ T.F Analysis is done by poles & zeros.

→ T.F is a polonomial in 's' in which it has Numerator & dinominator....

Ex:-




At $t=0^-$ :-

$i(0^-) = i_L(0^-) = 0$ Amps....

At $t = 0^+$ :-

$i_L(0^+) = \frac{1}{L}\int_{t=0^-}^{0^+} v\,dt = 0$ Amps

$V = i.R + L\frac{di}{dt}$

$\frac{di}{dt} = -\frac{Ri}{L} + \frac{V}{L}$.

$\boxed{\frac{di}{dt} = -\frac{R}{L}.i + \frac{V}{L}}$ $\xrightarrow{\text{Amp/sec}}$ state Equation:..

↓

The $\frac{di}{dt}$ provides the state of this N/W:..

* The state of any electrical N/W depends upon the Energy storage elements.

1) state Equation :- → This is the first equ that we write for any state model.

$$\boxed{\dot{x}(t) = Ax(t) + Bu(t)}$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \vdots\\ \dot{x}_n\end{bmatrix} = \begin{bmatrix} A \end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ \vdots\\ x_n\end{bmatrix} + \begin{bmatrix} B \end{bmatrix}\begin{bmatrix}u_1\\ u_2\\ \vdots\\ u_n\end{bmatrix}$$

2) OUTPUT Equation:- → This is the second equ that we write for any state model

$$\boxed{y(t) = Cx(t) + Du(t)}$$

$$\begin{bmatrix}y_1\\ y_2\\ \vdots\\ y_n\end{bmatrix} = \begin{bmatrix} C \end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ \vdots\\ x_n\end{bmatrix} + \begin{bmatrix} D \end{bmatrix}\begin{bmatrix}u_1\\ u_2\\ \vdots\\ u_n\end{bmatrix}$$

TYPE-1:-

TO obtain state Mode from state model from Diff Eqn.

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 10U$$

$$[s^2+4s+8]\,Y(s) = 10\,U(s)$$

$$\boxed{\frac{Y(s)}{U(s)} = \frac{10}{s^2+4s+8}}$$

I. Define state variables

Let $y = x_1$

$$\frac{dy}{dt} = \dot{x}_1 = x_2$$

$$\frac{d^2y}{dt^2} = \dot{x}_2$$

$\therefore$ $\dot{x}_1 = x_2$

$\dot{x}_2 = 10U - 4x_2 - 8x_1$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -8 & -4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 10\end{bmatrix}U$$

output eqn:

$$y = x_1$$

$$\therefore y = [1\ 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix} + 0\cdot U$$

short cut method:-

we can directly written the state model by using this short cut method.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 1\\ -8 & -4\end{bmatrix}}_{A}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ 10\end{bmatrix}}_{B}U \quad 10 \to K$$

$$\frac{Y(s)}{U(s)} = \frac{10}{s^2+4s+8}$$

$\leftarrow$ reverse order & reverse sign age 81

Q:8 $\frac{Y(S)}{U(S)} = \frac{1}{S^4+5S^3+8S^2+6S+3}$ → reverse order & reverse sign

Sol:-

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & -6 & -8 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

TYPE-2 :-

Q:9: $G(S) = \frac{Y(S)}{U(S)} = \frac{2S+1}{S^2+7S+9}$

Sol:-

$$S^2Y(S) + 7SY(S) + 9Y(S) = 2SU(S) + 1\cdot U(S)$$

$$\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 9y = 2\frac{du}{dt} + u$$

Shortcut method:-

seperately write Numerator & dinominator.

$$\frac{1}{S^2+7S+9} \cdot (2S+1)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -9 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$$ ← for dinominator

$$Y = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$ ← for Numerator

→ reverse the Numbers in Numerator.

PHASE VARIABLE METHOD:-

$$\frac{Y_1(S)}{X_1(S)} \cdot \frac{X_1(S)}{U(S)} = \frac{1}{S^2+7S+9} \cdot (2S+1)$$

Where ever i/p is there compare with dinominator &

Where ever o/p is there compare with Numerator.

$$\frac{X_1(S)}{U(S)} = \frac{1}{S^2+7S+9} \Rightarrow S^2X_1(S)+7SX_1(S)+9X_1(S)=U(S)$$

$$\frac{d^2x_1(t)}{dt}+7\frac{dx_1}{dt}+9x_1=u$$

Let $\frac{dx_1}{dt}=x_2$

$\dot{x}_2=\frac{dx_2}{dt^2}$

$\dot{x}_1=x_2$

$\dot{x}_2=\frac{d^2x_1}{dt^2} \Rightarrow \dot{x}_2=u-7x_2-9x_1$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -9 & -7\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u$$

OUTPUT Eqn :- $Y(S)=(2S+1)X_1(S)$

$=(2S)X_1(S)+X_1(S)$

$\Rightarrow y=2\frac{dx_1}{dt}+x_1.$

$y=2x_2+x_1$

$$\therefore\ y=\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+0\cdot u$$

Type-3:-

To obtain T.F from state model :-

$\dot{x}(t)=AX(t)+BU(t)$

$y(t)=CX(t)+DU(t)$

Apply Laplace Transform

$SX(S)-X(0)=AX(S)+BU(S)$

$Y(S)=CX(S)+DU(S)$

W.K.T for any T.F initial conditions are zero.

So $X(0)=0$

$SX(S)-AX(S)=BU(S)$

$X(S).[SI-A]=BU(S)$

$\Rightarrow X(S)=(SI-A)^{-1}\cdot BU(S)$

$$Y(s) = [C(sI-A)^{-1}\cdot B + D]\ U(s)$$

$$\boxed{\frac{Y(s)}{U(s)} = \underbrace{C\cdot(sI-A)^{-1}B+D}_{\text{Transfer matrix}}}$$

Q: 11

$$\dot{X} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T \cdot X$$

Sol:-

$$(sI-A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} s+1 & 0 \\ 0 & s+2 \end{bmatrix}$$

$$(sI-A)^{-1} = \frac{\text{Adj}(sI-A)}{|sI-A|} = \frac{1}{(s+1)(s+2)} \begin{bmatrix} s+2 & 0 \\ 0 & s+1 \end{bmatrix}$$

$$(sI-A)^{-1} = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$C\cdot[sI-A]^{-1} = \begin{bmatrix} 1 & 1 \end{bmatrix}_{1\times2} \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix}_{2\times2}$$

$$= \begin{bmatrix} \frac{1}{s+1} & \frac{1}{s+2} \end{bmatrix}_{1\times2}$$

$$\therefore C[sI-A]^{-1}\cdot B = \begin{bmatrix} \frac{1}{s+1} & \frac{1}{s+2} \end{bmatrix}_{1\times2} \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{2\times1}$$

$$= \frac{1}{s+1}$$

$$\therefore T.F = \frac{1}{s+1}$$

Q:8) $\dot{x}(t) = -2x(t) + 2u(t)$

$y(t) = 0.5\,x(t)$

Sol:- state var = 1 ⇒ since if more than 1 then we will get matrix. here no need to go for matrix. you can directly apply the Laplace transform.

$sX(s) = -2X(s) + 2U(s) \Rightarrow \frac{X(s)}{2}[s+2] = U(s)$

$Y(s) = 0.5\,X(s)$

$\therefore \frac{U(s)}{Y(s)} = \frac{X(s)\left[\frac{s+2}{2}\right]}{0.5\,X(s)}$

$\Rightarrow \frac{Y(s)}{U(s)} = \frac{0.5}{\frac{s+2}{2}} = \frac{1}{s+2}$

TYPE-4:-

Stability for State model

$\frac{Y(s)}{U(s)} = C(sI-A)^{-1}B + D$

$= C\,\frac{Adj(sI-A)}{|sI-A|}\,B + D$

$= \frac{C \cdot Adj(sI-A) \cdot B + |sI-A| \cdot D}{|sI-A|}$

W.K.T for T.F → Numerator part = zeros
Dinominator part = poles

So

zeros $= C(Adj(sI-A)) \cdot B + |sI-A| \cdot D$

$\boxed{\text{poles} = |sI-A| = 0}$

$(1+GH=0)$

** $\boxed{\text{Eigen values of system matrix } [A] = \text{closed loop poles}}$

we can comment on stability of a state model by using Eigen values of a state model.

for stable the Eigen values should be -ve

so these are in L.H.S of s-plane.

Conventional
Q(2)

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$

$$C = \begin{bmatrix} -17 & -5 \end{bmatrix} x + [1] r$$

Sol:-

$$\text{zeros} = C \cdot \text{Adj}(SI-A) \cdot B + |SI-A| \cdot D$$

$$= (-17 \ -5) \cdot \text{Adj}(SI-A) \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} + |SI-A| [1]$$

Now

$$SI-A = \begin{bmatrix} s & s-1 \\ s+20 & s+9 \end{bmatrix}$$

$$\text{Adj}(SI-A) = \begin{bmatrix} s+9 & \cancel{-s+20} \\ \cancel{-s+20} & s \end{bmatrix} = \begin{bmatrix} s+9 & -20 \\ 1 & s \end{bmatrix}$$

$$|SI-A| = s(s+9) + \cancel{(s+20)(s-1)}\ 20$$

$$\text{zeros} = [-17 \ -5] \cdot \begin{bmatrix} s+9 & \cancel{-20+s} \\ 1\cancel{(s+20)} & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + [(s)(s+9) + 20][1]$$

$$= -17 - 5s + s^2 + 9s + 20 = 0$$

$$s^2 + 4s + 3 = 0$$

$$s = -1, -3$$

$$\text{Poles} = |SI-A| = 0$$

$$s^2 + 9s + 20 = 0$$

$$s = -4, -5$$

Q(6)

(Ex) fan → linear system

this can be represented by two sets of state equ's.

both equ's are same.

so their Eigen values are also equal.

Ans (A): $|\lambda| = |\mu|$ and ~~setw~~; $X = W$

Type - 5 :-

Controllability & observability :-

To control the state variables → controllability
To Measure the state variables → observability

We can Test the controllability & observability of a system by using "KALMAN'S TEST"

| | |
|---|---|
| ** | $Q_c = [B \;\; AB \;\; A^2B \; \cdots \; A^{n-1}B]$ |
| ** | $Q_o = [C^T \;\; A^TC^T \;\; (A^T)^2C^T \; \cdots \; (A^T)^{n-1}C^T]$ |
| *** | $\lvert Q_c \rvert \neq 0 \;,\; \lvert Q_o \rvert \neq 0$ |

Q: 2:

sol:-

$A = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}$

$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$C = [1 \;\; 1]$

$Q_c = [B \;\; AB]$

$= \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$

$\lvert Q_c \rvert = -1 \neq 0$

$\therefore AB = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

$A^TC^T = \begin{bmatrix} 0 & 2 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

$\therefore Q_o = [C^T \;\; A^TC^T]$

$= \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$

$= -1$

∴ The system is controllable & observable

Q: 10

sol:

$Q_o = [C^T \;\; A^TC^T]$

$A^TC^T = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} b \\ 0 \end{bmatrix} = \begin{bmatrix} b \\ 2b \end{bmatrix}$

$\therefore Q_o = \begin{bmatrix} b & b \\ 0 & 2b \end{bmatrix}$

$\lvert Q_o \rvert = 2b^2 \neq 0$

Ans: (C)

→ observable for all Non zero values of 'b'

Type - 6 :-

Solution of state equation :-

$\dot{x}(t) = Ax(t) + Bu(t)$

(a) free Response $[u(t) = 0]$ :-

$\dot{x}(t) = Ax(t)$

$\dot{x}(t) - Ax(t) = 0 \rightarrow (1)$

The solution of this eq^n is

$x(t) = Ke^{At} \rightarrow (2)$

Apply Laplace transform to this eq^n (1)

$sX(s) - X(0) - AX(s) = 0$

$X(s)[sI - A] = X(0)$

$X(s) = [sI - A]^{-1} \cdot X(0)$

$\phi(s) = (sI - A)^{-1} \rightarrow$ Resolvant Matrix

$x(t) = \{ L^{-1}(sI - A)^{-1} \} \underset{K}{x(0)}$

$\therefore x(t) = e^{At} x(0)$

$\boxed{\therefore \phi(t) = e^{At} = L^{-1}(sI - A)^{-1}}$ ***

"STATE TRANSITION MATRIX"

b) forced Response :-

$sX(s) - X(0) = AX(s) + BU(s)$

$X(s)[sI - A] = X(0) + BU(s)$

$X(s) = (sI - A)^{-1} \cdot X(0) + (sI - A)^{-1} \cdot B \cdot U(s)$

$\boxed{x(t) = \{ L^{-1}(sI - A)^{-1} \cdot \} x(0) + L^{-1} \{ (sI - A)^{-1} B U(s) \}}$

$\Rightarrow$ *** $\boxed{x(t) = e^{At} x(0) + L^{-1} \{ \phi(s) \cdot B U(s) \}}$

Q:(4)
chapter(8)
sol:-

$x(t) = e^{At} x(0)$

$e^{At} = L^{-1}(SI - A)^{-1}$

$(SI - A) = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} S-1 & 0 \\ -1 & S-1 \end{bmatrix}$

$(SI - A)^{-1} = \frac{1}{(S-1)^2 + 0} \begin{bmatrix} S-1 & 0 \\ 1 & S-1 \end{bmatrix}$

$L^{-1}(SI - A)^{-1} = L^{-1}\begin{bmatrix} \frac{1}{S-1} & \cancel{0}/(S-1)^2 \\ 1/(S-1)^2 & \cancel{1}/(S-1) \end{bmatrix}$

$= \begin{bmatrix} e^t & 0 \, \cancel{te^t} \\ te^t & e^t \end{bmatrix}$

$\therefore x(t) = e^{At} x(0)$

$= \begin{bmatrix} e^t & \cancel{te^t}\,0 \\ te^t & e^t \end{bmatrix}_{2\times2} \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{2\times1}$

$= \begin{bmatrix} e^t \\ te^t \end{bmatrix}$

Q:5:

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

shortcut method:-

$e^{At} = \left[ L^{-1}(SI - A)^{-1} \right] = \phi(t)$

one of the property of S.T.M is $\phi(0) = e^{A(0)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

so without solving the pblm just apply this property

Properties of S.T.M:-

- $\phi(0) = I$.
- $\phi^{-1}(t) = \phi(-t)$
- $\phi^{k}(t) = \phi(kt)$
- $\phi(t_1 + t_2) = \phi(t_1)\,\phi(t_2)$

Type 7 :-

## STATE DIAGRAMS :-

1) controllable form :-

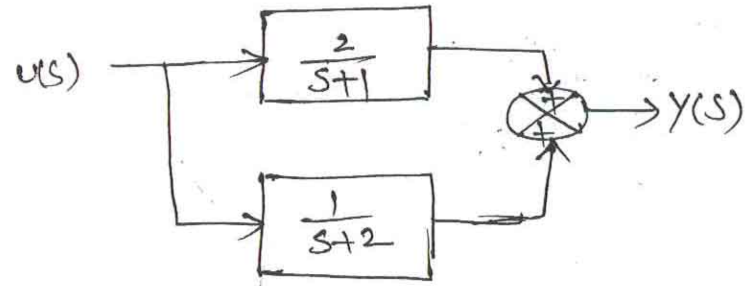
$$\frac{Y(S)}{U(S)} = \frac{3S+5}{S^2+3S+2}$$

sol :-

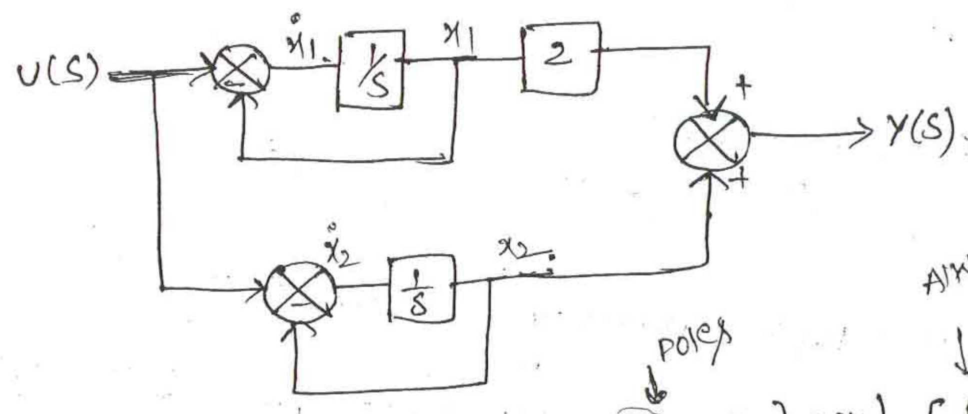
$$\frac{Y(S)}{U(S)} = \frac{3S+5}{S^2+3S+2} = \frac{3S+5}{(S+1)(S+2)}$$

$$\frac{Y(S)}{U(S)} = \frac{2}{S+1} + \frac{1}{S+2}$$

$$Y(S) = \frac{2 \cdot U(S)}{S+1} + \frac{U(S)}{S+2}$$




integrator based dgm is shown as




Always unit matrix

poles

$$\dot{x}_1 = -x_1 + u$$

$$\dot{x}_2 = -2x_2 + u$$

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = 2x_1 + x_2 \ldots$$

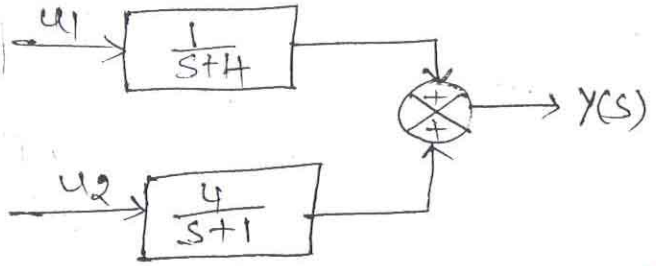
$$y = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0 \cdot u$$

Residues becomes 'C' matrix

NOTE :-

** In controllable form the system matrix 'A' is a Diagonal matrix with Diagonal Elements as Eigen values. and the matrix 'C' The Matrix 'B' is a unit Matrix. Elements are "Residues".

Pb)



the state model of the block dgm is ?

sol:-

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0 \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

2) OBSERVABLE form:-

In observable form we take directly the T.F, instead of partial fractions.

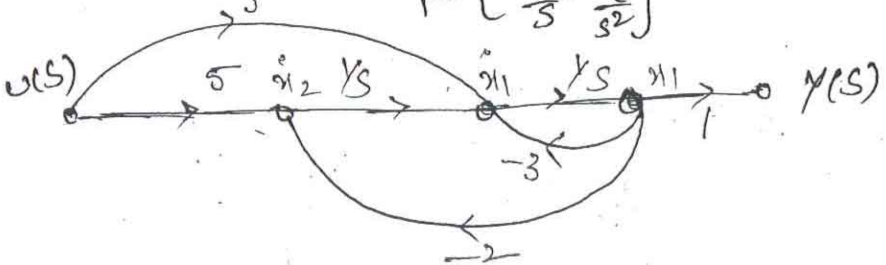
$$\frac{Y(S)}{U(S)} = \frac{3S+5}{S^2+3S+2}$$

$$= \frac{\cancel{S}\left[\frac{5}{\cancel{S}}+3\right]}{S^{\cancel{2}}\left[1+\frac{3}{S}+\frac{2}{S^2}\right]}$$

$$= \frac{\left(\frac{3}{S}+\frac{5}{S^2}\right)}{\left[1+\frac{3}{S}+\frac{2}{S^2}\right]} \quad //...$$

This should be write in the form of mason's Gain formula.

$$= \frac{\frac{3}{S}+\frac{5}{S^2}}{1-\left[-\frac{3}{S}-\frac{2}{S^2}\right]} \quad //...$$

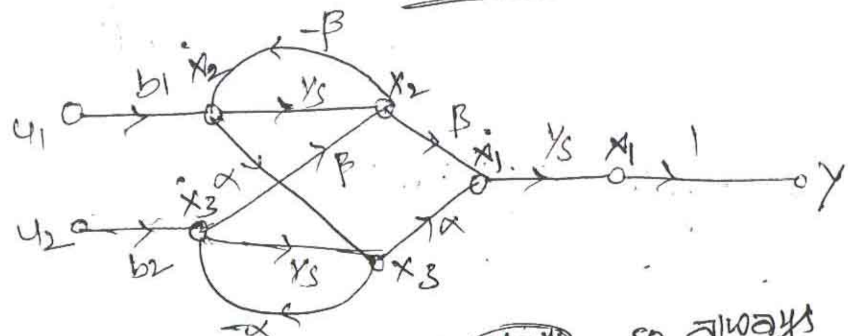


$$\dot{x}_1 = -3x_1 + x_2 + 3u$$
$$\dot{x}_2 = -2x_1 + 5u$$
$$y = x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Pb)

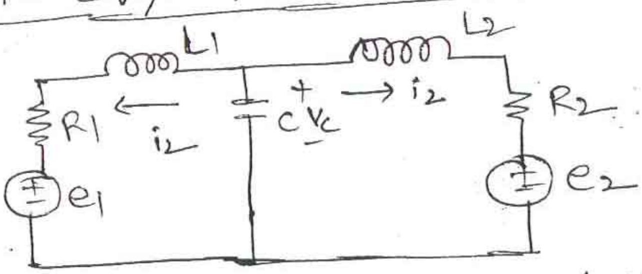



Sol:- Generally we are taken ($y=x_1$), so always first integration from o/p side is taken as $\dot{x}_1 \xrightarrow{1/s} x_1$.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & \beta & \alpha \\ 0 & -\beta & \alpha \\ 0 & \beta & -\alpha \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ b_1 & 0 \\ 0 & b_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

State Equ's of Electrical N/w's :-

Pb)




Sol:- first we have to take initial condi of this N/w.

Apply KVL to loop(1):-

$$L_1 \frac{di_1}{dt} + i_1 R_1 + e_1 - V_c = 0$$

$$\frac{di_1}{dt} = -\frac{i_1 R_1}{L_1} - \frac{e_1}{L_1} + \frac{V_c}{L_1} \rightarrow (1)$$

KVL to loop(2):-

$$L_2 \frac{di_2}{dt} + i_2 R_2 + e_2 - V_c = 0$$

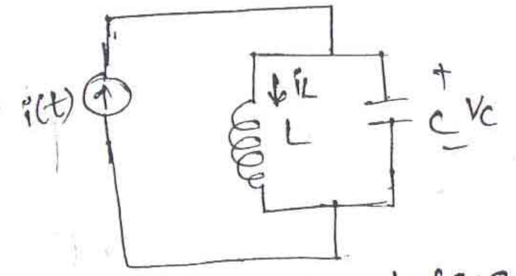
$$\Rightarrow \frac{di_2}{dt} = -\frac{i_2 R_2}{L_2} - \frac{e_2}{L_2} + \frac{V_c}{L_2}$$

$$\begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \\ \frac{dv_c}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{L_1} & 0 & \frac{1}{L_1} \\ 0 & -\frac{R_2}{L_2} & \frac{1}{L_2} \\ -\frac{1}{C} & -\frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ v_c \end{bmatrix} + \begin{bmatrix} -\frac{1}{L_1} & 0 \\ 0 & -\frac{1}{L_2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

conventional

Q:(1)...

(5)




sol:-

first we have to define state varii.

KCL:- $i(t) = i_L + C \cdot \frac{dv_c}{dt}$

$$\Rightarrow \frac{dv_c}{dt} = \frac{i(t)}{C} - \frac{i_L}{C} \rightarrow (1)$$

KVL:- $L\frac{di_L}{dt} - v_c = 0$

$$\frac{di_L}{dt} = \frac{v_c}{L} \rightarrow (2)$$

$$\begin{bmatrix} \frac{dv_c}{dt} \\ \frac{di_L}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{C} \\ \frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} v_c \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{C} \\ 0 \end{bmatrix} i(t)$$

(pb) ex Q:8:-

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [-0.5 \quad -3 \quad -5] x + 0 \cdot u$$

Eigen values $\rightarrow$ $|sI - A| = 0$

$$(sI - A) = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & -3 & -5 \end{bmatrix} \Rightarrow \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 0.5 & 3 & s+5 \end{bmatrix}$$

$|sI - A| = 0$

$$\begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 0.5 & 3 & s+5 \end{vmatrix} = 0$$

$$s(s^2 + 5s) + 1[0.5] = 0 \Rightarrow s =$$

$$\Rightarrow s^3 + 5s^2 + 0.5 = 0$$

Q.7 $\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$

$u = [-0.5 \;\; -3 \;\; -5] x + v$

Sol:- $\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \left[ [-0.5 \;\; -3 \;\; -5] x + v \right]$

$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.5 & -3 & -5 \end{bmatrix} x + v$

$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} x + v$

$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}$

$|sI - A| = 0$

$\Rightarrow \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 0 & 2 & s+3 \end{vmatrix} = 0$

$\Rightarrow s(s(s+3)+2) + 1(0) + 0(\;) = 0$

$\Rightarrow s^3 + 3s^2 + 2s = 0$

$\Rightarrow s = 0, -1, -2$

Pblms on

**Polar plot :-**

Pb) $G(s) = \dfrac{s+1}{s^2(s-2)}$

Sol:- While drawing the polar plot it is better to write the T.F in Time constant form.

To map 's':-

Polar plot have to be drawn.

$= \dfrac{1[1+s]}{-2s^2[1-0.5s]}$

$= \dfrac{-0.5(1+j\omega)}{(j\omega)^2[1-0.5j\omega]}$

$$|G(j\omega)| = \frac{0.5\sqrt{1+\omega^2}}{\omega^2\sqrt{1+(0.5\omega)^2}}$$

$$\angle G(j\omega) = \frac{[-180^\circ][\tan^{-1}\omega]}{[180^\circ][-\tan^{-1}0.5\omega]}$$

$$= -360 + \tan^{-1}\omega + \tan^{-1}[(0.5)\omega]$$

| $\omega$ | 0 | .... | $\infty$ |
|---|---|---|---|
| $\lvert G(j\omega)\rvert$ | $\infty$ | | 0 |
| $\angle G(j\omega)$ | $-360^\circ$ | | $-180^\circ$ |

To map $S_2$:- $S_2$ will becomes to origin.

To map $S_3$:- Draw the mirror image of $S_1$.

To map $S_4$:-

$$G(s) = \frac{1}{(s^2)(-2)} = \frac{0.5}{s^2(-1)}$$

*** $\boxed{-1 = e^{j\pi}}$ → correction factor

when ever T.F is like $G(s) = \frac{K(s+1)}{s(s-2)}$ (or) $(s-1)$ (or) $(s-9)$ .... we have to take a correction factor of $e^{j\pi}$, otherwise it will not encloses the polarplot. then it will becomes wrong!

so always, when $(-1)$ is comes we have to take correction factor.

Ex: $G(s) = \frac{(s+1)}{s(1-0.5s)}$

No need to take correction factor, since

$$G(s) = \frac{1}{s(1)}$$

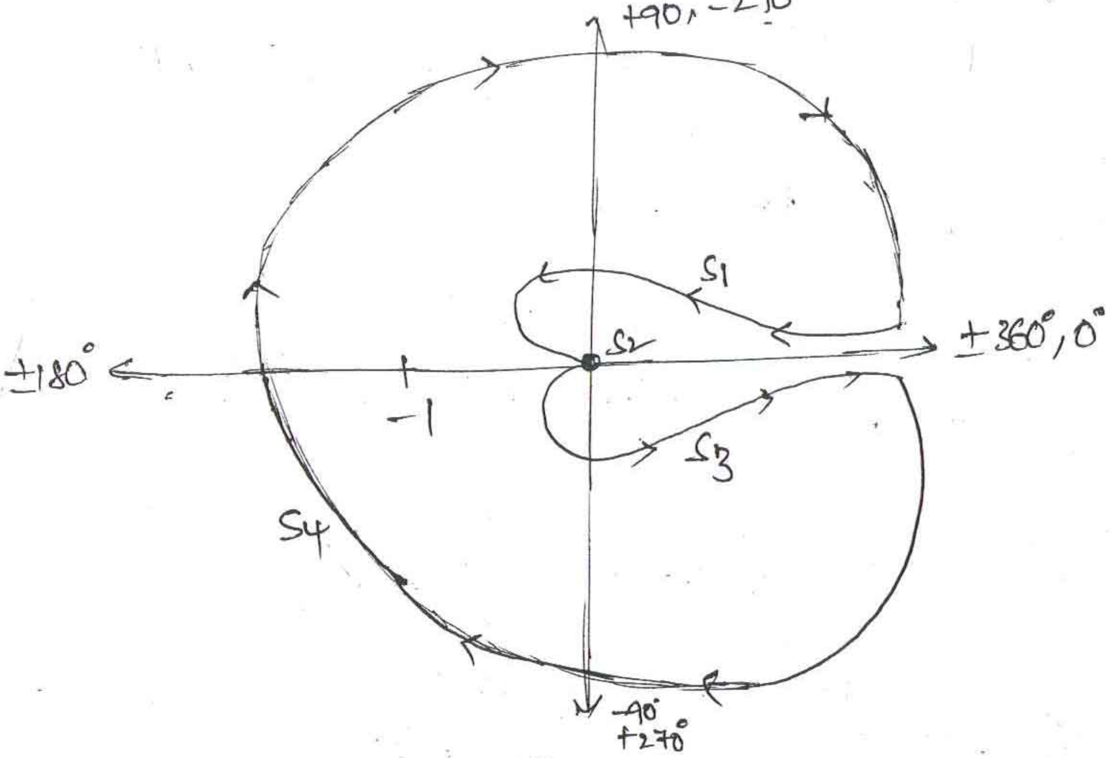
when ever $(-1)$ is comes in denominator, then only we have to take.

$$= \frac{0.5}{\lim_{R\to\infty} R^2 e^{j2\theta} \cdot e^{j\pi}}$$

$$= \infty \cdot e^{-j(2\theta+\pi)}$$

$$\theta = -\frac{\pi}{2} \rightarrow +\frac{\pi}{2}$$

$$\infty \cdot e^{-j0} \rightarrow \infty e^{-j2\pi} \ldots$$




$$-1 = 1 - Z$$

$$\Rightarrow \boxed{Z=2} \text{ unstable} \ldots$$

Pb) $G(S) = \frac{K(1+S)^2}{S^3}$ find range of 'K' for stability.

Sol:-

TO map $S_1$:-

→ Polar Plot ...

$$G(j\omega) = \frac{K(1+j\omega)^2}{(j\omega)^3}$$

$$|G(j\omega)| = \frac{K(1+\omega^2)}{(\omega)^3}$$

$$\angle G(j\omega) = -270° + 2\tan^{-1}\omega$$

| $\omega$ | 0 | 1 | $\infty$ |
|---|---|---|---|
| $\lvert G(j\omega)\rvert$ | $\infty$ | $2K$ | 0 |
| $\angle G(j\omega)$ | $-270$ | $-180$ | $-90$ |

$$-270 + 2\tan^{-1}\omega = -180^\circ$$

$$\tan^{-1}\omega = 45^\circ$$

$$\omega = 1 = \omega_{pc} = 1\ \text{rad/sec}$$

Gain $x = \dfrac{K(1+1)}{1} = 2K$

To map $S_4$:-

$$G(s) = \frac{K}{s^3} = \lim_{R\to\infty} \frac{K}{(Re^{j\theta})^3}$$

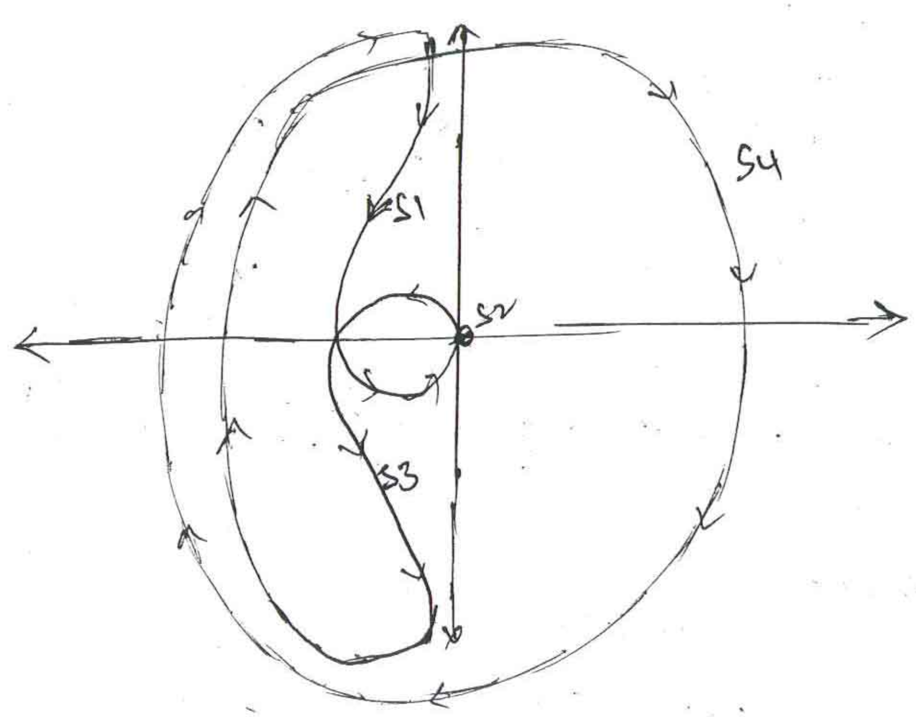
$$= \infty \cdot e^{-j3\theta}$$

$$[\theta \Rightarrow -\tfrac{\pi}{2} \text{ to } +\tfrac{\pi}{2}]$$

$$\infty e^{+270} \text{ to } \infty e^{-270}$$

(or)

$$\infty e^{j3\pi/2} \longrightarrow \infty e^{-j3\pi/2}$$



chapter-6
Q:4 (conventional)

Pb) i)

$$G(s) = \frac{s}{1-0.2s}$$

To map $S_1$:- we have to draw polar plot.

$$G(j\omega) = \frac{j\omega}{1-0.2\omega}$$

$$|G(j\omega)| = \frac{\omega}{\sqrt{1+(0.2\omega)^2}} = \frac{\omega}{0.2\omega\sqrt{\frac{1}{(0.2\omega)^2}+1}}$$

$$\angle G(j\omega) = 90° + \tan^{-1}(0.2)\omega$$

At $\omega=\infty$:

$$\frac{1}{0.2\sqrt{0+1}} = 5$$

| $\omega$ | 0 | | $\infty$ |
|---|---|---|---|
| $\lvert G(j\omega)\rvert$ | 0 | | 5 |
| $\angle G(j\omega)$ | 90° | | 180° |

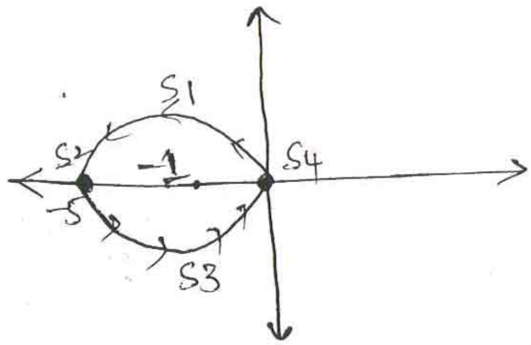
To map $S_2$:-

$$G(s) \cong \frac{\cancel{s}}{-0.2\cancel{s}} \cong -5 ..$$

To map $S_3$:- Just draw mirror image

To map $S_4$:-

$$G(s) = s$$
$$= \lim_{R\to 0} Re^{j\theta}$$
$$= 0.e^{j\theta} ..$$




Pb 2)

$$G(s) = \frac{(s+2)}{(s+1)(s-1)}$$ → Note:- one pole is in RHS, so unstable. This is not correct, since we are calculating the stability for CLTF: $1+GH=0$ ⇒ stable..

Sol:- we have to map only 3 sections...

To map $S_1$:-

Polar Plot

$$G(s) = \frac{-2[1+0.5s]}{(1+s)(1-s)}$$

$$G(j\omega) = \frac{-2(1+0.5j\omega)}{(1+j\omega)(1-j\omega)}$$

$$|G(j\omega)| = \frac{2\sqrt{(1+(0.5\omega)^2}}{(1+\omega^2)}$$

$$\angle G(j\omega) = \frac{[-180^\circ][\tan^{-1}(0.5\omega)]}{[\tan^{-1}\omega][-\tan^{-1}\omega]}$$

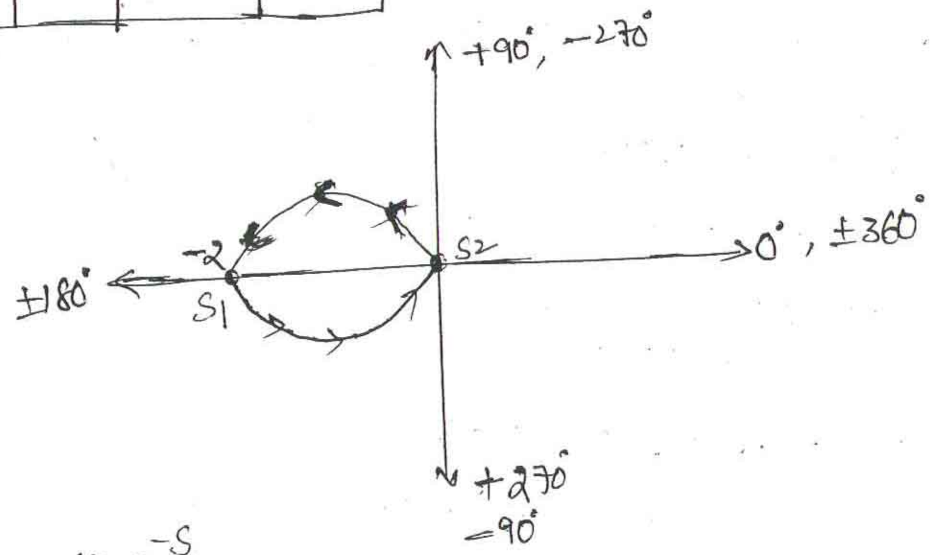
$$= -180^\circ + \tan^{-1}(0.5\omega)$$

| $\omega$ | 0 | ... | $\infty$ |
|---|---|---|---|
| $\lvert G(j\omega)\rvert$ | 2 | | 0 |
| $\angle G(j\omega)$ | $-180^\circ$ | | $-90^\circ$ |

$-180^\circ + \tan^{-1}(0.5\omega) = -180^\circ$

$\tan^{-1}\omega = 0$

$\Rightarrow \boxed{\omega = 0}$




pb) $G(s) = \dfrac{10e^{-s}}{s(s+2)}$

sol:- 4 sections have to be map.

To map $S_1$:-

polar plot

$$G(s) = \frac{5e^{-s}}{s(s+2)}$$

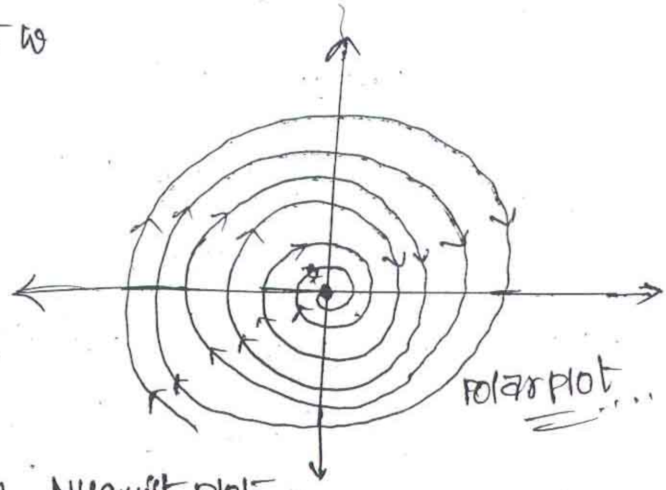
$$|G(j\omega)| = \frac{5e^{-j\omega}}{j\omega(j\omega+2)} = \frac{5\times1}{\omega\sqrt{1+(0.5\omega)^2}}$$

$$\angle G(j\omega) = \frac{(0^\circ)[-57.3\omega]}{[90^\circ][\tan^{-1}0.5\omega]}$$

$\angle G(j\omega) = -90^\circ - 57.3\omega - \tan^{-1} 0.5\omega$

| $\omega$ | 0 | -- | $\infty$ |
|---|---|---|---|
| $\|G(j\omega)\|$ | $\infty$ | | 0 |
| $\angle G(j\omega)$ | $-90^\circ$ | | $-\infty^\circ$ |

$-90, -180, -270, -360, -450, -540 \ldots\ldots -\infty$



Polar plot...

It is not possible to draw Nyquist plot, since $\infty$ circles have to draw in reverse direction...

unstable.....

Note :-
→ Nyquist plot in frequency domine is similar to R-H criteria in time response.....

let us consider the following example:

$$G(s) = \frac{s+2}{(s+1)(s-1)}$$

$$1+G(s)\,H(s)=0$$

$$1+\frac{s+2}{(s+1)(s-1)}=0$$

$$(s+1)\cdot(s-1)+(s+2)=0$$

$$\Rightarrow s^2+s+2=0\ldots\ldots$$

| | | | |
|---|---|---|---|
| + | $s^2$ | 1 | 1 |
| + | $s^1$ | 1 | 0 |
| + | $s^0$ | 1 | |

The no.of sign changes in RH criteria will give 'Z'.

$$N = P-Z$$

$$N = 1-0$$

$$\boxed{N=1}$$

Note:- with drawing the Nyquist plot we can say the no.of encirclements, by using this technique...

# Bode plot

In Bode plot; mag" chart are asymptotic plots i.e straight lines.

→ phase chart are free line chart..

1) system Gain:-

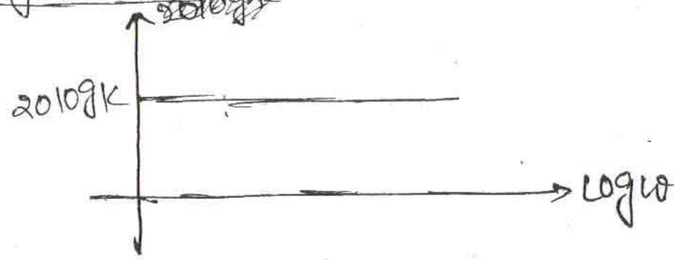
$$F(s) = \pm K$$

$$F(j\omega) = K + j0$$

$$|F(j\omega)| = \sqrt{(\pm K)^2 + 0^2}$$

Its db value is $20\log K$.

magnitude plot:




* we are not taking '0' on x-axis, since we can't define. so we will take 0.1, 0.2 (or) any other.

$$\text{slope}(m) = 0 \text{ db/dec}$$

phase plot:-

$$\angle F(j\omega) = K + j0 = 0^\circ \text{ for all } '\omega'$$

$$= -K + j0 = -180^\circ \text{ for all } '\omega'$$

2) Integral & Derivative factors:- (pole/zeros at origin)

↳ $s^{\pm n}$ .... [+ → zeros, − → poles]

$$f(j\omega) = (j\omega)^{\pm n} = (0 + j\omega)^{\pm n}$$

$$|F(j\omega)| = \left[\sqrt{0^2 + \omega^2}\right]^{\pm n} = [\omega]^{\pm n}$$

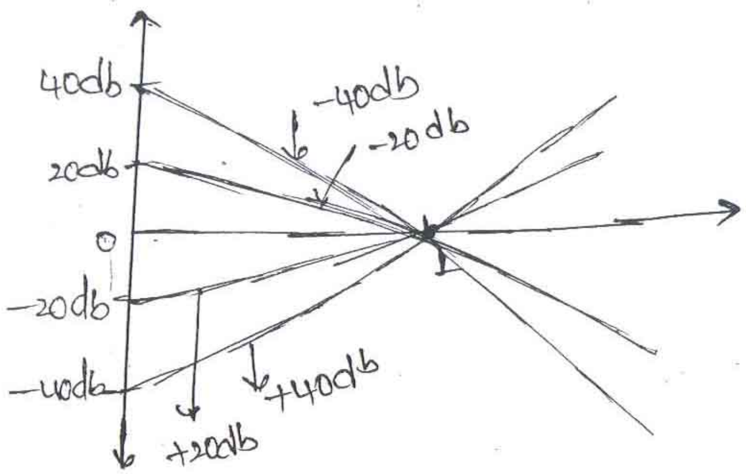
Its db value is

$$= \pm 20n\log\omega$$

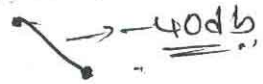
$$= \underbrace{\pm 20n}_{m} \; \underbrace{\log\omega}_{x} \rightarrow (1)$$

$\boxed{\text{slope} = \pm 20 \times n}$ db/dec

$\pm 20 n \log w = 0 \Rightarrow \log w = 0 \Rightarrow \boxed{w = 1}$ rad/sec




Note:- If you want to draw the Bode magnitude plot in this place (—), first we have to draw the plot for 0 to 1 & then draw a parallel line to that in this place. it will look like




first order factors :- $(1 \pm sT)^{\pm 1}$

Always
+ → zeros
− → poles

$F(jw) = (1 \pm jw)^{\pm 1}$

$|F(jw)| = \left[\sqrt{1 + (wT)^2}\right]^{\pm 1}$

Its db value is → $20 \log \left[\sqrt{1 + (wT)^2}\right]^{\pm 1}$

$20 \log \left[\sqrt{1 + (wT)^2}\right]^{\pm 1}$

$\pm 20 \log \sqrt{1 + (wT)^2} \rightarrow (1)$

This can not be compare with $y = mx$. since 'x' is always in the form of 'log w' only

Asymptotic Approximations :-

case (i) :- Low frequency

$1 >> (wT)^2$

$\pm 20 \log \sqrt{1} = 0 \, db$

case (ii) :- High frequency, → $(wT)^2 >> 1$, ...

$\pm 20 \log \sqrt{(wT)^2}$

$\pm 20 \log (wT) \rightarrow (2)$

$[\pm 20 \log w \pm 20 \log T]$

$mx + c$

$$\boxed{\text{slope }(M) = \pm 20 \text{ db/dec}}$$

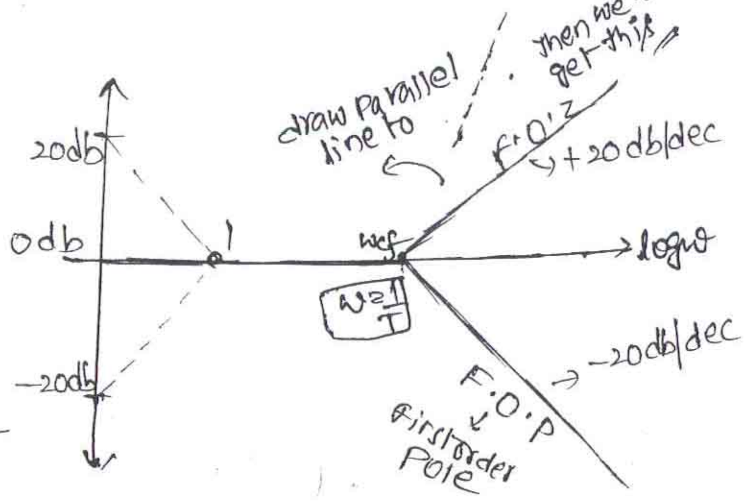
Corner frequency :- $(\omega_{cf})$

$0 = \pm 20\log(\omega T)$

$\log(\omega T) = 0$

$\omega T = \log^{-1}(0) = 1$

$\boxed{\omega = \omega_{cf} = 1/T}$ rad/sec




Ex:- $(s \pm 2)^{\pm 1}$

$\left[1 + \frac{s}{2}\right]^{\pm 1}$

$T = 1/2$ ; $\omega_{cf} = 2$ rad/sec

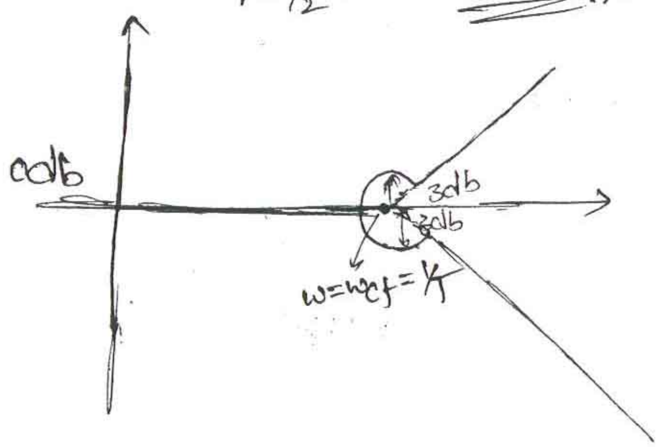
At $\omega = \omega_{cf} = 1/T$

$\pm 20\log\sqrt{1 + (\frac{1}{T} \times T)^2}$

$\pm 20\log(\sqrt{2})$

$= \boxed{\pm 3\text{db}}$

Error At $\omega_{cf}$




Quadratic factors :-

↳ representation ⇒ $(s^2 + 2\zeta\omega_n s + \omega_n^2)^{\pm 1}$

$\omega_n^2\left[\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1\right]^{\pm 1}$

put $s = j\omega$

$\left[1 - \left(\frac{\omega}{\omega_n}\right)^2 + j\, 2\zeta\left(\frac{\omega}{\omega_n}\right)\right]^{\pm 1}$

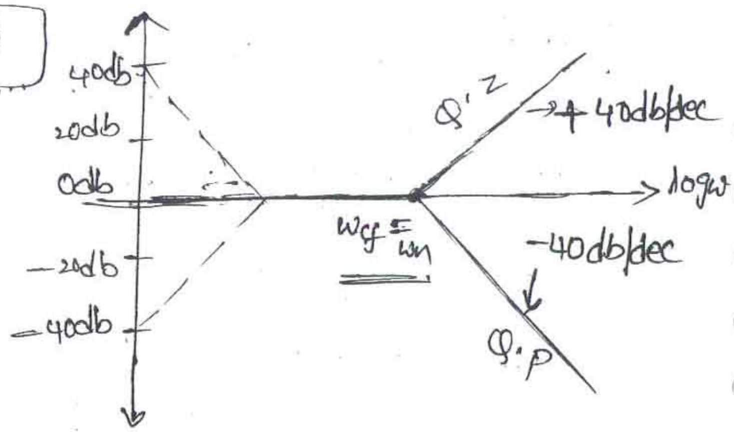
$$\boxed{\text{slope }(M) = \pm 40 \text{ db/dec}}$$

Corner Frequency, $\omega_{cf} = \omega_n$ rad/sec

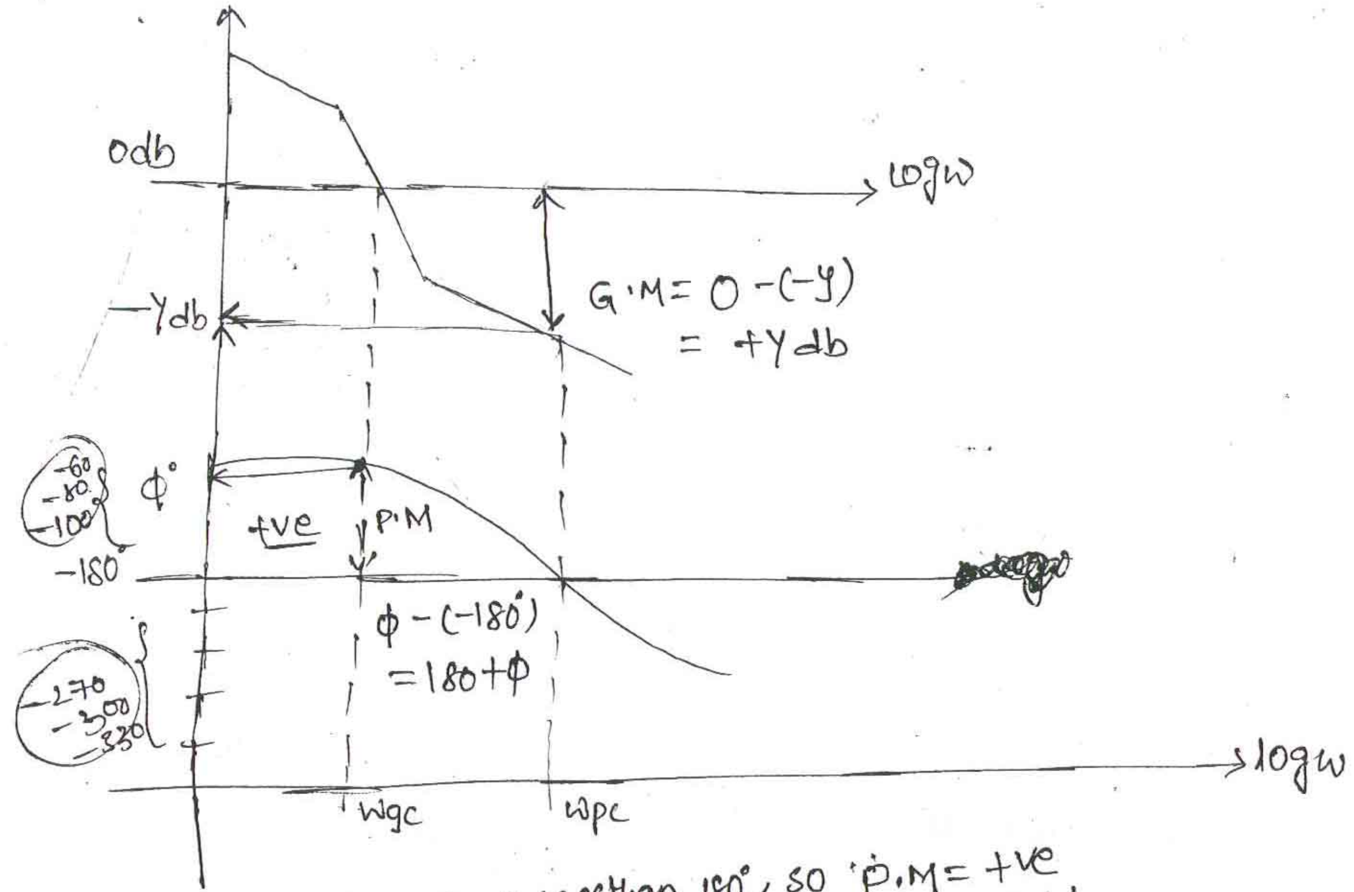
Ex:- $(s^2 + 4s + 16)^{\pm 1}$

$\omega_n^2 = 16 \Rightarrow \omega_{cf} = \omega_n = 4$ rad/sec

Error at $\omega_{cf} = \pm 20\log 2\zeta$

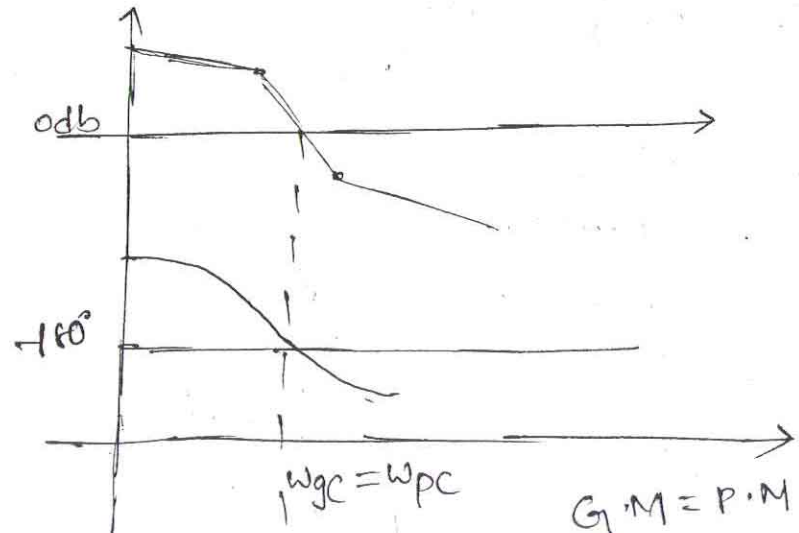



## To find G.M & P.M from Bodeplot :-

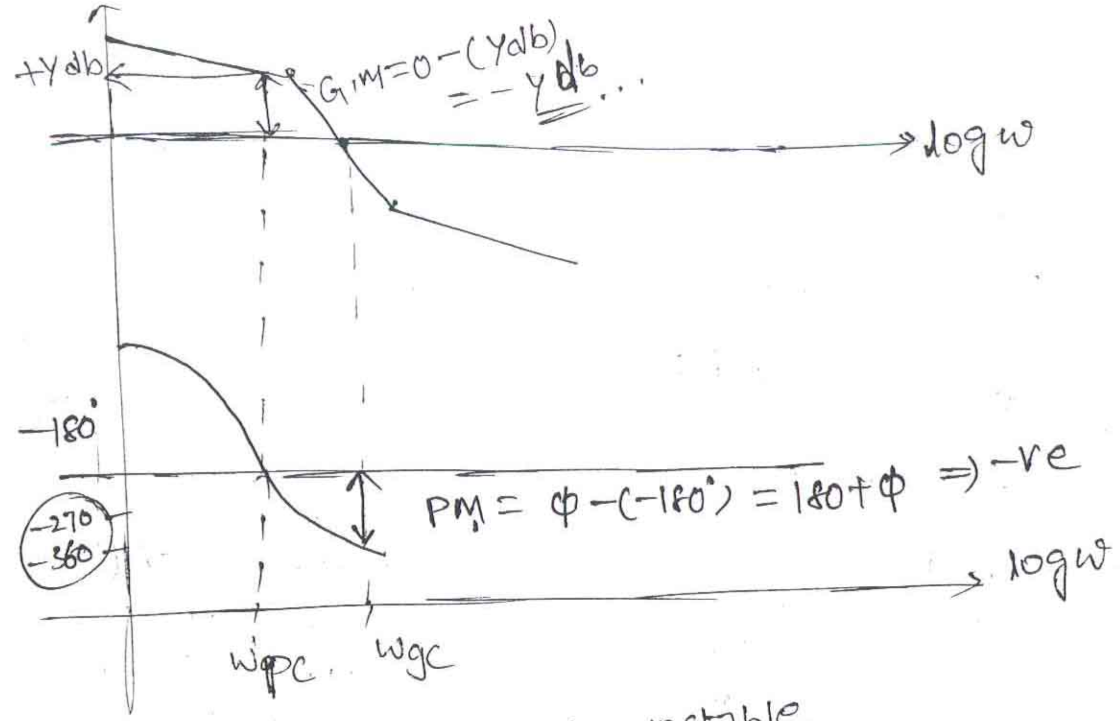



since $\phi$ is -ve, & less than 180°, so P.M = +ve.

$\boxed{w_{gc} < w_{pc}}$ → G.M = +ve, P.M = +ve } stable




G.M = P.M = 0 marginally stable




$w_{pc} < w_{gc} \Rightarrow$ unstable....

## Bodeplot for lead-lag compensator :-

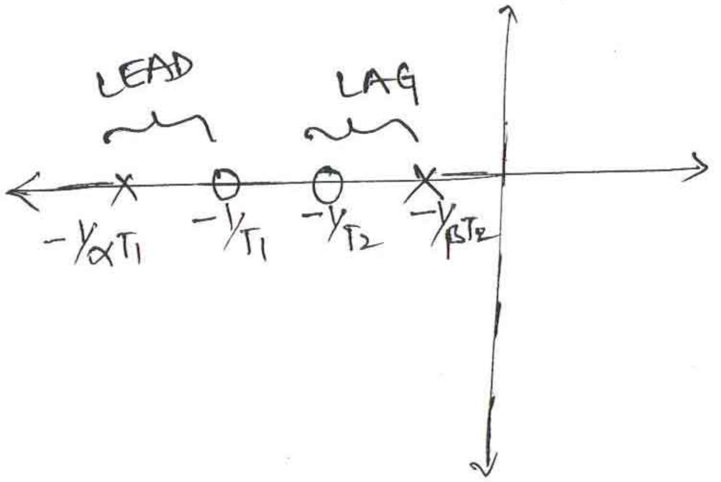
T.F of a lead-lag (or) lag-lead N/W is

$$F(S) = \frac{(V_o(S))}{V_i(S)} = \frac{\alpha(1+T_1S)(1+T_2S)}{(1+\alpha T_1 S)(1+\beta T_2 S)}$$

Always the T.F should be in time constant form for any plot. Already this is time constant form only.

put $S = j\omega$..

$$F(j\omega) = \frac{\alpha(1+j\omega T_1)(1+j\omega T_2)}{(1+j\omega\alpha T_1)(1+j\omega\beta T_2)}$$




$$\angle F(j\omega) = \tan^{-1}\omega T_1 - \tan^{-1}\omega\alpha T_1 + \tan^{-1}\omega T_2 - \tan^{-1}\omega\beta T_2 \ldots$$

## MAGNITUDE TABLE :-

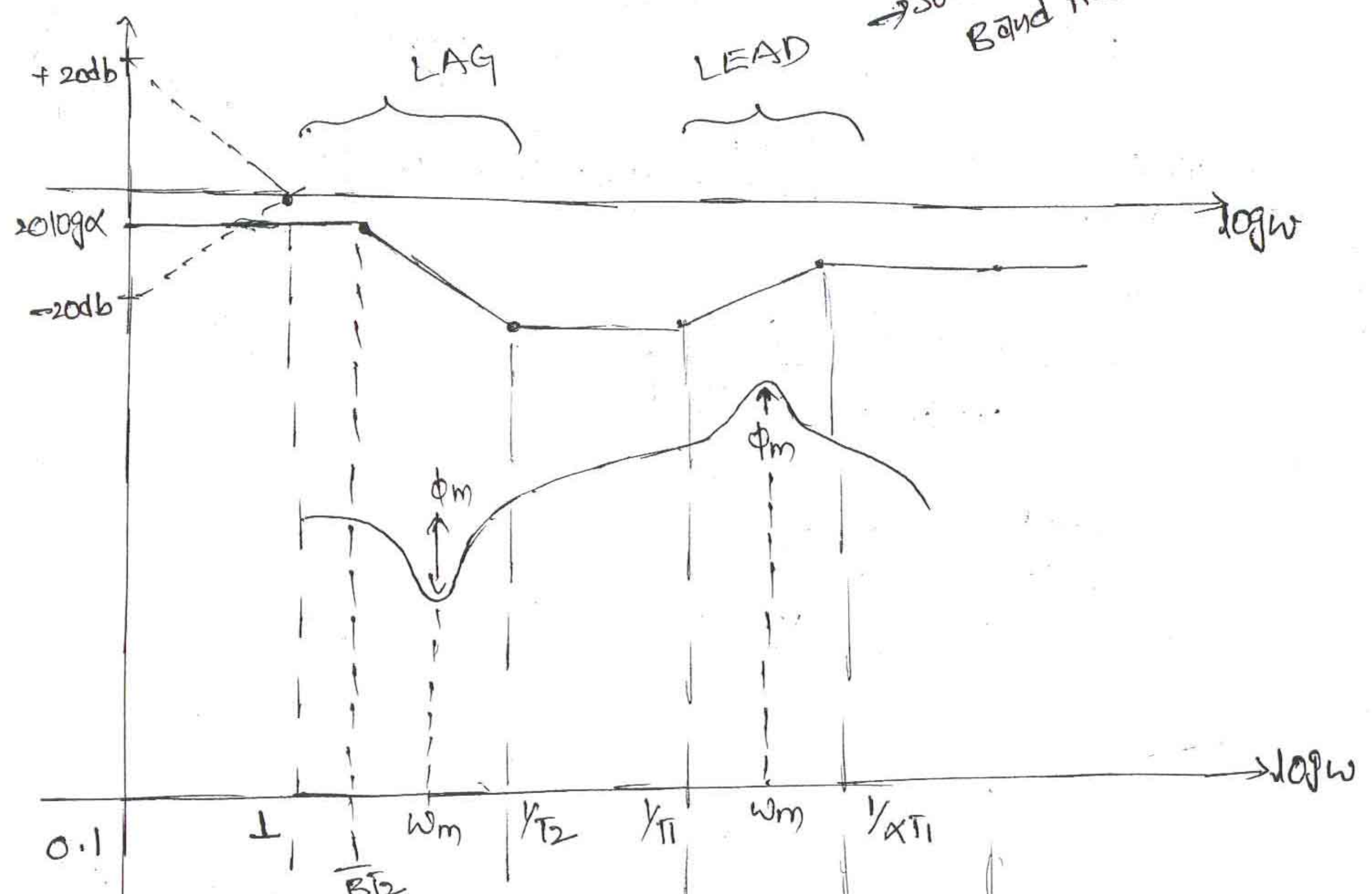
| S.NO | Factor | C.F | Magnitude |
|---|---|---|---|
| 1. | $k = \alpha$ | — | $20\log\alpha$ |
| 2. | $(j\omega)^{\pm n}$ | — | NIL |
| 3. | $\frac{1}{1+j\omega\beta T_2}$ | $\frac{1}{\beta T_2}$ | $-20$ db/dec |
| 4. | $1+j\omega T_2$ | $\frac{1}{T_2}$ | $+20$ |
| 5. | $1+j\omega T_1$ | $\frac{1}{T_1}$ | $+20$ |
| 6. | $\frac{1}{1+j\omega\alpha T_1}$ | $\frac{1}{\alpha T_1}$ | $-20$ |

0 db/dec, $-20$, 0, $+20$, 0 db/dec

NOTE :-

LAG → Low pass
LEAD → High pass
LAG-LEAD → Band reject
LEAD-LAG → Band pass

→ So this is a Band Reject filter ...




$$\omega_m = \sqrt{\frac{1}{\beta T_2} \times \frac{1}{T_2}} \; ; \quad \omega_m = \sqrt{\frac{1}{T_1} \times \frac{1}{\alpha T_1}}$$

$$= \frac{1}{T_2\sqrt{\beta}} \qquad\qquad = \frac{1}{T_1\sqrt{\alpha}}$$

charll of LEAD compensator:-

1) The maxll phase lead occurs at the Geometric mean of the two corner freq''s.

$$F(j\omega) = \phi = \tan^{-1}(\omega T_1) - \tan^{-1}(\omega \alpha T_1)$$

$$\tan\phi = \tan\left[\tan^{-1}(\omega T_1) - \tan^{-1}(\omega\alpha T_1)\right]$$

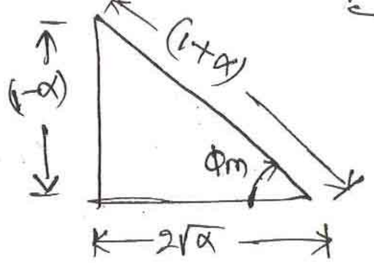
$$\tan\phi = \frac{\omega T_1 - \omega\alpha T_1}{1+(\omega T_1)^2\alpha} = \frac{\omega T_1(1-\alpha)}{1+(\omega T_1)^2\cdot\alpha}$$

At $\boxed{\omega = \omega_m = \frac{1}{T_1\sqrt{\alpha}}}$ ; $\phi = \phi_m$

$$\tan\phi_m = \frac{\frac{1}{\cancel{T_1}\sqrt{\alpha}}\cdot\cancel{T_1}(1-\alpha)}{1+\left[\frac{1}{\cancel{T_1}\sqrt{\alpha}}\times\cancel{T_1}\right]^2\cdot\alpha}$$

$$\tan\phi_m = \frac{1-\alpha}{2\sqrt{\alpha}}$$

$$\Rightarrow \boxed{\phi_m = \tan^{-1}\left(\frac{1-\alpha}{2\sqrt{\alpha}}\right)}$$




$$\sin\phi_m = \frac{1-\alpha}{1+\alpha}$$

$$\boxed{\phi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)}$$

$$(1+\alpha)\sin\phi_m = 1-\alpha$$

$$\alpha + \alpha\sin\phi_m = 1-\sin\phi_m$$

$$\alpha[1+\sin\phi_m] = (1-\sin\phi_m)$$

$$\boxed{\alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m}}$$

2) with regard to filtering property it acts as a "High pass filter".

3) The phase lead compensator shifts the gain cross over frequency to higher values where the desired phasemargin is acceptable. Hence it is effective when the slope of the uncompensated system near the gain cross over frequency is low.

4) chars of phase lag compensator:-

1) At regard to filtering property it acts as a low pass filter.

2) The phase lag compensator shifts the gain cross over freq to lower values where the desired phasemargin is acceptable. Hence it is Effective when the slope of the uncompensated system near the gain crossover freq is high.

3) The Maximum phaselag occurs at the Geometric mean of the two corner-freq's.

$$\angle F(j\omega) = \phi = \tan^{-1}(\omega T_2) - \tan^{-1}(\omega\beta T_2)$$

$$\tan\phi = \tan\left[\tan^{-1}(\omega T_2) - \tan^{-1}(\omega\beta T_2)\right]$$

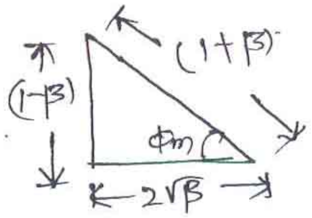
$$\tan\phi = \frac{\omega T_2 - \omega\beta T_2}{1+(\omega T_2)(\omega\beta T_2)} = \frac{\omega T_2(1-\beta)}{1+(\omega T_2)^2\beta} \;//$$

At $\omega = \omega_m = \frac{1}{T_2\sqrt{\beta}}$ ; $\phi = \phi_m$

$$\tan\phi_m = \frac{\frac{1}{\cancel{T_2}\sqrt{\beta}} \cdot \cancel{T_2}(1-\beta)}{1+\left[\frac{1}{\cancel{T_2}\sqrt{\beta}}\cdot \cancel{T_2}\right]^2 \cdot \beta}$$

$$\tan\phi_m = \frac{1-\beta}{2\sqrt{\beta}}$$

$$\boxed{\phi_m = \tan^{-1}\left(\frac{1-\beta}{2\sqrt{\beta}}\right)} \;//....$$




$$(1+\beta)\sin\phi_m = 1-\beta$$

$$\sin\phi_m \cdot \beta + \beta = 1 - \sin\phi_m$$

$$\boxed{\beta = \frac{1-\sin\phi_m}{1+\sin\phi_m}} \;//...$$

$$\Rightarrow \phi_m = \sin^{-1}\left(\frac{1-\beta}{1+\beta}\right) //..$$

$$\boxed{\phi_m = \sin^{-1}\left[\frac{1-\beta}{1+\beta}\right]} \;//....$$

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Q: 14 $G(S) = \frac{K}{s^2}$ , $H(S) = 1$

sol:- Here we get 3rd order system, so we will go for dominant pole concept. but we don't know 'K' value. so we can't go for this method. it is a double integrator. so 180° lag will be there. so it is very slow.

note:- for a single integrator 90° lag...

so for such system we can't use PI controller & lag compensator.

Note:- Since the plant has 180° phase lag, lead compensator should be used to give the needed specifications.

(d & c) ✗ confidently say both are wrong.

Ans: (a)....

page 76

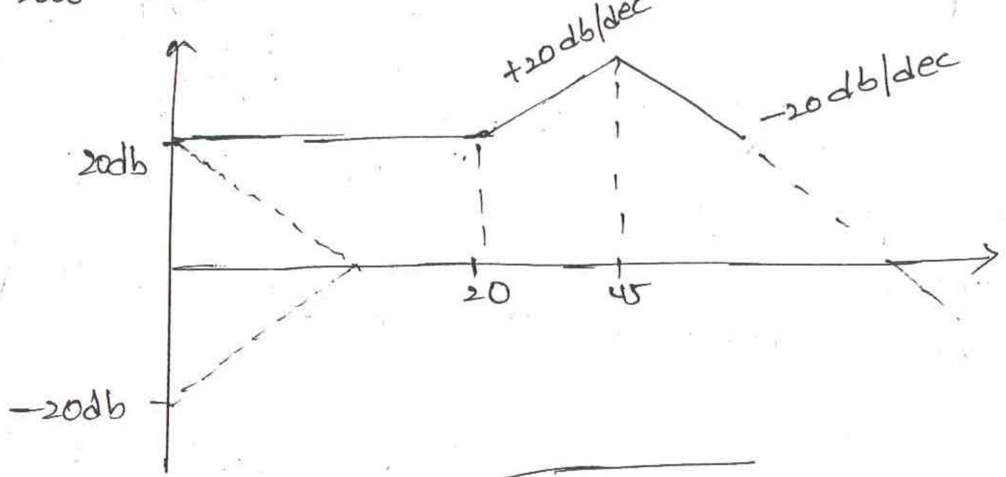
Q: 1 $$G(S) = \frac{10^3(s+20)}{s^2+210s+2000}$$

$$= \frac{10^3 \times 20 \left[1+\frac{s}{20}\right]}{2000\left[\frac{s^2}{2000} + \frac{210}{2000}s + 1\right]}$$

$$= \frac{10\left(1+\frac{j\omega}{20}\right)}{1+\frac{210(j\omega)}{2000} - \frac{\omega^2}{2000}}$$

$$= .....$$

| Factor | C.F | magnitude |
|---|---|---|
| $K$ | — | $20\log 10 = 20\,dB$ |
| $(j\omega)^{\pm n}$ | — | Nil |
| $1+\frac{j\omega}{20}$ | 20 | $+20$ db/dec |
| $\frac{1}{1-\frac{\omega^2}{2000}+\frac{j\omega(210)}{2000}}$ | 44.5 $\simeq 45$ | $-40$ db/dec |




$$|M|_{at\ \omega=1000} = \frac{\sqrt{(10)^2+\left(\frac{10\omega}{20}\right)^2}}{\sqrt{\left(1-\frac{\omega^2}{2000}\right)^2+\left(\frac{210}{2000}\omega\right)^2}}$$

Now substitute $\omega=1000 \Rightarrow |M|$ we will get...

(or)

from Graphical manner also we can calculate....

Here starting slope $=0 \Rightarrow$ Type 0 system.

for a Type 0 system $K_v, K_a = 0$

*** $K_p$ can be directly obtain from Graph.

$$20\log K_p = 20$$

$$\Rightarrow K_p = \log^{-1}(1) = 10$$

$$\boxed{\therefore K_p = 10}$$

from the basic definition also we can find.

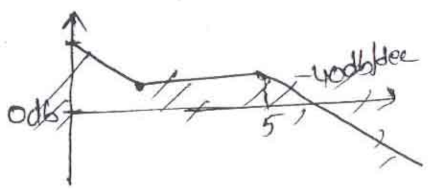
P6) $G(s) = \dfrac{50(s+1)}{s(s+2)(s+5)}$

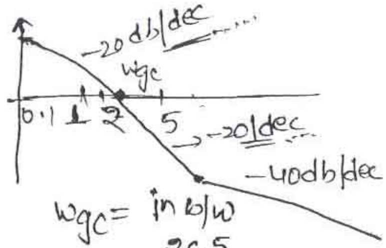
$= \dfrac{50(1+s)}{s \cdot 2(1+0.5s)\,5(1+0.2s)}$

$= \dfrac{5(1+j\omega)}{j\omega(1+0.5j\omega)(1+0.2j\omega)}$

Note:- Here $\dfrac{(s+1)}{(s+2)}$ forms lead compensator.

without compensator



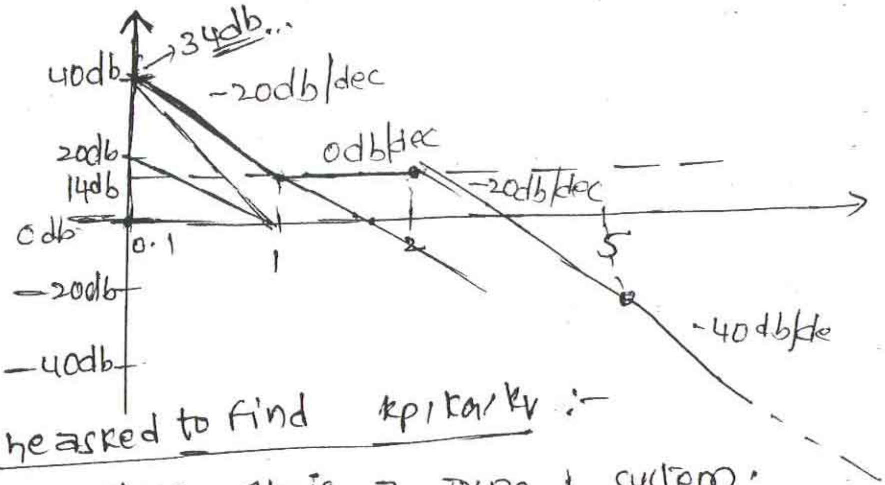




$\omega_{gc}$ = in b/w 2&5

when we will use compensator as shown in fig: wgc is occur in b/w 2&5 only. but wgc in this case is higher...

| factor | C.F | Magnitude |
| --- | --- | --- |
| $K=5$ | — | $20\log 5 = 14$db |
| $1/j\omega$ | — | $-20$db/dec |
| $1+j\omega$ | 1 | $+20$ db/dec |
| $\dfrac{1}{1+0.5j\omega}$ | 2 | $-20$db/dec |
| $\dfrac{1}{1+0.2j\omega}$ | 5 | $-20$ db/dec |

$-20$, 0db, $-20$, $-40$db/dec




$y = -20\log\omega + 14$

$= -20\log(0.1) + 14$

$= 20 + 14$

$= 34$db...

Easy way to calculate 'K' for corresponding gain:-

$y = mx + c$ → $20\log K$

(or) $20\log K - 20\log\omega = |M|$

If he asked to find $k_p, k_a, k_v$ :-

since it is a Type 1 system:

$k_p = \infty$, $k_a = 0$

$\therefore$ $\boxed{K_v = \omega}$ → This can be getting by extending the initial slope line. it cuts the 0db axis at perticular '$\omega$'. That corresponding value is called "$K_v$"...

Note:-

$\dfrac{y_2 - y_1}{x_2 - x_1} = m$

$\dfrac{-y_2 + 14}{\log 1 - \log(0.1)} = -20$db

$y_2 = 14 + 20\log(0.1)$

Q(8) page(72)

Ans : (b)

$w_2 = \sqrt{w_1 w_3}$ → G.M is applicable not Arithemetic mean...

(P5)

$$G(s) = \frac{3(s+1)(s+700)}{s^2(s^2+18s+400)}$$

Sol:

$$G(j\omega) = \frac{5.2\,(1+j\omega)\cdot\left(1+\frac{j\omega}{700}\right)}{(j\omega)^2\left[1-\frac{\omega^2}{400}+\frac{j18\omega}{400}\right]}$$

$$\angle G(j\omega)\Big|_{\omega<20} = \frac{[0^\circ]\,[\tan^{-1}\omega]\,[\tan^{-1}(\omega/700)]}{180\left[\tan^{-1}\left(\frac{18\omega}{400-\omega^2}\right)\right]}$$

$$\Rightarrow -180+\tan^{-1}\omega+\tan^{-1}(\omega/700)-\tan^{-1}\left(\frac{18\omega}{400-\omega^2}\right)$$

$$\angle G(j\omega)\Big|_{\omega>20} = \frac{(0^\circ)\cdot[\tan^{-1}\omega]\cdot\tan^{-1}(\omega/700)}{(180^\circ)\left[180^\circ-\tan^{-1}\left(\frac{18\omega}{400-\omega^2}\right)\right]}$$

$$= -180+\tan^{-1}\omega+\tan^{-1}\frac{\omega}{700} - 180^\circ+\tan^{-1}\left(\frac{18\omega}{400-\omega^2}\right)$$

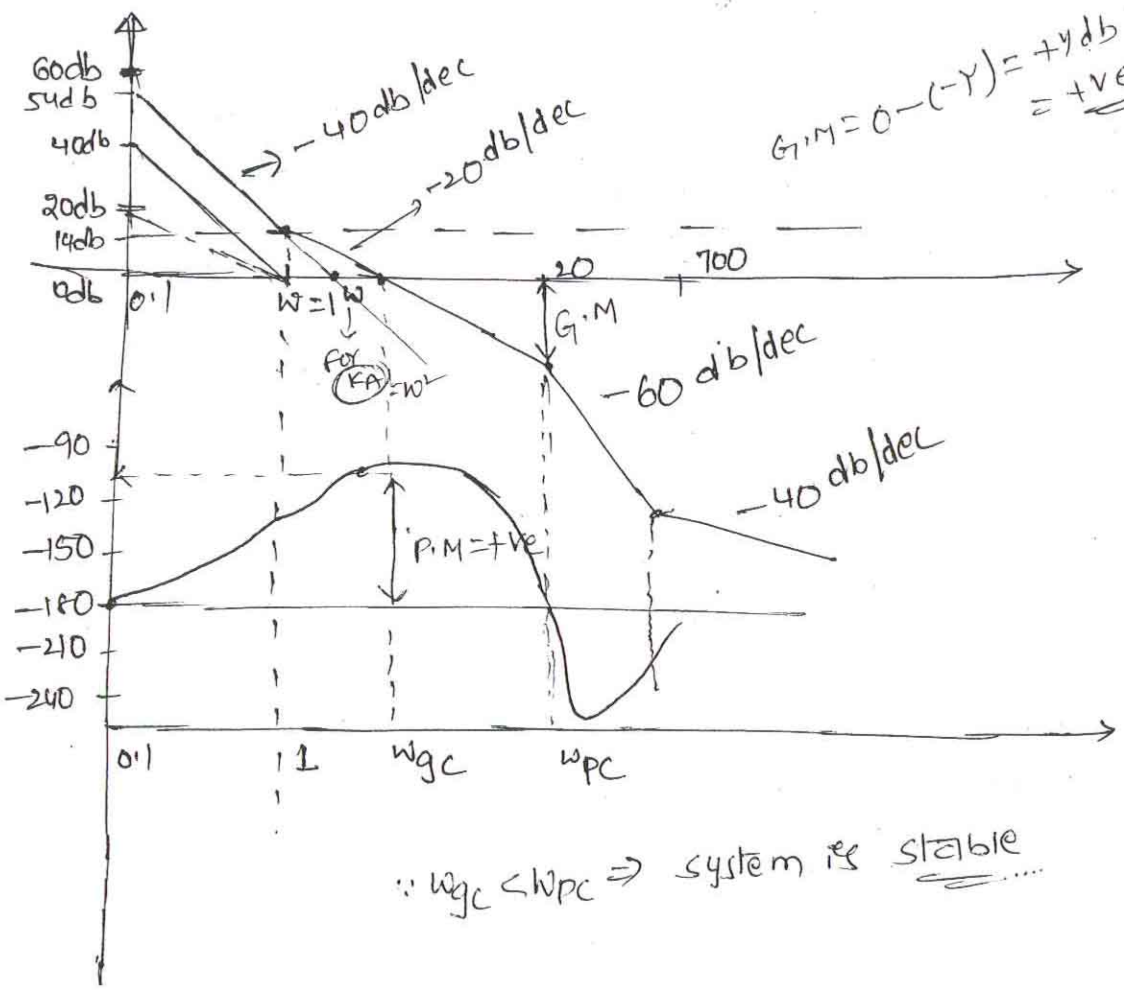
NOW PHASE ANGLE TABLE is given as :-

| $\omega$ | $-180^\circ$ | $+\tan^{-1}\omega$ | $+\tan^{-1}\left(\frac{\omega}{700}\right)$ | $-\tan^{-1}\left(\frac{18\omega}{400-\omega^2}\right)$ | $\phi_R$ |
|---|---|---|---|---|---|
| 0.1 | −180 | 5.7° | 0.008° | −0.25° | −174° |
| 1 | −180 | 45° | 0.08° | −2.5° | −137° |
| 5 | −180 | 78.6° | 0.4° | −13.5° | −114° |
| 20 | −180 | 87° | 1.6° | −90° | −181° |
| 100 | −180 | 89.4° | 8° | −169.4° | −251° |
| 700 | −180 | 89.9° | 45° | −178.5° | −223° |

Magnitude Table :-

| factor | C.F | Magnitude |
| --- | --- | --- |
| 1) $K = 5.2$ | — | $20\log 5.2 = 14db$ |
| 2) $\frac{1}{(j\omega)^2}$ | — | $-40db/dec$ |
| 3) $1 + j\omega$ | 1 | $+20 db/dec$ |
| 4) $\frac{1}{1 - \frac{\omega^2}{400} + \frac{j\omega}{400}}$ | 20 | $-40db/dec$ |
| 5) $1 + j\omega/700$ | 700 | $+20db/dec$ |

-40, -20, -60, -40




$\because \omega_{gc} < \omega_{pc} \Rightarrow$ system is stable

since it is type 2 system,

$K_p = K_v = \infty$

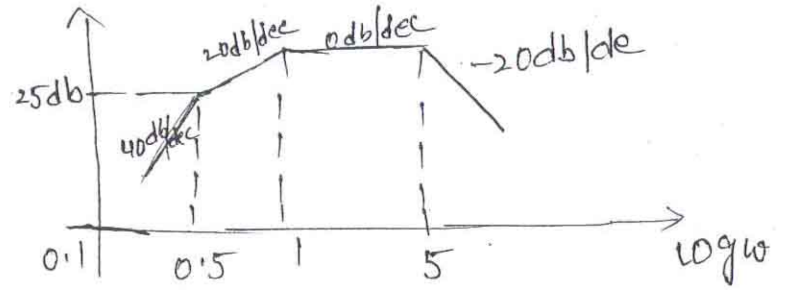
*** $\boxed{K_A = \omega^2}$

$Y = -40\log\omega + 14$

$Y = -40\log(0.1) + 14$

$Y = 54db$

(16) Inverse Bode plots




Sol:-

NOTE:-

1) Observe the starting slope, This will give the information of poles (or) zeros at origin.

2) For the starting slope write the equation $y = mx + C$

$C = 20\log K$

3) At every corner frequency observe the change in slope This will give the information of first order & (OR) Higher order factors...

$$G(s) = \frac{K s^2 \cancel{(1+s)}}{(1+sT_1)(1+sT_2)(1+sT_3)}$$

$w_1 = 0.5;\ w_2 = 1;\ w_3 = 5$

$\boxed{\because w = 1/T} \Rightarrow T_1 = \frac{1}{0.5};\ T_2 = 1;\ T_3 = 1/5$

$20\log K - 20\log(w) = 25\ db$

$20\log K - 20\log(0.5) = 25$

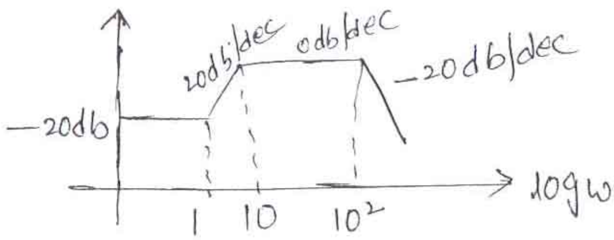
$\Rightarrow K = \ldots\ldots$

$$G(s) = \frac{(70)\, s^2}{\left(1+\frac{s}{0.5}\right)(1+s)(1+s/5)}$$

$$= \frac{70\, s^2 \times 5}{(1+2s)(1+s)(s+5)}$$

$$= \frac{350\, s^2}{(s+1)(s+5)(2s+1)} \ /\ldots$$

(Pb)




Sol:-

Initial slope = 0 db/dec.

so it is type 0 system.

$$G(s) = \frac{K(1+sT_1)}{(1+sT_2)(1+sT_3)}$$

$T_1 = \frac{1}{1}$ ; $T_2 = 1/10$ ; $T_3 = 1/10^2$

$20\log K - 20\log(\omega) = -20db$

$20\log K - 20\log(1) = -20db$

$\cancel{20}\log K = -\cancel{20}$

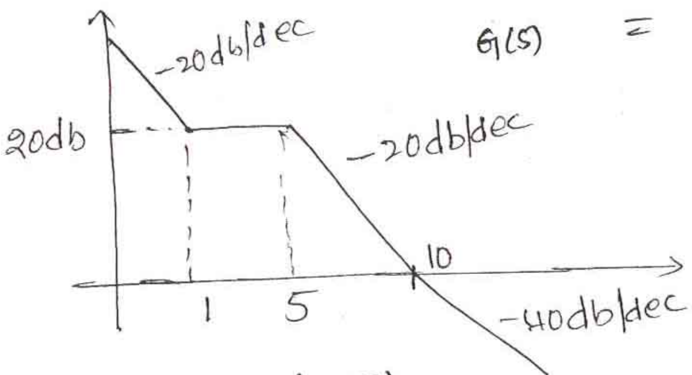
$K = \log^{-1}(-1)$

$K = 0.1$

$$\therefore G(s) = \frac{(0.1)(1+\frac{s}{1})}{(1+\frac{s}{10})(1+\frac{s}{100})} = \frac{(0.1)(1+s)\times 10\times 100}{(s+10)(s+100)}$$

$$G(s) = \frac{100(s+1)}{(s+10)(s+100)}$$ //....

(Pb)




Sol:-

$$G(s) = \frac{K(1+sT_1)}{s(1+sT_2)(1+sT_3)}$$

$20\log K - \overset{(0)}{\cancel{20}}\log\omega = 20$

$20\log K - \overset{(0)}{\cancel{20}}\log(1) = 20$

$20\log K = 20$

$\boxed{K=10}$ //....

$T_1 = 1/1$

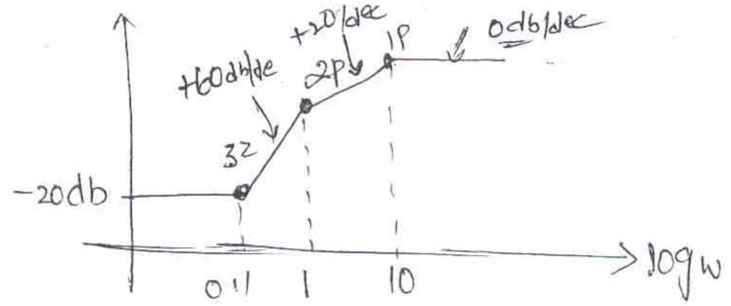
$T_2 = 1/5$

$T_3 = 1/10$

$$\therefore G(s) = \frac{10(1+s/1)}{s(1+s/5)(1+s/10)}$$

$$= \frac{10(1+s)\times 5\times 10}{s(5+s)(10+s)}$$

$$G(s) = \frac{500(s+1)}{s(s+5)(s+10)}$$ ....




Note:-
$20\log K - 20(n)\log w = |M|db$
poles

$20\log K + 20n\log w = |M|$
zeros

Sol:- initial slope = 0

$$G(s) = \frac{K(1+sT_1)^3}{(1+sT_2)^2(1+sT_3)}$$

$$20\log K - \overset{20\times(0)}{\cancel{20}}\log(0.1) = -20 \Rightarrow 20\log K = -20$$

$$\boxed{K = 0.1}$$

$\cancel{20\log K + 20 = -20}$

(or) $\cancel{\log K = -2}$

$\boxed{\cancel{K = \frac{1}{100}}} = \cancel{0.01}$

$y = mx + c$

$-20 = (0) + 20\log K$

$\Rightarrow \boxed{K = 0.1}$

$T_1 = 10$

$T_2 = 1$

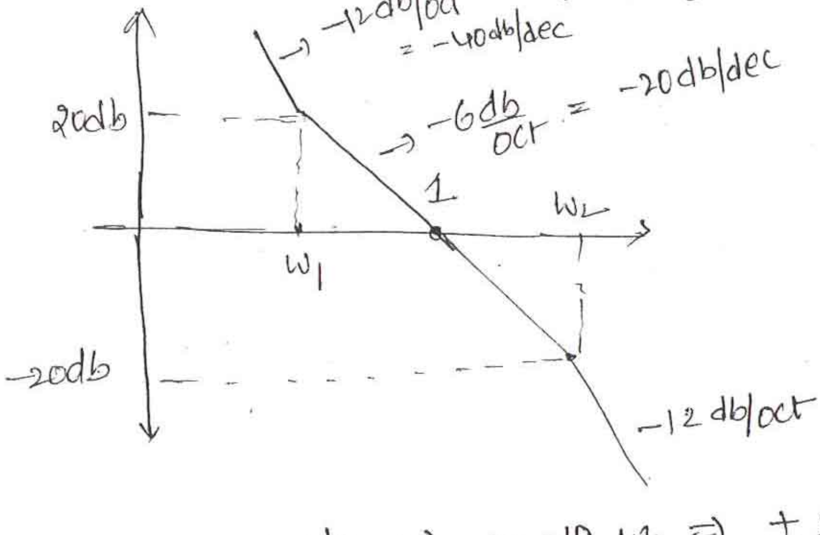
$T_3 = 0.1$

$$G(s) = \frac{0.1\left(1 + \frac{s(10)}{\cancel{10}}\right)^3}{(1+s)^2\left(1+\frac{s}{10}\right)^1}$$

$$= \frac{\cancel{(10)}\ (\cancel{1+10s})^3}{\cancel{10}(s+1)^2(s+10)}$$

$$= \frac{(10s+1)^3}{(s+1)^2(10+s)}$$

(Pb)




Decade $\Rightarrow$ $w_2 = 10\,w_1 \Rightarrow \pm 20\times n\log 10 \Rightarrow$ slope = $\boxed{\pm 20n\ db/dec}$

Octave $\Rightarrow$ $w_2 = 2w_1 \Rightarrow \pm 20(n)\log 2 \Rightarrow$ slope = $\boxed{\pm 6n\ db/dec}$

for $2^{nd}$ line : slope = $-6$db/oct

$Y = mx + c$ (or)

$20 = (-20)\log w_1 + 0$

$\Rightarrow w_1 = 0.1$ rad/sec

$(x_1, y_1) = (w_1, 20)$
$(x_2, y_2) = (1, 0)$

$\therefore -20 = \frac{0-20}{1-w_1}$

$-20 + 20\log w_1 = -20$

$w_1 =$

for $w_2$:-

At $w = w_2$

$20 = -2\log w_2 + 0$

$\Rightarrow w_2 = 10$ rad/sec

To find k:-

At $w = 0.1$

$y = mx + c$

$20 = 40\log(0.1) + c$

$\Rightarrow c = -20$

$\boxed{k = 0.1}$ //...

$\therefore G(s) = \frac{0.1\,(1+10s)}{s^2\,(1+0.1s)}$

## M & N Circles

Nicol charts :-

→ The Nicol chart consists of magnitudes and phase angles of a closed loop system represented as a family of circles known as M & N circles.

→ The Nicol chart gives information about closed loop frequency response of a system.

→ Let the closed loop T.F

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

let $G(s) = x + jy$

$$\frac{C(s)}{R(s)} = \frac{x+jy}{1+(x+jy)}$$

The magnitude [M] : $M = \frac{\sqrt{x^2+y^2}}{\sqrt{(1+x)^2+y^2}}$

$$M^2[(1+x)^2+y^2] = x^2+y^2$$

$$M^2x^2 - x^2 + M^2y^2 + 2xM^2 + M^2 = 0$$

$$x^2[M^2-1] + y^2[M^2-1] + 2xM^2 + M^2 = 0 \quad \to ①$$

In eq① . If $\underline{M=1}$

$2x+1$

$\Rightarrow$ It represents form a straight line passing through $-\frac{1}{2}, 0$.

If $M \neq \Rightarrow$ eq① represents a family of circles.

$$x^2+y^2+2x\frac{M^2}{M^2-1} + \frac{M^2}{M^2-1} = 0$$

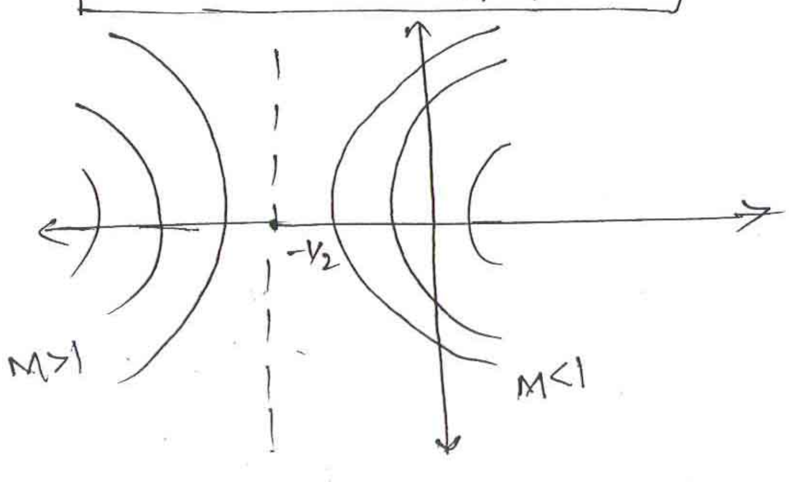
$$x^2+y^2+2x\frac{M^2}{M^2-1} + \frac{M^2}{M^2-1} + \frac{M^2}{(M^2-1)^2} = \frac{M^2}{(M^2-1)^2}$$

$$x^2 + 2\cdot\frac{xM^2}{M^2-1} + \frac{M^4}{(M^2-1)^2} + y^2 = \frac{M^2}{(M^2-1)^2}$$

$$\left[x + \frac{M^2}{M^2-1}\right]^2 + y^2 = \frac{M^2}{M^2-1}$$

centre $= \dfrac{-M^2}{M^2-1}, 0$

Radious $= \dfrac{M}{M^2-1}$




for $M<1 \Rightarrow$ circles is in rightside.

for $M>1 \Rightarrow$ " " leftside.

## N – circles :–

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)}$$

let $G(S) = X + jY$

$$\frac{C(S)}{R(S)} = \frac{X+jY}{1+X+jY}$$

let $\alpha$ = phase angle of C.L. system

$N = \tan\alpha$ represents a family of circles.

$$\alpha = \tan^{-1}\frac{Y}{X} - \tan^{-1}\frac{Y}{1+X}$$

$$N = \tan\alpha = \tan\left[\tan^{-1}\frac{Y}{X} - \tan^{-1}\frac{Y}{1+X}\right]$$

$$N = \frac{\frac{Y}{X} - \frac{Y}{1+X}}{1 + \frac{Y^2}{X(1+X)}}$$

$$X^2 + X + Y^2 - \frac{Y}{N} = 0$$

Adding the Team $\frac{1}{4} + \left(\frac{1}{2N}\right)^2$ on bothsides.

$$X^2 + X + \frac{1}{4} + Y^2 - \frac{Y}{N} + \left(\frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

$$\left[X + \frac{1}{2}\right]^2 + \left[Y - \frac{1}{2N}\right]^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

centre $= \frac{1}{2}, \frac{1}{2N}$

radious $= \sqrt{\frac{1}{4} + \left(\frac{1}{2N}\right)^2}$

** All the N-circles will intersect the real axis b/w -1 & origin.

Q:5
page 7

$$x^2 + 2.25x + y^2 = -1.125$$

$$x^2 + 2 \times \frac{M^2}{M^2-1} + y^2 + \frac{M^2}{M^2-1} = 0$$

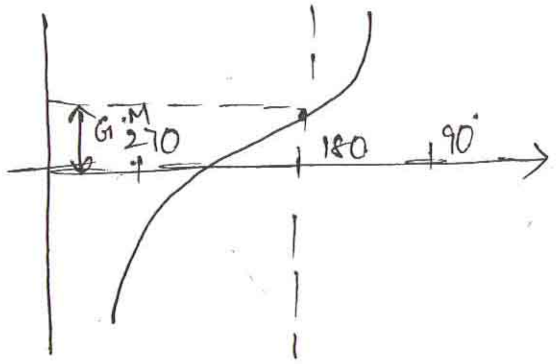
$$\frac{M^2}{M^2-1} = 1.125$$

$$\Rightarrow \boxed{M = 3}$$

Q:10
Ans:- Ans (a)

Q:6 Ans:- (b)
options 2 → since it consists of mag & phase circles.
3 → it doesn't consists of frequ circles.

Q:12




$G.M = 0 - (Y)$
$= -Y\,db$
so $GM = -ve$
so P.M also $-ve$....

System unstable.

Q:24 Ans :- (d)

$\frac{dk}{ds} = 0 \Rightarrow 4s^3 + 24s^2 + 72s + 80 = 0$

$\Rightarrow s = \underline{-2}, \; \underline{-2 \pm j2.45}$

for real values k = ___ → +ve valid B.A.P, as shown in rule no: (B.A.P ⑥).

for complex poles it is more complex to check the value of K & then to say whether the complex pole is valid or not.

**NOTE:-** using Angle condition check the validity of complex B.A.P

$$G(s)H(s)\Big|_{s=-2+j2.45} = \frac{K}{(-2+j2.45)(-2+j2.45+4)(-2+j2.45+2+j4)(-2+j2.45+2-j4)}$$

$$= \frac{K+j0}{(-2+j2.45)(2+j2.45)(0+j6.45)(-j1.5)}$$

$$= \frac{\angle 0}{[130°][50°][90][-90]} = \underline{-180°}$$

Therefore the complex B.A.P is valid B.A.P.

so the easy way to check whether the complex B.A.P is valid or not.

$\therefore$ NO: of $\underline{B.A.P = 3}$ ....

Rule no ⑦ :- Intersection of root locus:

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 36 | K |
| $s^3$ | 8 | 80 | 0 |
| $s^2$ | 26 | K | 0 |
| $s^1$ | $\frac{2080-8K}{26}$ | 0 | 0 |
| $s^0$ | K | | |

$\frac{2080-8K}{26} = 0$

$\Rightarrow K = K_{mar} = 260$

$A(s) = 26s^2 + K = 0$

$\Rightarrow \underline{s = \pm j3.16}$ ....

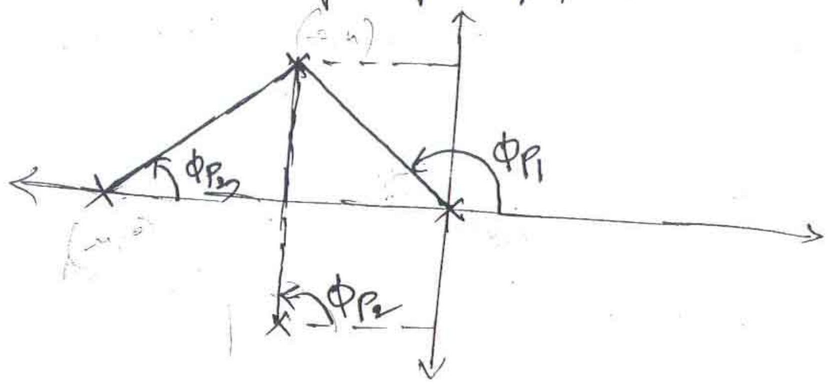
Intersection of Asymptote to the jw axis :-

$\tan(45) = \frac{Y}{2}$

$\Rightarrow Y = 2$

$\therefore \boxed{Y = \pm j2}$ //...

When complex poles are terminating at ∞, then there will be angle of departure.



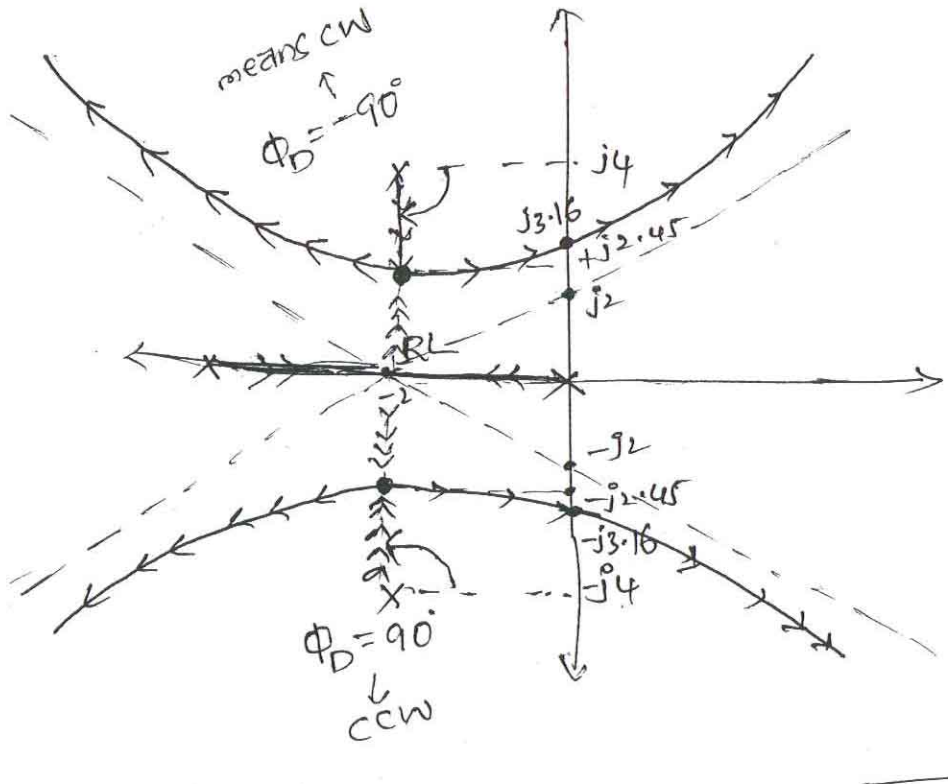

$$\phi_{P_1} = 180 - \tan^{-1}\left(\frac{4-0}{0-(-2)}\right)$$
$$= 180 - 63.4$$
$$= \underline{116.6^\circ}$$

$$\phi_{P_2} = 90$$
$$\phi_{P_3} = \tan^{-1}\left[\frac{4-0}{-2-(-4)}\right]$$
$$= \underline{63.4}$$

$$\therefore \phi = \Sigma\phi_z - \Sigma\phi_p$$
$$= 0^\circ - (116.6^\circ + 90^\circ + 63.4^\circ)$$
$$= \underline{-270^\circ}$$

$$\phi_D = 180 + \phi$$
$$= 180 + (-270) = \underline{-90} = \phi_D$$

Now the complete root locus of the given T.F is,




The condition for stability is $\boxed{0 < K < 260}$.